Differential Equations

MATH 6340 WINTER 2023
YORK UNIVERSITY

Instructor: Huaiping Zhu

Topics

- General properties of Differential Equations
  Existence, uniqueness, dependence on initial data and parameters, extensibility.
- Linear Systems and Stability
  General theory of linear systems, Periodic coefficients and Floquet Theory, Stability of linear and nonlinear systems.
- Nonlinear Systems: Local Theory
  Linearization, Invariant manifolds, Hartman-Grobman Theorem, Normal form theory.
- Nonlinear Systems: Global Theory
  Limit sets and attractors, periodic sets and limit cycles; Poincare map, Poincare-Bendixson Theory, Lotka-Volterra system, Liénard systems.
- Bifurcation theory of nonlinear systems:
  Saddle-node bifurcation, transcritical bifurcation, pitchfork bifurcation, Hopf bifurcation, homoclinic-bifurcations, co-dimension 2 and 3 bifurcations.
- Nonlinear dynamics and applications in physics, biology and finance.

Textbook

Lawrence Perko, Differential Equations and Dynamical Systems, 4th Edition. Lecture Notes (Book Draft).

References

- J.K. Hale, Ordinary Differential Equations, Krieger, Malabar, Florida, 1980.
- J.K. Hale and H. Kocak Dynamics and Bifurcations, Springer 1991.
- M. W. Hirsch, S. Smale, and R. Devaney, Differential Equations, Dynamical Systems, and an Introduction to Chaos. 3rd Edition, 2013.

Grade Policy: Assignments (40%) + Final exam (60%)

Final Exam: 180 minutes. Invigilated in-person.

For Compressive Exam: There will be 6 questions, total marks 120. I will consider 60%, or equivalently, 72 a pass.

GS MATH 6622 3.0: Generalized Linear Models

(WINTER, 2022 - 23)
YORK UNIVERSITY

Instructor: Wei Liu, office N601B, Ross Building, liuwei@yorku.ca

Lectures: 11:30am – 1:00pm on Tuesdays and Thursdays, MC 216 and LSB 101

Office hours: by appointment

Class Webpage: eClass (please check it out frequently!)

It is vital to be aware of course announcements, and adhere to dates and deadlines for the course. It is students' responsibility to be aware of all in-class announcements and eClass announcements.

Materials from the course are copyrighted and not to be distributed to individuals outside of the course whether as a physical or online copy at any time before the course has started, during the tenure of the course or after the course is completed. Individuals caught disobeying this will be charged.

Textbook

A. J. Dobson and A. G. Barnett (2008). An Introduction to Generalized Linear Models (third edition). Chapman and Hall/CRC Press.

References

Agresti (2013). Categorical Data Analysis (third edition). Hohn Wiley & Sons.

Faraway (2016). Extending the Linear Model with R: Generalized Linear, Mixed Effects, and Nonparametric Regression Models. Chapman and Hall/CRC Press.

McCullagh and Nelder (1989). Generalized Linear Models (second edition). Chapman and Hall/CRC Press.

Venables and Ripley (2002). Modern Applied Statistics with S (fourth edition). Springer.

Course description

Generalized Linear Models (GLMs) are an extension to linear models where the response variable is not continuous. These models are commonly used for analysis where the response variables are binary, categorical, proportions, or counts. GLMs play a crucial role in statistics and can be widely applied in industry, government, and banks as well as other areas. Tentative topics will include: review of linear models; formulation of GLMs; link functions; iterative least squares algorithms; deviance and asymptotic theory; model selection; residuals and quasi-likelihood; generalized linear mixed models.

Evaluation

Three assignments via Crowdmark (30%) , a group project and presentation (25%), and a final exam (45%). The Ph.D. students should obtain at least 60/100 to pass the comprehensive exam. It is the students' responsibility to

- be aware of the due time. NO extension for any submission. 0 grade for late submission.
- make sure your assignment is uploaded fully and completely before the due time. Try to make your picture a small size. Large files may cause uploading problem.
- make sure the content you uploaded is clear, readable and markable. The unclear submission will not be graded.
- Missed final exam: A student who becomes ill, has a personal/family emergency or a religious observance may ask for a later date for their final exams. To do this, students must request deferred standing, no later than one week after the missed examination or the last day of classes. For details, please visit: Deferred Standing.
- A simple scientific calculator without statistical functions can be used for the final exam. Graphical or programmable calculators are NOT allowed.
- It is expected that all students are aware of their individual responsibilities under the University Policies and Regulations which will be strictly adhered to in this class. No form of academic dishonesty will be tolerated.

