York University
IPHYS 2010 HW2 (Winter 2020): Rocket Science
Due Jan.27, 2020

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1 Overview

This assignment focuses around the theme of *rocket science*. That is, the basic concepts relevant to sending a rocket to, oh say, Mars. Questions will deal with basic kinematics, conservation of momentum as the basic principle of rocket thrust, deriving the "rocket equation", and aspects of gaseous ignition inside a rocket engine. Also, there will be computational components that will require you to write some basic computer code to produce several plots. You are free to use any language you prefer (though solutions will be given in Matlab) and further, you are encouraged to work together in groups.

1. (6 pts) An important consideration in launching a rocket is *escaping* the gravitational potential energy of earth.

   (a) Briefly explain how gravitational potential energy *escape speed* (v_{esc}) are related.

   (b) Consider a rocket that is launched vertically upward at $\sqrt{2}v_{esc}$ from planet Earth, whose mass is M_E and approximate radius is R_E. Derive an expression for its speed as a function of the distance from the planet [i.e. $v(r)$]. [*Hint: Remember conservation of energy!.*]

   (c) Find numerical values for M_E and R_E, and write a script to plot $v(r)$. Make sure to clearly comment your code, and that your plot is clearly labeled.

   (d) Like the above problem, a popular first-year physics text (Wolfson) states "*A rocket is launched vertically upward at 3.1 km/s*". Briefly explain conceptually what is wrong with the basis of these statements.

2. (8 pts) Estimate the energy required to send a rocket of mass M_o to Mars. Clearly state assumptions made and outline all the relevant calculations in detail.

   [*Hint: This is a "Fermi-type" problem. There are many paths one could take and possible considerations to make, such as gravitational potential energy tied to Earth/Mars/Moon/Sun, engine efficiency, etc.*]

3. (4 pts) Rockets move about via *thrust*. That is, say in outer space, there is nothing to "push against" so to induce an external force and thereby accelerate something else is required. So the relevant forces have to come from "inside" the system (and hence thrust). A fundamental principle underlying thrust is the *conservation of momentum* (COM).

   (a) State the law of COM. Identify several different interdisciplinary scenarios where COM plays an important role (e.g., kinesiology, car crashes, etc.).

(b) Explain conceptually why COM is relevant for rockets. For example, the momentum of what? Does a rocket's mass change with time? How is such related back to COM? Feel free to use equations as needed.

(c) Examine Figs.1a and 1b. Briefly outline what parallels there are between the two different scenarios shown there. Additionally, what differences are there?

4. (12 pts) Consider a rocket in deep space (like that in Fig.1b), where there are negligible external forces such as gravity or drag. The rocket, with mass M, burns fuel at a constant rate μ until all the fuel is used up (call that the *burnout time*, or t_B). Assume that the velocity $v_{\rm rel}$ of the ejected gas particles relative to the rocket is constant.

   (a) Draw a free-body diagram and set up the equation of motion via Newton's 2nd law). Specifically, you should find that

   $$M\frac{d\mathbf{v}}{dt} = \mathbf{F}_T \tag{1}$$

   where $\mathbf{F}_T$ is the thrust force. How does $\mathbf{F}_T$ depend upon the prescribed variables?

   (b) Determine the change of the velocity [i.e., $v_{tB}-v_0$] of the rocket during the time interval $t \in [0, t_B]$ as a function of the change of the rocket's mass during that interval. Note that your answer does not have an explicit time dependence and velocity depends upon the *mass ratio* (a useful measure of time into the burn of a rocket).

   A common approach to solving this is to use integrals, but you should try to solve it without integrals. Use only your knowledge of derivatives and solutions to the (differential) equation of the form $x' = a + bx$ (which you saw earlier in the semester).

   [*Hint: Your answer should be the so-called* **rocket equation**.]*

   (c) Comment briefly on how time factors into your answer to the last part. How does the thrust change as a function of time?

   (d) Rearrange your answer to the last part to express the rocket's mass as a function of its velocity.

   (e) A rocket, weighing 30000 lbs. before liftoff, is fired vertically upward. At burnout, it weighs 10000 lbs. Gases are exhausted at a rate of 10 slugs/s, at a velocity of 5000 ft/s (relative to the rocket). Assume both those two quantities are constant. What is the thrust force? Additionally, what is the rocket's speed and kinetic energy at t_B?

   (f) At what mass ratio is the kinetic energy (T) of the rocket, including fuel, maximal? Calculate the velocity, mass, kinetic energy of the rocket, and the time at which that occurs [assuming $v(0) = 0$].

   (g) In terms of mass ratios, is a rocket like a car? Why or why not?

   (h) Now consider the rocket in the *constant* gravity field near Earth, where the goal is to get off the surface. Draw the relevant free-body diagram. Set up the equation of motion (i.e., what modifications to Eqn.1 need to be made?). Which condition has to be satisfied to allow the rocket to take off?

