

# Comprehensive Exam

## Course information

Title **Mathematical Modelling**

Course number **MATH 6931**

Semester **Fall**

Exam is:  Closed-book  Open-Book

If open-book, provide a detailed rationale

N/A closed book

Is Exam invigilated?  Yes  No

If No, provide a detailed rationale

N/A invigilated

## Evaluation method

The written exam will be 5 questions. Non-comprehensive students will choose 4 out of 5 problems to complete. The comprehensive exams will require all 5 problems to be completed.

# Comprehensive Exam

Does the duration of exam exceed 3 hours?  Yes  No  
 If Yes, provide a detailed rationale

Since comprehensive students have an additional question to complete, they will be provided an additional half hour making the exam 3.5 hours. They are welcome to finish the exam within the regular 3 hours if they so choose.

## Outline of topics to be covered

1. Introduction (philosophy of modelling, structure of a model, scaling and non-dimensionalization)
2. ODE Models (Predator-prey systems, linear stability, disease dynamics, chemical kinetics, inverse modelling and parameter fitting, Euler method.)
3. Stochastic Models (Birth-death processes, Markov Chains, Random Walk, Gillespie Algorithm)
4. PDE Models (Traffic models, characteristics and shocks, conservation laws)

Passing threshold (% of the exam): 65

Faculty	Signature	Date
Name Iain Moyles	Iain Moyles <small>Digitally signed by Iain Moyles                      Date: 2022.10.18 13:18:36 -04'00'</small>	10/18/22

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# Comprehensive Exam

## Course information

Title: Partial Differential Equations

Course number: 6350 3.0

Semester: Fall 2022

Exam is:  Closed-book  Open-Book

If open-book, provide a detailed rationale

Is Exam invigilated?  Yes  No

If No, provide a detailed rationale

## Evaluation method

6 equally-weighted questions from the topics (see below) will have to be answered on examination booklets. Answers have to be written out clearly and justified fully.

# Comprehensive Exam

Does the duration of exam exceed 3 hours?  Yes  No  
 If Yes, provide a detailed rationale

**Outline of topics to be covered**

Topics: The Convolution, The Fourier Transform, Tempered Distributions, Symbols, Pseudo-Differential Operators, Asymptotic Expansions, The Product of Two Pseudo-Differential Operators, The Formal Adjoint of a Pseudo-Differential Operator, The Parametrix of an Elliptic Pseudo-Differential Operator,  $L^p$ -Boundedness of Pseudo-Differential Operators, Sobolev Spaces, Global Regularity of Elliptic Partial Differential Equations, Weak Solutions of Pseudo-Differential Equations

Reference: M. W. Wong, An Introduction to Pseudo-Differential Operators, Third Edition, World Scientific, 2014

Passing threshold (% of the exam): 50%

Faculty	Signature	Date
Name M. W. Wong	M. W. Wong	10/18/22

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# Comprehensive Exam

## Course information

Title **General Topology**

Course number **6540**

Semester **Fall**

Exam is:  Closed-book  Open-Book

If open-book, provide a detailed rationale

Is Exam invigilated?  Yes  No

If No, provide a detailed rationale

## Evaluation method

Written exam. Approximately 6 questions on various topics. Part marks for partial solutions is possible.

# Comprehensive Exam

Does the duration of exam exceed 3 hours?  Yes  No  
 If Yes, provide a detailed rationale

### Outline of topics to be covered

topological spaces, basis for topology, continuous functions (equivalent def's for metric) product topology (including infinite), metric spaces, quotient spaces. complete metric spaces, Baire spaces connectedness, compactness, compactness in metric spaces and applications, generalizations of compactness: countable compactness, sequential compactness, local compactness etc..., countability axioms, separation axioms, normal spaces, Urysohn Lemma and Tietze Extension Theorem, Tychonov Theorem, Urysohn Metrization and Nagata-Smirnov Metrization, compactifications including the one-point and Stone-Cech compactification, paracompactness, net convergence, filter convergence, ultrafilters, function spaces, Stone-Weierstrass Theorem

#### References

- [1] Dugundji, James; Topology, Allyn and Bacon series in advanced mathematics. Allyn and Bacon (Boston) (1966).
- [2] Munkres, James R.; Topology, second edition. Prentice Hall (2000)
- [3] Willard, Stephen; General Topology, Addison-Wesley (1970).

