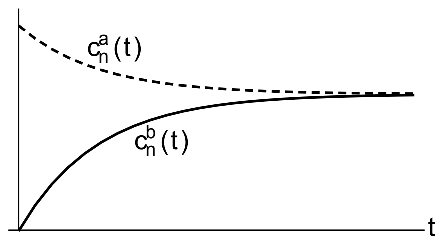
York University BPHS 4080 (Winter 2020) - HW 2

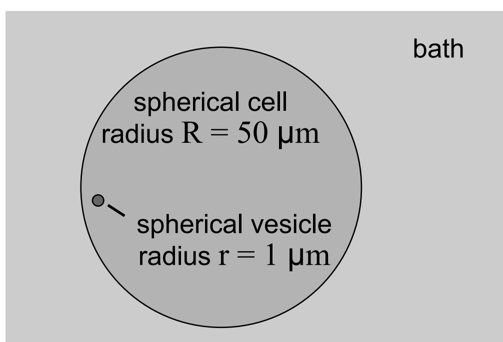
Due Date: Jan. 27, 2020 1:30 PM

Questions

1. A solute n diffuses through a membrane that separates two compartments that have different initial concentrations. The concentrations in the two compartments as a function of time, $c_n^a(t)$ and $c_n^b(t)$, are shown in the figure below. The volumes of the two compartments are V_a and V_b. Is $V_a > V_b$ or is $V_a < V_b$? Explain.



2. All cells are surrounded by a cell membrane. The cytoplasm of most cells contains a variety of organelles that are also enclosed within membranes. Assume that a spherical cell with radius $R = 50\mu m$ contains a spherical organelle called a vesicle, with radius $r = 1\mu m$, as shown in the following figure.

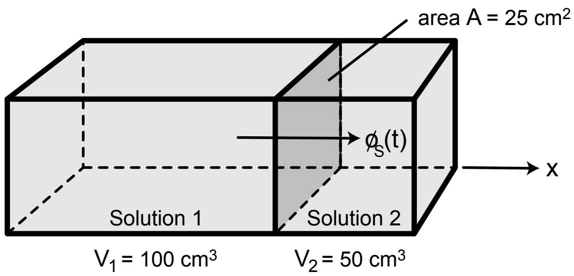


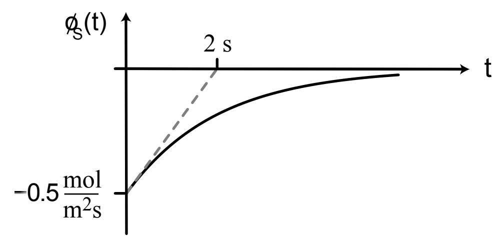
Assume that the membranes surrounding the cell and vesicle are uniform lipid bilayers with identical compositions and the same thickness d = 10 nm. Assume that solute X is transported across both the cell and vesicle membrane via the dissolve and diffuse mechanism. Assume that X dissolves equally well in the bath and in the aqueous interiors of the vesicle and cell. Assume that the solute X dissolves 100 times less readily in the membrane (i.e. the partitioning coefficient is 0.01). Assume the diffusivity of X in the membranes in $10^{-7} cm^2/s$. Initially, the concentration of X is zero inside the cell and inside the vesicle. At time $t = 0$, the cell is plunged into a bath that contains X with concentration 1 mmol/L.

a. Estimate the time that is required for the concentration of X in the cell to reach 0.5 mmol/L. Find a numerical value or explain why it is not possible to obtain a numerical value with the information that is given.

b. Estimate the time that is required for the concentration of X in the vesicle to reach 0.5 mmol/L. Find a numerical value or explain why it is not possible to obtain a numerical value with the information that is given.

3. Two solutions of an uncharged solute S have volumes $V_1 = 100cm^3$ and $V_2 = 50cm^3$ and are separated by a thin membrane (area $A = 25cm^2$) permeant to S and impermeant to water.