*Kudos if you didn't Google this! But since you likely did, who was Konstantin Tsiolkovsky?

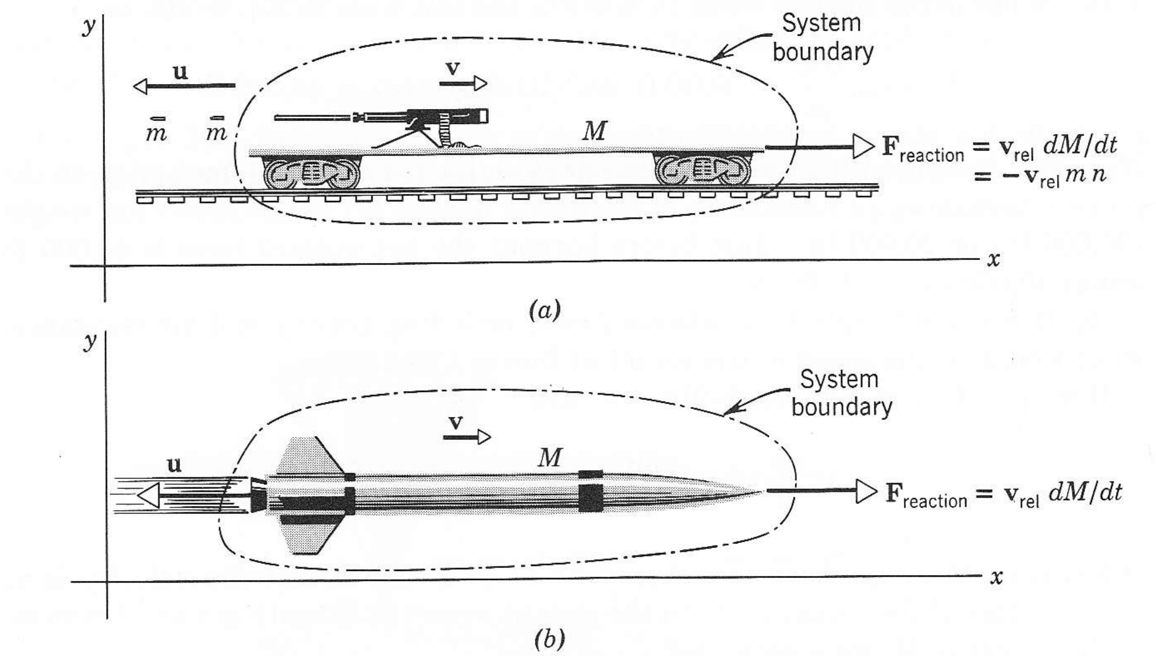

Fig. 9–11 (*a*) Example 8. A machine gun is fixed to a car that rolls with negligible friction. The gun fires bullets of mass m at a rate (number per unit time) n, the velocity of the bullets with respect to the gun being $\mathbf{u} - \mathbf{v}$. At the instant shown some bullets have already left the system. The velocities indicated for the car and the bullets are those that would be measured by an observer in a reference frame fixed to the rails as shown. The reaction force on the system is $\mathbf{F} = -mn\mathbf{v}_{\text{rel}} = (dM/dt)\mathbf{v}_{\text{rel}}$. (*b*) A rocket moves through space with negligible external forces. Gas particles are ejected from the exhaust, the particles having a velocity $\mathbf{u} - \mathbf{v}$ with respect to the rocket. The rate at which mass is expelled at the exhaust is $-dM/dt$. The reaction force on the rocket is $\mathbf{F} = (dM/dt)\mathbf{v}_{\text{rel}}$. The velocities indicated for the rocket and exhaust gases are relative to the ground.

Figure 1: From Resnick and Halliday (1966).

[*Hint: Ch.7.8 of Hawkes (Physics for Scientists and Engineers: An Interactive Approach) might be helpful! Try heading over to Steacie!*]

5. (6 pts) This question builds off your derivation of the rocket equation in the last part and asks you to computationally explore several aspects via a code. Along with your answers, you should turn in any relevant code. Make sure it is concise and commented (so to make clear what is what).

   (a) Write a code that plots the velocity of the rocket as a function of the mass ratio for several different exhaust velocities (v_{rel}).

   (b) Pick a set of parameters (e.g., v_{rel}) and determine at what point the rocket's velocity exceeds that of the exhaust velocity. Comment (e.g., How does such depend upon v_{rel}? Do the relevant values seem *special* in some way?). Additionally, how would such appear to an observer on the ground?

   [*Hint: A book entitled* **Rocket and Spacecraft Propulsion** *by Martin J.L. Turner might be helpful. You should be able to find a soft copy via York's subscription to SpringerLink (if you have trouble, ask a librarian in Steacie!).*]