Instructions:

- Read all instructions carefully.
- Clearly write your name and student number above **BEFORE** you start the exam. Also, have your student ID out and on your desk (you may be asked for an invigilator to see it before/during/after the exam).
- Once the test begins, the instructor and invigilators will not be able to answer questions. You will need to interpret things as best you can and answer accordingly.
- Show all work clearly in order to get full credit. Points can be taken off if it is not clear to see how you arrived at your answer (even if the final answer is correct).
- Calculators can be used for this exam. Use of phones/tablets/computers/smart watches/etc... is not permitted.
- Sketch all relevant graphs and explain all relevant mathematics. Circle/box your final answers.
- Please keep your written answers brief; be clear and to the point.
- Feel free to use scratch paper (some is included at the back, feel free to detach it). You will not be graded upon what is on the scratch paper, though you must turn it in with your exam.
- You are allowed a formula sheet (8.5x11 in) to bring with you. It must be a single hard copy sheet of paper (though you can write on front and back). **You must turn such in with your exam.**
- This test has 3 problems (plus an extra credit problem) and is worth 100 points. It is your responsibility to make sure that you have done all the problems!
- Make sure to turn your test in as requested at the end of the exam period. Failure to do such can lead to a failing grade.
1. (30 points)
For the following questions, circle the appropriate choice for True or False. **No explanation in necessary.**

- **True or False** - Speed and velocity are the same thing.
- **True or False** - Centripetal acceleration \((a)\) is related to the square of the angular velocity.
- **True or False** - Parabolas are related to cubic functions.
- **True or False** - Einstein’s theory of general relativity is an approximation of Newton’s law of gravity.
- **True or False** - If a plane flying at constant velocity drops a package, it will be directly over the package when it hits the ground.
- **True or False** - Velocity is the integral of displacement with respect to time.
- **True or False** - Newton’s 2nd law is \(F = \frac{dp}{dt}\) where \(p\) is momentum.
- **True or False** - If the rate of change of a function is proportional to itself, then the function is exponential.
- **True or False** - For uniform circular motion, the acceleration vectors all point in the same direction.
- **True or False** - A feather and a rock both move towards Earth with the same acceleration.
- **True or False** - A ball dropped versus throw up both move towards Earth with the same acceleration.
- **True or False** - Arc length is \(r\theta^2\), where \(r\) is the radius and \(\theta\) is the angle extended.
- **True or False** - The basic idea underlying rocket thrust is a. reduction of mass and b. conservation of momentum.
- **True or False** - Stiction refers to the friction of sliding bodies.
- **True or False** - Kinetic friction tends to be greater than static friction.
- **True or False** - A falling cylinder would always feel a smaller drag force than a sphere.
- **True or False** - A cylinder falling “end down” and another falling “side down” will have the same drag coefficients.
- **True or False** - Work is the energy transferred between systems via a continuous force.
- **True or False** - Conservation on energy indicates that energy does not change.
- **True or False** - A change in energy is always negative.
- **True or False** - For a thrown ball, it is possible to have the velocity and acceleration vectors to point in the same direction at the instant the ball is thrown.
• True or False – Uniform motion can effectively be described as “unchanging change”.
• True or False – The product of two vectors is always a scalar.
• True or False – Work has the same units as energy.
• True or False – When compressing a spring, the work done on the spring is positive.
• True or False – Integrals can arise from Riemann sums when the rectangles get infinitesimally large.
• True or False – A falling object experiencing drag will asymptotically approach terminal velocity.
• True or False – The First Law of Thermodynamics embodies the distinction between “good” versus “bad” energy.
• True or False – Rolling friction tends to be larger than kinetic or static friction, thus giving tires their “grip”.
• True or False – Drag forces can only be proportional to velocity or its square.
2. (35 points)
A 1.0 kg physics book is connected by a string to a 500 g coffee cup (see figure below). The book is given a push up the slope and released with a speed \( v = 3.0 \text{ m/s} \). The coefficients of friction are \( \mu_s = 0.50 \) and \( \mu_k = 0.20 \).

a. Draw the appropriate free-body diagram.

b. Determine how far does the book slides.

Known:
- \( m_c = 0.5 \text{ kg} \)
- \( m_B = 1.0 \text{ kg} \)
- \( v_o = 3.0 \text{ m/s} \)
- \( \mu_s = 0.50 \)
- \( \mu_k = 0.20 \)

\( \square \) Book will slide upwards w/ some const. acceleration (call it \( a_1 \)). Assume book is initially at \( x'_0 \) and comes to rest at \( x'_1 \). Then

\[
V_{fx'}^2 = V_{ox'}^2 + 2a_1 (x'_1 - x'_0) = 0 = 3.0^2 + 2a_1 (x'_1)
\]

\[ \rightarrow \quad a_1 x'_1 = -4.5 \text{ m}^2/\text{s}^2 \] (note: We are trying to determine \( x'_1 \))

\( \square \) We can determine \( a_1 \) by considering all forces acting on the book:

\[
\sum F_x' = N_B - W_B \cos (20) = 0 \quad \Rightarrow \quad N_B = (1.0)(9.8) \cos (20) = 9.2 \text{ N}
\]
\[(\Sigma F_B)_{x'} = T + f_k + W_B \sin(\theta) = -m_B q_1 \]  
(Note: \(f_k = k_N N_B\))

d Also consider the forces acting on the cup: 
negative since book slows along \(y'\)

\[(\Sigma F_c)_{y} = [T - W_c = m_c q_1]\] (since cup and book must have same acceleration!)

