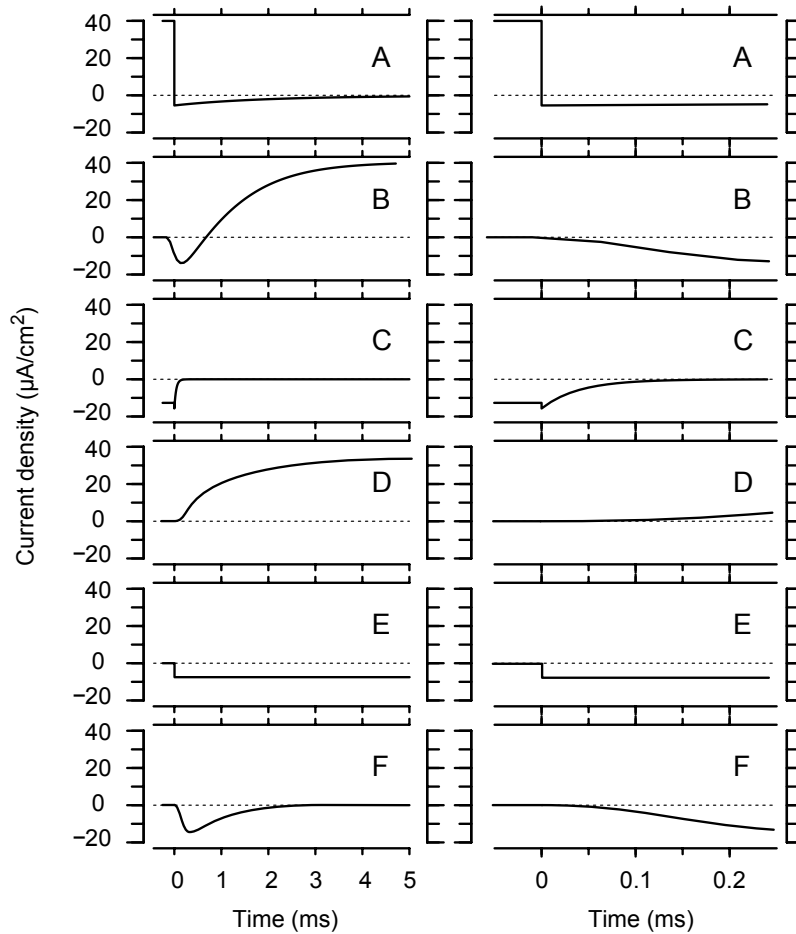


York University BPHS 4080 (Winter 2020) - HW 6

Due Date: Apr. 3, 2020 1:30 PM (but anytime before 4/15 is fine too!)

Questions

1. Various current densities were calculated from the Hodgkin-Huxley model of a squid giant axon for voltage clamp conditions, and results are shown in the following figure.



Default values of the parameters (as listed on page 191 of volume 2 of the text) are used. Each result is shown twice. Plots on the left show results on a 5 ms scale. The early part of the response is replotted on the right with a 0.2 ms scale.

- a) Which row shows the sodium current density that results when V_m is stepped from -50 mV to -75 mV at $t = 0$? Explain.
- b) Which row shows the potassium current density that results when V_m is stepped from -50 mV to -75 mV at $t = 0$? Explain.

c) Which row shows the leakage current density that results when V_m is stepped from -50 mV to -75 mV at $t = 0$? Explain.

2. A bipolar stimulus electrode, as seen in the figure below, has two electrodes, a source (E) and a sink (D). Current is injected at the source (D) electrode and removed at the sink (E) electrode. The medium is effectively unbounded. For this system, ignore electrode-electrolyte voltages, resistances, or other effects, and assume that each electrode is not large enough to modify the field produced by the other; ignore any complications from whatever structures hold the electrodes in place - their effects approach those of a mathematical dipole.

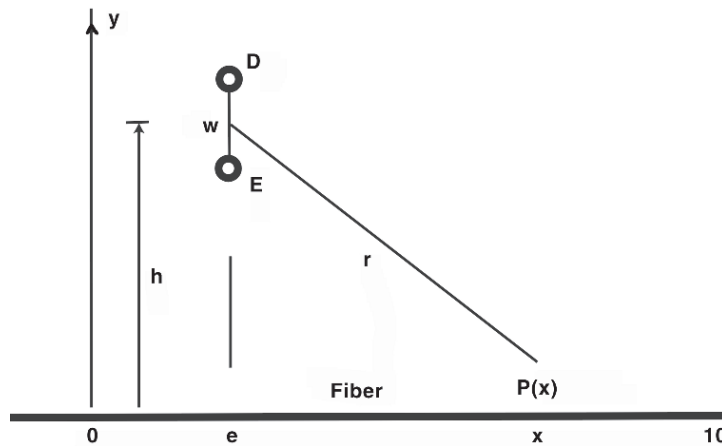


Figure 1: An image depicting an electrode above a fiber; the inter-electrode axis is normal to the fiber axis. Distance h is the vertical axis distance to the center of the electrode pair, whereas w is the distance separating the electrodes and r is the distance from the center of the pair to position x along the fiber, which is considered to lie at $y = 0$ along the x axis. Note that $P(x) \equiv \phi(x)$.

a) Each electrode is a spherical conductor that has a radius of 0.15 cm. The two electrodes are separated by $w = 4$ cm, center to center. The stimulus produces a rectangular current pulse of duration 4 ms and current amplitude 25 mA. The medium has a resistivity of $50 \Omega \cdot \text{cm}$. What voltage between the electrodes is required to produce this stimulus current?

b) Consider the geometry of a bipolar stimulus oriented normal to a fiber; the stimulus electrodes and fiber lie within an unbounded space of resistivity $50 \Omega \cdot \text{cm}$. Suppose that the height $h = 1$ cm and separation $w = 0.2$ cm. Electrode coordinate e (seen labeled on the x -axis) is at 0 cm. For this example, allow the electrode closest to the x -axis be the sink, and the more distant electrode the source. If the stimulus current equal to 25 mA is applied, creating a potential $\phi(x)$ along the x -axis, what $\frac{d^2\phi(x)}{dx^2}$ at $x = 0.3$ cm be equal to?