Passing threshold (% of the exam): 50

Faculty	Signature	Date
Name Paul Szeptycki		10/28/22

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# Comprehensive Exam

## Course information

Title Applied Algebra

Course number Math 6121

Semester Fall 2022

Exam is:  Closed-book  Open-Book  
If open-book, provide a detailed rationale

Is Exam invigilated?  Yes  No  
If No, provide a detailed rationale

## Evaluation method

The comprehensive exam instructions are typically:  
This is a 3 hours exam  
Give detailed justifications and explanations where appropriate. Attention to detail and clarity of exposition are important.  
Total: 6 problems of 10 points; required for passing: 32 points.

# Comprehensive Exam

Does the duration of exam exceed 3 hours?  Yes  No  
 If Yes, provide a detailed rationale

## Outline of topics to be covered

Linear Algebra (Recall crash course, Graduate level): Direct sum and tensor product has corresponding operations on basis and linear transformations.  
 Group Theory and representation Theory: Recall: Group, morphism, subgroup, G-sets (and G-morphisms), Isomorphisms Theorems and quotient groups.; Jordan-Holder Theorem; Sylow Theorem;  
 Representation of finite groups and characters (over C): Maske’s Theorem; Schur’s lemma; Structure of the space of G-endomorphisms; Structure of the inner space of characters on G  
 Final Theorem: the number of irreducibles representations for G equal the number of conjugacy classes of G  
 Preliminary notions in ring: definitions; Euclidian domain; Principal ideal domain; Polynomial rings  
 Grobner basis with emphasis on algorithmic aspect and computational geometry solving polynomial system of equations (with some application to robotics and computational geometry)  
 Modules over PID (Advanced linear algebra); Chinese Remainder Theorem; Classification of finitely generated modules over PID; Classification of finitely generated abelian groups; rational canonical form; Jordan canonical form;  
 (1) D. S. Dummit and R. M. Foote, "Abstract Algebra" Willey (2004). ISBN: 978-0-471-43334-7.  
 (2) T. W. Hungerford, "Algebra", GTM Springer (2003). ISBN: 978-0-387-90518-1.  
 (3) D. A. Cox, J. Little and D. O’shea, "An Introduction to Computational Algebraic Geometry and Commutative Algebra" UTM Springer (2007). ISBN: 978-0-387-35650-1.  
 (4) B. Sagan, "The Symmetric Group", Springer-Verlag GTM (2001) ISBN: 978-1-4757-6804-6  
 Reference (1) and (2) are good for the general material. Reference (3) is for the Grobner basis, and the reference (4) is for the group representation only (chap 1).

Passing threshold (% of the exam): 53.3333

Faculty	Signature	Date
Name Nantel Bergeron	 <small>Digitally signed by Nantel Bergeron                      Date: 2022.10.18 17:42:17 -04'00'</small>	10/18/22

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# Comprehensive Exam

## Course information

Title **Mathematical Statistics**

Course number **6620**

Semester **F**

Exam is:  Closed-book  Open-Book

If open-book, provide a detailed rationale

Is Exam invigilated?  Yes  No

If No, provide a detailed rationale

## Evaluation method

short-answer questions

# Comprehensive Exam

Does the duration of exam exceed 3 hours?  Yes  No  
 If Yes, provide a detailed rationale

The final exam for MATH 6620 is three hours long. The comprehensive asks more questions and of greater difficulty, so additional time is warranted.

### Outline of topics to be covered

1. probability spaces (e.g. chapter 1 in [S])
2. change of variables: 1-1 or not, univariate or multivariate (e.g. chapter 1 in [S])
3. some fundamental distributions including non central chi-square (e.g. chapter 1 in [S])
4. mathematical inequalities used in statistics (e.g. Jensen, Gibbs) (e.g. chapter 1 in [S])
5. different notions of convergence and convergence theorems (e.g. chapter 1 in [S])
6. sufficiency, completeness, ancillarity (e.g. chapter 2 in [S])
7. exponential families (e.g. chapter 2 in [S])
8. statistical decision theory: loss functions, Bayes' estimators, minimax (e.g. chapter 4 in [S])
9. UMVUE (e.g. chapter 3 in [S])
10. hypothesis testing: UMP, MLR (e.g. chapter 6 in [S])

