York University BPHS 3090 (Winter 2014) - HW 5

Due Date: Mar. 13, 2020 1:30 PM

Questions

1. Assume that an action potential is traveling at constant velocity, v, in the positive z-direction along an axon. Assume that the core conductor model is valid so that

$$\frac{\partial^2 V_m(z,t)}{\partial z^2} = (r_i + r_0)\, K_m(z,t)$$

The waveshape of the action potential at one point in space, $z = z_0$ is shown in the figure below

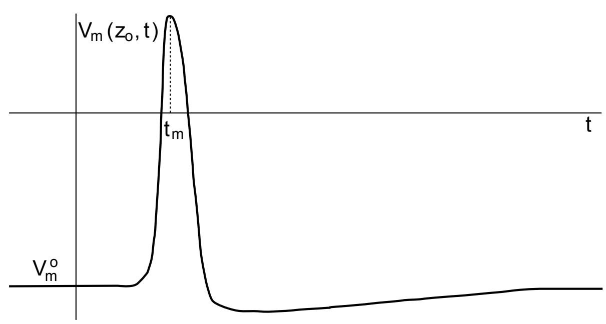

Figure 1: Waveform of a propagated action potential

a) Sketch $K_m(z,t)$ on the same time scale as $V_m(z_0,t)$.

b) Prove that one cannot account for $K_m(z,t)$ by assuming that the membrane can be represented by the equivalent circuit for an incremental element of length δz shown in the figure below. g_m and c_m are constant conductances and capacitances per unit length. [HINT: Consider the polarity of the current through the parallel combination of g_m and c_m prior to the time of occurrence of the peak of the action potential, t_m.]

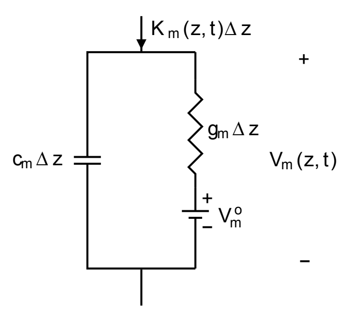

Figure 2: Equivalent network for the membrane of an axon.

2. A cylindrical fibre's membrane has a certain radius r. The extracellular volume is outside the membrane extending to a radius of twice this amount. It can be noted that the membrane itself is considered to have negligible thickness relative to r. The membrane resistance at rest is 2 $k\Omega \cdot \text{cm}^2$, the membrane capacitance is 1.2 μF / cm^2, the intracellular resistivity is 100 $\Omega\cdot$cm, the extracellular resistivity is 40 $\Omega\cdot$cm and the radius is 50 μm. Find each of the following values listed below along with their corresponding units, and ensure to use the linear core-conductor model.

a) What is the membrane resistance per unit length?

b) What is the membrane capacitance per unit length?

c) What is the intracellular resistance per unit length?

d) What is the extracellular resistance per unit length?

3. The following two experiments are performed on a squid giant axon:

- *Experiment #1:* The axon is placed in a large volume of sea water, and the size of the *transmembrane* action potential is measured by means of an intracellular micropipette and is found to have a peak-to-peak value of 100 mV. The conduction velocity is 36 m/s.
- *Experiment #2:* The axon is placed in oil and the transmembrane potential is still found to 100 mV peak-to-peak. The peak-to-peak size of the *extracellular* action potential is 75 mV.

Estimate the expected conduction velocity in Experiment # 2. State your assumptions.

4. A cylindrical cell has a diameter of 500 μm and a length equal to $L = 4\ cm$. After brief experimentation, it is discovered that the cell has the following cable parameters: A membrane conductance $g_m = 100\ \frac{\mu S}{cm}$, a membrane capacitance $c_m = 150\ \frac{nF}{cm}$, and an internal resistance $r_i = 10\ \frac{k\Omega}{cm}$ such that $r_0 \ll r_i$. Two experiments are later conducted:

- *Experiment #1:* The cell is impaled, as seen in the top image in the following figure, with a micropipette at its center so that the membrane potential could be measured
- *Experiment #2:* Axial electrodes are impaled, as seen in the bottom image in the following figure, along the length of the cell to record the potential across the membrane - these electrodes have a resistance per unit length $r = 5\ \frac{\Omega}{cm}$.

a) Determine the cell space constant and the membrane time constant

b) Given the set-up of the experiment 1 explained above, determine the measured potential across the membrane at $z = 0$ (i.e determine $\nu_m(0,t)$)

c) Given the set-up of the experiment 2 explained above, determine the measured potential across the membrane at $z = 0$ (i.e determine $\nu_m(0,t)$)

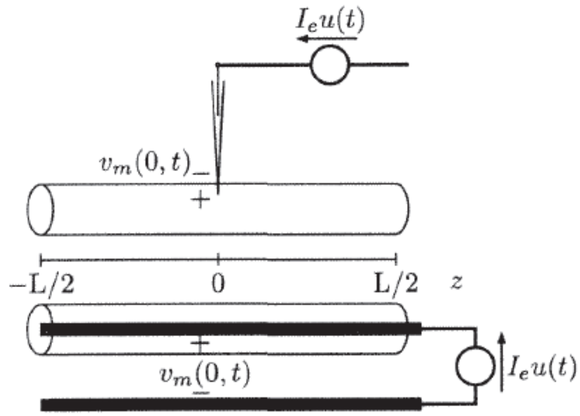

Figure 3: Image denoting the two conducted experiments with the given cylindrical cell with (top) a micropipette and with (bottom) an axial electrode

5. In a first year physics course, one learns that an electrical network is an interconnection of electrical elements within a given circuit consisting of a closed loop to allow for a return path for the current.. In other words, the current has a return path such that the charge can 'flow' throughout the circuit. Below is a figure from Weiss' book denoting an experiment in which the membrane potential is monitored as pulses of current are passed through a given cell. Does this system represent a 'circuit'? If so, sketch what the circuit looks like. If not, explain why.

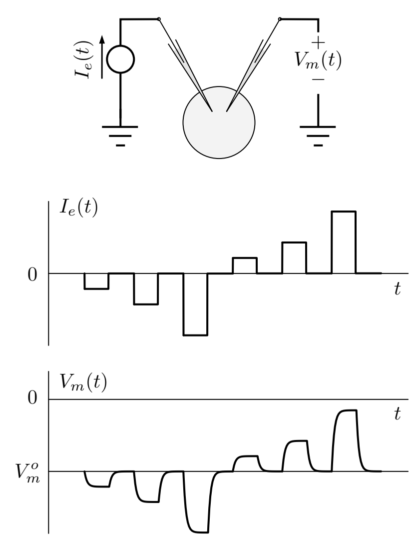

Figure 1.1