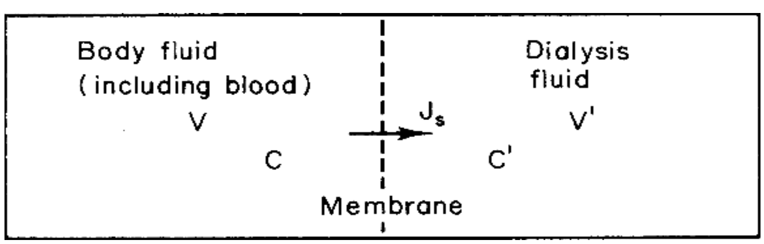


The flux of S through the membrane obeys Fick's law for membranes. At time $t = 0$, the concentration of S in solution 1 is $c_1(0) = 100mol/m^3$. The initial concentration of S in solution 2 is not known. The flux of S through the membrane in the positive x direction is found to be an exponential function of time as shown in the plot.

a. Determine the concentration $c_1(t)$ of S in solution 1 and the concentration $c_2(t)$ of S in solution 2 as functions of time, assuming that the solutions are well-stirred. Sketch $c_1(t)$ and $c_2(t)$ on suitably labeled axes.

b. Determine numerical values for the final concentrations of S: $c_1(\infty)$ and $c_2(\infty)$. If it is not possible to determine numerical values, list the other information that would be needed to determine a numerical value.

4. Dialysis is commonly used in hospitals to remove urea from a patients plasma when their kidneys begin to fail. It is known that along with being found in the plasma, urea can also be found in the interstitial brain fluid and the cerebrospinal fluid in the same concentration as the plasma.



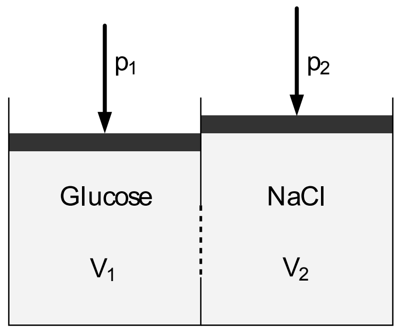
If the solute transport by diffusion is given as $J_s = \omega RT(C - C')$, where C denotes the concentration of the solute in the blood, C' denotes the concentration in the dialysis fluid, ω denotes the Solute Permeability (in $mol \cdot N^{-1} \cdot s^{-1}$), R denotes the Gas Constant and T denotes the temperature of the system (in Kelvins):

a. Determine the rate of change of the number of solute molecules N and the rate of change of the concentration C of the solute in the blood (assuming that the surface area of the membrane can be denoted as S)

b. Determine the solution to the differential equation for the rate of change of the solute concentration C; what is the time constant?

c. During dialysis conducted at room temperature, typical cellophane wrappers are used to introduce the dialysis fluid. If 40 liters of dialysis solution is used, and the wrapper has an area of $3m^2$ and a solute permeability of $2.45 \cdot 10^{-9} mol \cdot N^{-1} \cdot s^{-1}$, how long would the dialysis session run?

5. The following figure shows an experimental apparatus for testing a semipermeable membrane.



Volumes V_1 and V_2 contain well-stirred aqueous solutions of glucose and NaCl, respectively. These volumes are separated by a membrane that is permeable only to water. Assume that the pistons are ideal (i.e. they are frictionless, and faithfully transmit the pressure P_1 and P_2 to V_1 and V_2 respectively). Also assume that effects of gravity are negligible.

a. When the system is in equilibrium, what will be the relation between the hydraulic pressure (P_1 and P_2 and solute concentrations in each compartment?

b. Volumes V_1 and V_2 are initially equal, with $V_1 = V_2 = 1L$. At time $t = 0$, the concentration of glucose in compartment 1 is 0.01 mol/L and the concentration of NaCL in compartment 2 is 0.01 mol/L. If $P_1 = P_2$, what is the final volume of compartment 2? Sketch V_2 as a function of time for $t \geq 0$.

6. The water content of plants is very high (up to 90% by weight) but this water is in flux; water is absorbed through the roots, rises as sap, and evaporates from the leaves. The total water content of a plant can be replaced many times per hour. The mechanims that determine water flow include: gravity, osmosis, and capillarity. In this problem we will consider the effet of gravity and osmosis. Assume that water flow is steady and is due to the gravitational, osmotic, and *other* forces. Let the pressure due to gravity be p_g; and the osmotic pressure be π. Let the pressure due to other sources in the trees be p_o (e.g., capillary forces); assume these sources are not appreciable in the medium that is in contact with the roots. Consider two trees:

1. The General Sherman giant sequoia (*Sequoia gigantea*) which is located in Sequoia National Park in California stands 272 feet high and has a diameter of 36.5 feet at its base; you can drive your car through a tunnel that has been cut through the base. The tree was estimated to be 3,800 years old.

2. The Red mangrove (*Rhizophora mangle*) grows in the tropics to a height of as much as 80 feet. It is found in tidial creeks and estuaries and it can grow with its roots in seawater. Yet the sap has the composition of fresh water.