Email Policy

- It is strongly recommended that you use your York University email account.
- Please always put MATH 6622 on the subject line.
- Please identify yourself with your name and student number at the end of email.
- Important Dates

MATH 6652 3.0 Numerical Solutions to Differential Equations: Syllabus for comprehensive exam

WINTER 2022-23
YORK UNIVERSITY

Evaluation

The final exam for Math 6652 is to serve as the comprehensive exam in numerical analysis. As per the course outline, the final exam is weighted as 60% of the course grade. The exam will have 6 questions; four questions will be evaluated out of 15 marks while two questions will be evaluated out of 20 marks (total 100 marks). A passing grade for the exam will be 60/100 marks. The exam will be written in person at the University and invigilated by the course instructor.

The main themes stressed in the exam problems are consistency, stability, and accuracy of numerical schemes applied to parabolic, elliptic and hyperbolic partial differential equations. One problem at the end of the exam is designed to ensure the students have learned basic skills in Matlab computing, reflecting the practical/lab component of the course.

Principal Text

K.W. Morton and D.F Mayers, Numerical Solution of PDEs, Cambridge University Press, 1994.

Other References

J.C. Strikwerda, Finite Difference Schemes and Partial Differential Equations, Second Ed., SIAM, Philadelphia, 2004.

C. Johnson, Numerical Solution of Partial Differential Equations by the Finite Element Method, Cambridge University Press, Cambridge, 1987.

Comprehensive Examination Topics:

Section 1: Finite Difference Formulations

1.1 Taylor series expansion

1.2 Mixed partial derivatives

1.3 Finite difference equations

Section 2: Parabolic Equations in One Space Dimension

2.1 Explicit methods: FTCS, Richardson, and Dufort-Frankel

2.2 Implicit methods: Laasonen, Crank-Nicolson, beta (theta) formulation

2.3 Truncation error, consistency, and convergence

Section 3: Parabolic Equations in Two and Three Space Dimensions

3.1 Standard finite difference methods

3.2 ADI method in two and three dimensions

3.3 Approximate factorization

3.4 Truncation error, consistency, and convergence

3.5 General problems

Section 4: Stability Analysis

4.1 Discrete perturbation stability analysis

4.2 Fourier stability analysis

4.3 Stability of explicit and implicit methods

4.4 Multi-dimensional problems

Section 5: Linear Second Order Elliptic Equations in One and Two Space Dimensions

5.1 Explicit iterative methods: Jacobi, point Gauss-Seidel, point successive over-relaxation.

5.2 Implicit iterative methods: Line Gauss-Seidel, Line successive over-relaxation, ADI method.

5.3 Stability, error analysis, and consistency.

Section 6: Hyperbolic Equations in One Space Dimension

6.1 Explicit methods for linear equations: FTFS, FTCS, first upwind, Lax, Leapfrog, Lax-Wendroff.

6.2 Implicit methods for linear equations: BTCS, Crank-Nicolson.

6.3 Multi-step methods for linear equations: Richtmyer, Lax-Wendroff, MacCormack.

6.4 Explicit methods for non-linear equations: Lax, Lax-Wendroff, MacCormack.

6.5 Implicit methods for non-linear equations: Beam and Warming.

6.6 Linear damping.

6.7 Flux corrected transport.

graduate studies | YORK UNIVERSITÉ UNIVERSITY

Office of the Dean
York University

230 York Lanes — 4700 Keele Street
Toronto, ON, Canada M3J 1P3

Tel: (416) 736-5521
yorku.ca/gradstudies

Graduate Program in Mathematics and Statistics

Comprehensive Exam

Course information

Title: Analytic Number Theory

Course number: MATH 6110

Semester: winter 2022-2023

Exam is: ☑ Closed-book ☐ Open-Book

If open-book, provide a detailed rationale

Is Exam invigilated? ☑ Yes ☐ No

If No, provide a detailed rationale

Evaluation method

The exam will consist of a mix of definition questions, proofs of central results from analytic number theory (largely following Apostol's presentation), and proofs of novel variations of these results.

Office of the Dean
York University
230 York Lanes — 4700 Keele Street
Toronto, ON, Canada M3J 1P3
Tel: (416) 736-5521
yorku.ca/gradstudies

Graduate Program in Mathematics and Statistics

Comprehensive Exam

Does the duration of exam exceed 3 hours? If Yes, provide a detailed rationale	☐ Yes	☑ No

Outline of topics to be covered

- elementary number theory
- arithmetic functions, average values, and asymptotics
- Euler and Abel summation and related techniques
- equivalent forms of the prime number theorem, and related asymptotics
- Dirichlet characters and primes in arithmetic progressions
- the Riemann zeta function and Dirichlet L-functions (definitions, basic analytic properties)
- the analytic proof of the prime number theorem

Main reference: “Introduction to Analytic Number Theory,” T.M. Apostol, Springer 1976

Passing threshold (% of the exam): 65%

Faculty	Signature	Date
Name Patrick Ingram		10 Feb 2023, rev 6 Mar

This form must be submitted electronically by email to: gradmath@yorku.ca.