\(\Rightarrow q_1 \quad (m_c + m_B) = -g [m_c + m_B \sin(\theta)] - k_N N_B\)

\(\Rightarrow q_1 (1.0 + 0.5) = -(9.8) [0.5 + 1.0 \cdot \sin(\theta)] - (0.20)(9.21) \quad \Rightarrow \quad q_1 = -6.7 \text{ m/s}^2\)

d Back to earlier: 
\[X_1' = \frac{-4.5}{q_1} = \frac{-4.5}{-6.7} \approx 0.7 \text{ m} \quad \Rightarrow \quad X_1' \approx 0.7 \text{ m}\)

**c. At the highest point, does the book stick to the slope, or does it slide back down?**

Max Value of static friction is 
\[(f_s)_{\text{max}} = \mu_s N_B = (0.50)(9.21) = 4.6 \text{ N}\]

Now we consider whether the force \((f_s)\) needed to keep the book in place is larger or smaller than \((f_s)_{\text{max}}\):

- When cup is at rest, \(T = W_c = m_c g\)
- Then the total forces acting along \(x'\) for the book are:

\[(\Sigma F_B)_{x'} = f_s - T - W_B \sin(\theta) = f_s - m_c g - m_B \sin(\theta) = 0\]

\(\Rightarrow f_s = (m_c + m_B \sin \theta) g \approx 8.3 \text{ N}\)

\(\Rightarrow \text{Since } f_s > (f_s)_{\text{max}}, \text{ the book will slide down}\)
3. (35 points)
A projectile is fired with an initial speed of 30 m/s at an angle of 60° above the horizontal. The object hits the ground 7.5 s later.

(a) Sketch the relevant diagram, indicating the main quantities of interest.

(b) How much higher or lower is the launch point relative to the point where the projectile hits the ground?

- Note: $y_f$ could be positive or negative (that is what we need to find out!)

- Known: $v_0 = 30 \text{ m/s}$, $a_x = 0$, $a_y = -9.8 \text{ m/s}^2$, $t_0 = 0$, $x_0 = 0$, $y_0 = 0$, $t_f = 7.5 \text{ s}$

- Then $y_f = y_0 + v_{0y}(t_f - t_0) + \frac{1}{2} a_y (t_f - t_0)^2$

  $= 0 + 30 \cdot \sin(60°)(7.5 - 0) + \frac{1}{2}(-9.8)(7.5 - 0)^2 \approx -30.8 \text{ m}$

  $\Rightarrow$ Thus launch point is 381 m above landing site.

(c) To what maximum height above the launch point does the projectile rise?

- Similar to before, except that at peak [i.e. $(x_p, y_p)$], $v_{py} = 0$

  $v_{py}^2 = v_{0y}^2 + 2a_y (y_p - y_0) = 0 = [30 \cdot \sin(60°)]^2 + 2(-9.8)(y_p - 0)$

  $\Rightarrow y_p \approx 34 \text{ m}$
Extra Credit (10 Points):

You are a member of the CSL-T (Crime Scene Investigation – Toronto) team. You get a call that a body has been found in an apartment. You head immediately over to the crime scene and arrive at 3:02 AM. As per your training, one of the first things you do is measure the body temperature and find it to be 69.4°F. You notice the apartment has had its air conditioner (AC) blasting, and that it appears it has been on for a long time (presumably since before the murder). You measure the ambient temperature and find it to be 65.0°F. You then proceed to look for other clues. Fifty nine minutes after you arrived (and measured the body temperature), you measure the body temperature again and find it to be 67.68°F (ambient is still at 65.0°F). You announce to your partner that you have deduced the approximate time of death.

[Hint: Normal (healthy) body temperature is 98.6°F]

a. Briefly explain how you could estimate the time of death from the information provided above.

- Newton’s Law of Cooling: \[ \frac{dT}{dt} = \alpha (T_0 - T) \rightarrow T(t) = T_0 + C e^{-\alpha t} \]
- We should have enough pieces to deduce how much time has passed since death (Note: \( \alpha > 0 \))

b. What was the approximate time of death?

- Let \( t = 0 \) be the time of death and measures were made at \( t = t_D \) and \( t_D + 59 \) min.
- \( T(0) = 98.6 = T_0 + C e^{-\alpha \cdot 0} = T_0 + C = 65.0 + C \rightarrow C = 33.6°F \)
- \( T(\infty) = T_0 + C e^{-\infty} = T_0 \rightarrow T_0 = 65.0°C \) (ambient temp)
- \( T(t_D) = 69.4 = 65 + 33.6 \cdot e^{-\alpha t_D} \)
- \( \Rightarrow \alpha t_D = -\ln\left(\frac{4.4}{33.6}\right) = 2.03 \)
- \( T(t_D + 59) = 67.68 = 65 + 33.6 \cdot e^{-\alpha (t_D + 59)} \)
- \( \Rightarrow \alpha (t_D + 59) = -\ln\left(\frac{2.68}{33.6}\right) = 2.53 \)
- Two equations, two unknowns: \( \Rightarrow \alpha = \frac{2.03}{t_D} \)
- \( \alpha (t_D + 59) = 2.53 \rightarrow t_D = 239 \) min = 3 hrs. 59 min
- Body found @ 3:02 AM means death occurred @ 11:03 PM