FOUR-DAY SUMMER MATH BRIDGING FOR *STEM* SUCCESS

The nature of the problem

- $>$ 50% of incoming first-year Science majors at York have "learned" math by being given, and memorizing, a large number of facts, lists of allowed and not-allowed algebraic manipulation rules, and a very large number of mechanical solution templates covering a large range of specific problems
- Typical high-school testing for these students gives back problems almost identical to those provided as templates, rarely requiring students to solve "new" problems involving the same conceptual material associated with template problems, and easily solvable by any student with a conceptual understanding of the material underlying the template problems.
- As a result of this highly counterproductive approach to "teaching" and "testing", those students who've been taught math this way
  - actually lack a basic conceptual understanding of the intuitive meaning of the four arithmetic operations (especially division) and hence are unable to assess (and, in fact, don't understand that it's even possible to assess) whether or not a putative algebraic manipulation rule makes sense;
  - have typically never had the relation of algebra to basic arithmetic explained to them well enough that it will occur to them that putative algebraic manipulations can be explored numerically for plausibility;
  - very frequently, because they rely solely (and have very often been strongly encouraged to rely solely) on memory, mix up which algebraic manipulations are from the allowed list and which from the not-allowed list, often ending up performing manipulations that any student with an intuitive understanding of what was going on would immediately recognize made no sense;

  - as a result of being asked to almost entirely simply regurgitate sequences of manipulations they've been required to memorize as "methods" for solving various template problems, and hence having had essentially no practice in exploring problems unfamiliar in form but involving only material they are already (nominally) conceptually familiar with, have acquired essentially no problem-solving skills.

- Many incoming first-year Science majors also come from high schools where they were, often implicitly, but frequently also very explicitly, given the impression/told that

  - math is "really hard";
  - that only a tiny number of "really smart people" can actually understand it; and that, therefore,
  - they have to just memorize what they've been asked to memorize, learn to implement template solution sequences as rapidly as possible (often being told that writing down intermediate steps and making small sketches is bad because it will slow them down(!)), and to stop asking questions about why they're supposed to do what they're being asked to do in the way they're being told to do it.

  The result is that a large fraction of the incoming first-year Science major cohort consists of students who

  - try to get by relying entirely on memorization;
  - find math incredibly taxing, and massively time-consuming as a result;
  - are unable to handle problems which differ in only very minor ways from templates they have seen in class or in the textbook, even when these involve material conceptually identical to that of the template problems; and, particularly damagingly,
  - assume, by default, that they are unable to actually "do math" and hence are psychologically blocked from coming to recognize that they actually can.

The “Background Tutorial” initiative: basic design goals for attempting to deal with these issues

- Design worksheets to cover the key elementary “background” topics where incoming students have been found to display common shortcomings in their basic conceptual understanding.
- Design worksheet materials in such a way that students will recognize them as dealing with topics they have “previously studied”.
- Structure the worksheet questions in such a way that
  - they are easily solved by a student with a conceptual understanding of the underlying material, but
  - they have a form that will not typically match that of template problems the students are likely to have seen before, and hence are such that the studnets will not be able to solve them simply by pulling up out of memory template manipulation sequences they have previously memorized.
- Make the worksheets “iteratable”.
  - The idea here is to provide blocks of questions, each covering one or a small number of conceptual points, and allow students who fail to correctly answer all questions in a block the opportunity to listen to a conceptual explanation of the underlying ideas and then go back to retry the relevant block of questions again.
  - This structure allows each student to go at his/her own pace. Multiple iterations of the cycle of problem-solving followed by conceptual-explanation are easily possible, provided of course, the individual questions in each block are neither marked nor solved by the session leader.
  - It is crucial that the questions not be individually “marked” or solved by the session leader. The goal is for students to see that, once they get a conceptual understanding of the basic ideas underlying the block of questions being worked on,

they are themselves easily able to solve these questions. The iteration of the conceptual discussion process is crucial, both pedagogically and psychologically. The blocks of questions are designed (and tested by implementation) to be such that all students attending the sessions are able, after sufficient iteration, to successfully complete each block.

  - The worksheet materials are, by design, supposed to be such that the iterative process, when implemented, can be naturally described as representing a "typical learning process". The goal here is three-fold:
    * to manage potential student discouragement;
    * to have students (implicitly, through seeing it happen themselves, rather than being told it) discover that they themselves are able to actually understand mathematics and, using the conceptual understanding they acquired earlier in the sessions, handle problems they have not seen before; and
    * to have the students (again by discovering it themselves, rather than being told it) find how much easier and more efficient it is to approach mathematics through conceptual understanding rather than by engaging in vast amounts of memorization.

- For some topics known to be problematic for typical incoming students (roots are a good example), the worksheet material provide "precursors" among the problems primarily dealing with other conceptual matters, so there is an opportunity to identify the conceptual shortcomings in the understanding of the precursor material and briefly pass over this prior to the students meeting it again, in more detail, later in the sequence of worksheets.

- For practical reasons (to do with limited resources to support remediation, and to keep the number of session leaders required under control), all of the above needs to be structured in such a way that a reasonable number of students (practically this turns out to be 15 to 20 or less) can be handled by a single session leader in the effectively parallel one-on-one manner necessitated by the iterative conceptual-explanation structure of the sessions.

The "Background Tutorial" initiative: a few implementation strategy specifics

- To allow a single session leader to handle up to 15-20 student: all questions are True or False.
- To ensure a student cannot typically get all questions in a given block right without a conceptual understanding of the underlying material: typically at least six (usually more) questions per block.
- To avoid counterproductive student discouragment/frustration, the number of questions in most blocks is kept to below ~ 10. (The materials are also designed so that there are some optional block-breaks the session leader can introduce for students struggling with the iteration of a longer block.)
- To allow an instant assessment of any shortcomings in a student's conceptual understanding of the material underlying a given block of questions, the T or F answer possibilities for each question are arranged on the worksheets in an aligned column at the right. The students are to circle their choices. This organization is designed (and works very effectively) to allow "answer template strips" to be held up to the worksheets, making it very fast for the session leader to both assess the student's performance on the block in question and immediately determine what conceptual shortcomings (if any) the student might have, and hence what type of conceptual discussion the student will then most benefit from (for this to work effectively, especially with the pressures associated with multiple, parallel one-to-one discussion, it is crucial that the session leaders be extremely familiar with the worksheet materials, and how they are structured, in advance).
- A single error in a block of questions can often be the result of carelessness; as a result, students are told one of three things about their answers to the questions in a given block:
  - that all were answered correctly (in which case they can go on to the next block),
  - that there is only one error (if the session leader is essentially certain that this is a result of carelessness, the student can be directed to look amongst 3 or 4 questions for the error),

  - that there is more than one error (in which case the number of errors is NOT specified and the student, ideally immediately, takes part in a conceptual discussion of the relevant underlying ideas).

- Scattered throughout the worksheet are shorter "problem solving practice" blocks, designed to present the students with problems of types they will ideally not have seen before, but which are easily solved using the conceptual understanding they have achieved by completing the blocks previous to the new "problem solving practice" block. These are designed to give the student practice in how to approach and analyze such "new" problems, and to provide the session leaders with opportunities to introduce some of the most common strategies for approaching and analyzing such problems, all with a goal of breaking students of the habit of looking at a problem and, if they don't recognize it as corresponding to a template problem whose solution they have memorized, freezing, feeling there is nothing they can do to handle it, and calling up some random sequence of template operations in hopeful desperation.

Links to further information: bethune.yorku.ca/talks