The compositions of the relevant media are given below

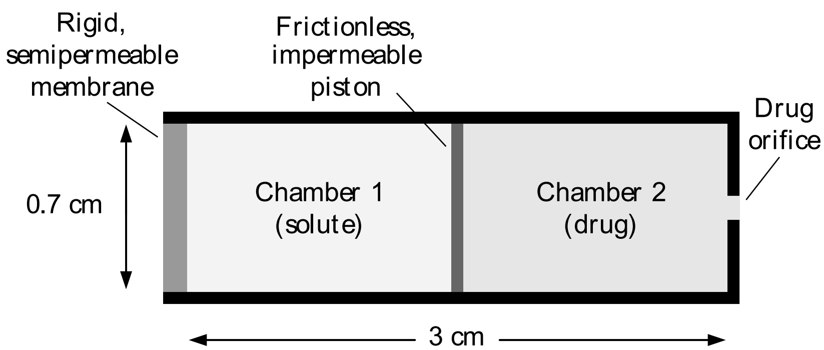
Ion	Concentration mmol/L			
	Giant Sequoia		Red Mangrove	
	soil	sap	seawater	sap
K^+	1	20	10	20
Na^+	2	0	450	0
Cl^-	30	2	530	2
NO_3^-	3	20	0	20
Ca^{2+}	7	1	10	1
Mg^{2+}	8	0	50	0

a. Draw a figure for both the General Sherman and the Red mangrove; ensure to denote all forces working in each system which aid/suppress the sap in rising within the trees, and to clearly state all assumptions that have been made.

b. Find a numerical bound on p_o (expressed in atmospheres) such that the sap will rise in each tree.

c. Which tree requires the larger value of p_o? Explain.

7. The following figure shows the design of a miniature pump that can be implanted in the body to deliver a drug. No batteries are required to run this pump!



The pump contains two cylindrical chambers filled with incompressible fluids: the two chambers together have a length of 3 cm and a diameter of 0.7 cm. Chamber 1 is filled with a solution whose concentration is 10 mol/L; the osmolarity of this solution greatly exceeds that of body fluids. Chamber 2 is filled with the drug solution. The two chambers are separated by a frictionless, massless, and impermeable piston. The piston moves freely and supports no difference in hydraulic chambers. The pump walls are rigid, impermeable and cylindrical with an orifice at one end for delivering the drug and a rigid, semipermeable membrane at the other end. The orifice diameter is sufficiently large that the hydraulic pressure drop across this orifice is negligible and sufficiently small so that the diffusion of drug though the orifice is also negligible. The semipermeable membrane is permeable to water onlly, and not permeable to the solute. Assume the T = 300 K.

a. Provide a discussion of 50 words or fewer for each of the following:

i. What is the physical mechanism of drug delivery implied by the pump design?

ii. What is (are) the source(s) of energy for pumping the drug?

iii. Assume there is an adequate supply of drug in the pump for the lifetime of the implanted subject and this it is necessary to provide a constant rate of drug delivery. Which fundamental factors limit the useful lifetime of this pump in the body?

b. When implanted in the body, the pump delivers the drug at a rate of 1 μL/h. Find the value of hydraulic conductivity $\mathcal{L}_V$, of the semipermeable membrane.

8. This problem will require the use of SoftCell, software used in MATLAB. To access the remote version of MATLAB provided by York University, go to `computing.yorku.ca/students/computer-labs/connecting-from-home-webfas/` and follow the instruction specific to your computer type. Next you need to download from the course website the SoftCell software and unzip it into an accessible directory. Next launch MATLAB using webfas and change the directory to the one containing SoftCell by clicking on the ellipsis next to current folder. In the command window now input "softcell" which should launch the SoftCell graphcial user interface. We are interested in the random walk module.

a. Now that the random walk simulator has been launched, go ahead and run a simulation using 100 steps and 50 particles. The number of steps can be changed by via the "Control" panel and the number of particles can be changed by clicking on the parameters button in the "Particle Set #1" panel (you can ignore the columns regarding region 2 and 3, ensure the initial distribution is set to pulsatile, the location at 200 and ensure that you click the update button to initialize your changes). Start the simulation, and after its completion, click the "Hist" button to observe both the expected and actual distribution of the random walk. What shape does the expected distribution appear to be? Does the actual distribution match the expected (make sure to run several simulations to get a better sample size)? Click on the "Graphs" button and view the graph for the mean particle location. Describe and explain the expected mean.

b. Next bias the random walk so that one direction is prefered over another. This can be done by varying the "Right Step Prob" in the "Parameters" window. Initially, what distribution do you predict will arise? What distribution arises from the simulation? Can you think of any systems where biased random walks are relevant? Make sure you include relevant information in answering this question (i.e., particle number, number of steps, biasing probabilities)