REFERENCES:

[S] Mathematical Statistics by Jun Shao (Springer Text in Statistics)  
 [CB] Statistical Inference by Casella and Berger (Duxbury Advanced Series)

Passing threshold (% of the exam): 60

Faculty	Signature	Date
Name Hanna Jankowski		11/1/22

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# Comprehensive Exam

## Course information

Title Applied Statistics I

Course number 6630

Semester Fall 2022

Exam is:  Closed-book  Open-Book

If open-book, provide a detailed rationale

Is Exam invigilated?  Yes  No

If No, provide a detailed rationale

## Evaluation method

The exam will be comprised of a 3-hr closed book written test and a 1-hr online crowdmark test using computing program to implement an algorithm.

The 3-hour theory exam will be administered in person and will be invigilated. The online part is open book and open notes. The online part is administered in the evening (8-9 pm) on the same day of the theory exam. There is no invigilation and students complete the online programming test using their own computer at home.

Twenty percent of the overall grade is the online programming part.

The passing threshold is 60% of the overall grade combining the theory and the online part.

# Comprehensive Exam

Does the duration of exam exceed 3 hours?  Yes  No  
 If Yes, provide a detailed rationale

The comprehensive exam has an online test in computing in addition to the written test in theory.

### Outline of topics to be covered

The exam will cover the topics in applied statistics including:  
 Maximum likelihood estimation using numeric method including continuous and discrete optimization  
 EM algorithm for missing data  
 Monte Carlo simulation methods  
 Markov Chain Monte Carlo methods  
 Bootstrap and Jackknife methods  
 The reference for the exam will be:  
 1) Chapters 1,2,3,4,6, 7, 9 of Computational statistics, Geof Givens and Jennifer Hoeting, Wiley

Passing threshold (% of the exam): 60%

Faculty	Signature	Date
Name Xin Gao	Xin Gao <small>Digitally signed by Xin Gao                      Date: 2022.10.19 10:54:17 -04'00'</small>	

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# Comprehensive Exam

## Course information

Title **Complex Analysis**

Course number **6300**

Semester **Fall 2022**

Exam is:  Closed-book  Open-Book

If open-book, provide a detailed rationale

Is Exam invigilated?  Yes  No

If No, provide a detailed rationale

## Evaluation method

There will be a closed book in-person written exam of 3 hours scheduled during the final exam period..

# Comprehensive Exam

Does the duration of exam exceed 3 hours?  Yes  No  
 If Yes, provide a detailed rationale

**Outline of topics to be covered**

Complex differentiability, power series, Cauchy-Riemann equations, Cauchy integral theorem, harmonic functions, residues, contour integration, theorems of Liouville, Jensen, Morera, Rouché, Hurwitz, meromorphic functions, Weierstrass products, Montel's theorem, Riemann  $\zeta$ -function, Weierstrass  $\mathfrak{P}$ -function, Riemann mapping theorem

Passing threshold (% of the exam): 60%

Faculty	Signature	Date
Name Peter Gibson		10/24/22

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# Comprehensive Exam

## Course information

Title Differential Dquations

Course number 6340

Semester Winter 2023

Exam is:  Closed-book  Open-Book

If open-book, provide a detailed rationale

Is Exam invigilated?  Yes  No

If No, provide a detailed rationale

## Evaluation method

The compressive exam will have 6 questions, total marks 120.  
I will consider 60%, or equivalently, 72 a pass.

# Comprehensive Exam

Does the duration of exam exceed 3 hours?  Yes  No  
 If Yes, provide a detailed rationale

## Outline of topics to be covered

- General properties of Differential Equations  
Existence, uniqueness, dependence on initial data and parameters, extensibility.
- Linear Systems and Stability  
General theory of linear systems, Periodic coefficients and Floquet Theory, Stability of linear and nonlinear systems.
- Nonlinear Systems: Local Theory  
Linearization, Invariant manifolds, Hartman-Grobman Theorem, Normal form theory.
- Nonlinear Systems: Global Theory  
Limit sets and attractors, periodic sets and limit cycles; Poincare map, Poincare-Bendixson Theory, Lotka-Volterra system, Liénard systems. .
- Bifurcation theory of nonlinear systems:  
Saddle-node bifurcation, transcritical bifurcation, pitchfork bifurcation, Hopf bifurcation, homoclinic-bifurcations, co-dimension 2 bifurcations.
- Nonlinear dynamics and applications in physics, biology and finance.