3. The Hodgkin-Huxley model of a space-clamped squid giant axon stimulated by a pulse of membrane current of amplitude $20 \mu\text{A}/\text{cm}^2$ and of duration 0.5 ms produces a membrane action potential. You can demonstrate this to yourself by running the space-clamped version of the Hodgkin-Huxley simulation software provided with this subject. Start by using the default parameters. If you change the membrane capacitance from $1 \mu\text{F}/\text{cm}^2$ to $20 \mu\text{F}/\text{cm}^2$, no action potential occurs.

a) Explain why no action potential occurs for the larger value of membrane capacitance. You may base your explanation on comparison of the membrane currents, conductances, and activation variables for the default parameters and for the increased capacitance condition. Alternatively, you may wish to perform additional computations with other values of membrane capacitance.

b) With membrane capacitance fixed at $20 \mu\text{F}/\text{cm}^2$, determine a set of parameters that produce an action potential with a waveform that is identical to the action potential obtained with the default parameters. To determine whether or not the action potential is identical to that obtained with the default parameters, you can superimpose plots and/or look at the parameters computed from the responses. You should use the computer as a tool to check your ideas and not as a substitute for thinking. You should avoid a strictly trial-and-error approach. There are simply too many parameters in the Hodgkin-Huxley model for you to explore them all randomly. When you have arrived at a satisfactory solution, explain why your parameter change produces the desired result.

4. Each of the panels in Figure 2 shows action potentials computed from the Hodgkin-Huxley model of a space-clamped axon in response to a current pulse of duration 0.5 ms and of amplitude $20 \mu\text{A}/\text{cm}^2$. In each panel the dashed curve shows the response for the standard parameters of the Hodgkin-Huxley model. Each of the solid curves was obtained by computing the response of the Hodgkin-Huxley model with identical parameters as the dashed curve except that one parameter of the model was changed. For each of the waveforms a)-d), determine which one of the following statements is consistent with the computation.

a) The leakage conductance was reduced from $G_L = 0.3$ to $0.01 \text{ mS}/\text{cm}^2$

b) The temperature was increased from 6.3 to 10°C

c) The membrane capacitance was increased from $C_m=1$ to $1.1 \mu\text{F}/\text{cm}^2$

d) The intracellular sodium concentration was reduced from 50 to $25 \text{ mmol}/\text{L}$.

5. A large unmyelinated axon is immersed in oil, and five different arrangements of electrodes for delivering current stimuli and for measuring potential responses are attached to the axon as shown below.

The stimulus current, a brief positive pulse at $t = 0$, is the same for each arrangement of electrodes. The pulse has a duration that is much shorter than the membrane time constant of the cell, and a strength that is low enough that the cells voltage response remains in its linear range of operation. The space constant of the cell is, λ_C . In arrangement d, the potential is recorded and the current is delivered at the same longitudinal position. In part e, the electrodes are much longer than λ_C .

For each of the different arrangements (parts a-e in the figure) determine which of the following waveforms for $v(t)$ represents the deviation of the measured potential from its resting value. For each waveform, the horizontal axis corresponds to $v(t) = 0$, and the vertical axis to $t = 0$. If no waveform applies, answer None. Explain the basis of your choice in each case.

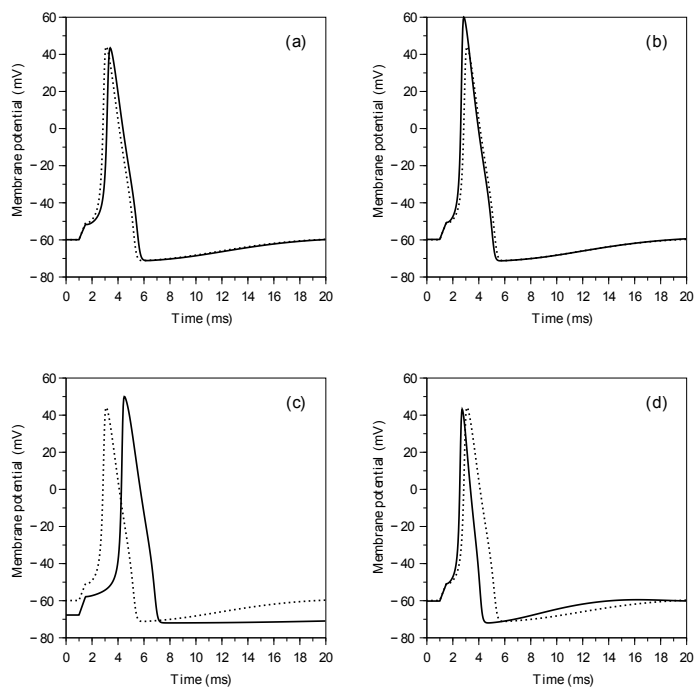


Figure 2: Waveform of action potentials computed from the Hodgkin-Huxley model

