PHYS 2010 (W20)
Classical Mechanics

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Tutorial V

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Ref. (re images):
16. One-to-Four

Which pattern does not belong?
A load of mass $m$ lies on a perfectly smooth plane, being pulled in opposite directions by springs 1 and 2, whose coefficients of elasticity are $k_1$ and $k_2$ respectively (Fig. 60). If the load be forced out of its state of equilibrium (by being drawn aside), it will begin to oscillate with period $T$. Will the period of oscillation be altered if the same springs be fastened not at points $A_1$ and $A_2$, but at $B_1$ and $B_2$? Assume that the springs are subject to Hooke's law for all strains.
Problem

(a) If \( z = Ae^{j\theta} \), deduce that \( dz = jz \, d\theta \), and explain the meaning of this relation in a vector diagram.

(b) Find the magnitudes and directions of the vectors \( 2 + j\sqrt{3} \) and \( (2 - j\sqrt{3})^2 \).
Problem

Find the equivalent electrical circuit for the hanging mass–spring shown in Figure 3-17a and determine the time dependence of the charge $q$ in the system.

![Diagram of a hanging mass-spring system and an equivalent electrical circuit.]

**FIGURE 3-17** Example 3.4 (a) hanging mass–spring system; (b) equivalent electrical circuit.
RLC circuit = Damped, Driven Harmonic Oscillator

<table>
<thead>
<tr>
<th>Mechanical</th>
<th>Electrical</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$ (force)</td>
<td>$V$ (potential)</td>
</tr>
<tr>
<td>$v$ (velocity)</td>
<td>$I$ (current)</td>
</tr>
<tr>
<td>$x$ (position)</td>
<td>$q$ (charge)</td>
</tr>
<tr>
<td>$m$ (mass)</td>
<td>$L$ (inductance)</td>
</tr>
<tr>
<td>$b$ (damping)</td>
<td>$R$ (resistance)</td>
</tr>
<tr>
<td>$k$ (spring)</td>
<td>$1/C$ (capacitance)</td>
</tr>
</tbody>
</table>
Consider the series RLC circuit shown in Figure 3-18 driven by an alternating emf of value $E_0 \sin \omega t$. Find the current, the voltage $V_L$ across the inductor, and the angular frequency $\omega$ at which $V_L$ is a maximum.

**FIGURE 3-18** Example 3.5. RLC circuit with an alternating emf.
A simple harmonic oscillator consists of a 100-g mass attached to a spring whose force constant is $10^4$ dyne/cm. The mass is displaced 3 cm and released from rest. Calculate (a) the natural frequency $\nu_0$ and the period $\tau_0$, (b) the total energy, and (c) the maximum speed.
Problem

Allow the motion in the preceding problem to take place in a resisting medium. After oscillating for 10 s, the maximum amplitude decreases to half the initial value. Calculate (a) the damping parameter $\beta$, (b) the frequency $\nu_1$ (compare with the undamped frequency $\nu_0$), and (c) the decrement of the motion.
Consider a simple harmonic oscillator. Calculate the *time* averages of the kinetic and potential energies over one cycle, and show that these quantities are equal. Why is this a reasonable result? Next calculate the *space* averages of the kinetic and potential energies. Discuss the results.

*Hint: Recall that*

\[ f_{avg} = \frac{1}{b-a} \int_a^b f(x) \, dx \]