Passing threshold (% of the exam):

Faculty	Signature	Date
Name Huaiping Zhu		10/30/22

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# Comprehensive Exam

## Course information

Title Functional Analysis I

Course number 6461

Semester F22

Exam is:  Closed-book  Open-Book

If open-book, provide a detailed rationale

Is Exam invigilated?  Yes  No

If No, provide a detailed rationale

## Evaluation method

The usual (the students work on the assigned problems for three hours, and I grade them afterwards`

# Comprehensive Exam

Does the duration of exam exceed 3 hours?  Yes  No  
 If Yes, provide a detailed rationale

## Outline of topics to be covered

Topological vector spaces. Normed spaces.

Banach spaces. Baire Category Theorem. Closed Graph Theorem. Open Graph Theorem. Classical Banach spaces ( $C(X)$ ,  $c_0$ ,  $\ell_p$ ,  $\ell_\infty$ ,  $L_p(X, \mu)$ ,  $L_\infty(X, \mu)$ ). Hilbert space.

Local convexity. Hahn-Banach theorems.

Weak topologies. Compactness. Dual spaces. Riesz representation theorems. Krein-Milman theorem. Stone-Weierstrass theorem.

Bounded linear operators. Operators on Hilbert space. Spectral theorem for compact, self-adjoint, operators.

Passing threshold (% of the exam): 60%

Faculty	Signature	Date
Name Ilijas Farah	<i>Ilijas Farah</i>	October 19, 2022

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# Comprehensive Exam

## Course information

Title Advanced Numerical Methods

Course number MATH6651

Semester Fall 2022

Exam is:  Closed-book  Open-Book

If open-book, provide a detailed rationale

Is Exam invigilated?  Yes  No

If No, provide a detailed rationale

## Evaluation method

The comp exam of MATH 6651 is three-hour and in-person exam, and will be invigilated. The exam is closed-book.

Students are allowed to bring "a formula sheet" (two letter pages but only two sides, for formulas that you prepared, wrote or typed) and a calculator is also allowed.

Passing threshold is 60% of the comprehensive exam.

# Comprehensive Exam

Does the duration of exam exceed 3 hours?  Yes  No  
 If Yes, provide a detailed rationale

**Outline of topics to be covered**

Chapter 1. Optimization Methods  
 Section 1 Search Methods in One Dimension (Golden and Fibonacci Methods)  
 Section 2 The Simplex Search Method in Two Dimensions  
 Section 3 The Steepest Descent Technique and Newton's Method in Multi-dimensions

Chapter 2. The Preconditioned Conjugate Gradient Methods  
 Section 1 The Gradient Methods and The Conjugate Direction Method  
 Section 2 The Conjugate Gradient Method  
 Section 3 The Preconditioned Conjugate Gradient Method  
 Section 4 The Nonlinear Conjugate Gradient Methods

Chapter 3. Numerical Methods for Initial Value Problems of ODEs  
 Section 1 The Euler's Method and Higher-order Taylor's Methods  
 Section 2 The Runge-Kutta Methods  
 Section 3 The Multistep Methods  
 Section 4 Stability Analysis  
 Section 5 Adaptive Step Technique  
 Section 6 Applications to Systems of ODEs and Higher-Order Equations

Chapter 4. Approximation Theory  
 Section 1 Orthogonal Polynomials and Least Squares Approximation  
 Section 2 Chebyshev Polynomials and the Economization  
 Section 3 Pade approximation  
 Section 4 Fourier Approximation and Fast Fourier Transforms

Chapter 5. Numerical Methods for Boundary-Value Problems of ODEs

Passing threshold (% of the exam):60/100

Faculty	Signature	Date
Name Dong Liang	 <p>Digitally signed by Dong Liang                      Date: 2022.10.28 20:47:47 -04'00'</p>	Oct 28, 2022

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