The syllabus must be approved by the PhD Committee

graduate studies | YORK UNIVERSITÉ UNIVERSITY

Office of the Dean
York University

230 York Lanes — 4700 Keele Street
Toronto, ON, Canada M3J 1P3

Tel: (416) 736-5521
yorku.ca/gradstudies

Graduate Program in Mathematics and Statistics

Comprehensive Exam

Course information

Title: Algebra II

Course number: Math 6122

Semester: Winter

Exam is: ☑ Closed-book ☐ Open-Book

If open-book, provide a detailed rationale

Is Exam invigilated? ☑ Yes ☐ No

If No, provide a detailed rationale

Evaluation method

In-person exam, two and half hours. There will be 10 problems (10% each). 55% is the minimal pass for the comprehensive exam.

Office of the Dean
York University
230 York Lanes — 4700 Keele Street
Toronto, ON, Canada M3J 1P3
Tel: (416) 736-5521
yorku.ca/gradstudies

Graduate Program in Mathematics and Statistics

Comprehensive Exam

Does the duration of exam exceed 3 hours? ☐ Yes ☑ No If Yes, provide a detailed rationale

Outline of topics to be covered
Further ring and module theory; more on ideals (primes, irreducible, maximal, etc), UFD, injective and projective modules; tensor products of modules; Introduction to algebraic geometry: varieties, radical ideals, Hilbert ' s nullstellensatz; Fields theory and Galois theorems: field extensions, splitting fields, automorphism group of fields, Galois correspondence, Galois groups of polynomials, solving polynomials with radicals. Introduction to category theory. Textbook: Algebra, GTM 73, by Thomas W. Hungerford

Passing threshold (% of the exam): 55 %

Faculty	Signature	Date
Name Yun Gao		02/10/2023

This form must be submitted electronically by email to: gradmath@yorku.ca.

Graduate Program in Mathematics and Statistics

Comprehensive Exam

graduate studies | YORK UNIVERSITÉ UNIVERSITY

Course information	
Title	Measure Theory
Course number	MATH 6280
Semester	Winter 2023

Exam is: ☑ Closed-book ☐ Open-Book

If open-book, provide a detailed rationale

Is Exam invigilated? ☑ Yes ☐ No

If No, provide a detailed rationale

Evaluation method

This will be a written exam during the April exam period. It will be exactly the same as the final exam of the course 6280 that is being offered in the Winter term.

Comprehensive Exam

graduate studies | YORK UNIVERSITÉ UNIVERSITY

Does the duration of exam exceed 3 hours? If Yes, provide a detailed rationale	☐ Yes	☑ No

Outline of topics to be covered

Sigma-algebras; measures; Lebesgue measure; measurable functions; integration; convergence theorems; Lp spaces; signed measures; decomposition theorems; product measures; differentiation of measures.

These topics correspond to the material in the course textbook:
Donald L. Cohn, "Measure Theory", Second Edition, Birkhauser 2013. Chapters 1-6, except sections 2.6, 4.4, and 6.1. (This book is available for free download to all York students through York Libraries.)
The material also appears in:
H.L. Royden and P.M. Fitzpatrick, "Real Analysis", Fourth Edition, Prentice Hall 2010. Chapters 2-8 and 17-20 (but only parts of some chapters).

Passing threshold (% of the exam): 60 %

Faculty	Signature	Date
Name Neal Madras	Neal Madras	1/29/23

This form must be submitted electronically by email to: gradmath@yorku.ca.

graduate studies | YORK UNIVERSITÉ UNIVERSITY

Office of the Dean
York University

230 York Lanes — 4700 Keele Street
Toronto, ON, Canada M3J 1P3

Tel: (416) 736-5521
yorku.ca/gradstudies

Graduate Program in Mathematics and Statistics

Comprehensive Exam

Course information	
Title	Differential Equations
Course number	6340
Semester	

Exam is: ☑ Closed-book ☐ Open-Book

If open-book, provide a detailed rationale

Is Exam invigilated? ☑ Yes ☐ No

If No, provide a detailed rationale

Evaluation method

There will be 6 questions to cover the selected topics from the syllabus.

Office of the Dean
York University
230 York Lanes — 4700 Keele Street
Toronto, ON, Canada M3J 1P3
Tel: (416) 736-5521
yorku.ca/gradstudies

Graduate Program in Mathematics and Statistics

Comprehensive Exam

Does the duration of exam exceed 3 hours? ☐ Yes ☑ No If Yes, provide a detailed rationale

Outline of topics to be covered
1.General properties of Differential Equations Existence, uniqueness, dependence on initial data and parameters, extensibility. 2.Linear Systems and Stability General theory of linear systems, Periodic coefficients and Floquet Theory, Stability of linear and nonlinear systems. 3.Nonlinear Systems: Local Theory Linearization, Invariant manifolds, Hartman-Grobman Theorem, Normal form theory. 4.Nonlinear Systems: Global Theory Limit sets and attractors, periodic sets and limit cycles; Poincare map, Poincare-Bendixson Theory, Lotka-Volterra system, Liénard systems. . 5.Bifurcation theory of nonlinear systems Saddle-node bifurcation, transcritical bifurcation, pitchfork bifurcation, Hopf bifurcation, homoclinic-bifurcations, co-dimension 2 and 3 bifurcations. 6.Nonlinear dynamics and applications in biology, physics and finance.
Passing threshold (% of the exam): 60%

Faculty	Signature	Date
Name Huaiping Zhu		1/27/23

This form must be submitted electronically by email to: gradmath@yorku.ca.

graduate studies | YORK UNIVERSITÉ UNIVERSITY

Office of the Dean
York University

230 York Lanes — 4700 Keele Street
Toronto, ON, Canada M3J 1P3

Tel: (416) 736-5521
yorku.ca/gradstudies

Graduate Program in Mathematics and Statistics

Comprehensive Exam

Course information

Title: Advanced Mathematical Statistics

Course number: 6621

Semester: W23

Exam is: ☑ Closed-book ☐ Open-Book

If open-book, provide a detailed rationale

Is Exam invigilated? ☑ Yes ☐ No

If No, provide a detailed rationale

Evaluation method

A comprehensive exam

Office of the Dean
York University
230 York Lanes — 4700 Keele Street
Toronto, ON, Canada M3J 1P3
Tel: (416) 736-5521
yorku.ca/gradstudies

Graduate Program in Mathematics and Statistics

Comprehensive Exam

Does the duration of exam exceed 3 hours? ☐ Yes ☑ No If Yes, provide a detailed rationale

Outline of topics to be covered

The topics of the course include:

Probability theory;

Limit theorems in mathematical statistics;

Asymptotic properties of the following statistics: (a) The usual statistics computed from a sample;(b) The statistics concocted as transformations of vectors of more basic statistics; (c) statistics arising in classical parametric inference and contingency table analysis; (d) U-statistics

References:

1) Shao, J. Mathematical Statistics, 2nd edition, Springer, 2003.

2) Lehmann, E.L. and Casella, G., Theory of Point Estimation, 2nd edition, Springer, 1998.

3) Lehmann, E.L. and Romano, J.P., Testing Statistical Hypotheses, 3nd edition, Springer, 2005.

4) Lehmann, E.L., Elements of Large-Sample Theory, Springer; Corrected edition (August 27, 2004).

Passing threshold (% of the exam): 60%

Faculty	Signature	Date
Name Yuehua Wu	Yuehua Wu Digitally signed by Yuehua Wu Date: 2023.01.27 13:15:54 -05'00'	

This form must be submitted electronically by email to: gradmath@yorku.ca.

graduate studies | YORK UNIVERSITÉ UNIVERSITY

Office of the Dean
York University

230 York Lanes — 4700 Keele Street
Toronto, ON, Canada M3J 1P3

Tel: (416) 736-5521
yorku.ca/gradstudies

Graduate Program in Mathematics and Statistics

Comprehensive Exam

Course information

Title Generalized linear models

Course number Math6622

Semester Winter

Exam is: ☑ Closed-book ☐ Open-Book

If open-book, provide a detailed rationale

Is Exam invigilated? ☑ Yes ☐ No

If No, provide a detailed rationale

Evaluation method

The Ph.D. students should obtain at least 60/100 to pass the comprehensive exam.

Office of the Dean
York University
230 York Lanes — 4700 Keele Street
Toronto, ON, Canada M3J 1P3
Tel: (416) 736-5521
yorku.ca/gradstudies

Graduate Program in Mathematics and Statistics

Comprehensive Exam

Does the duration of exam exceed 3 hours? If Yes, provide a detailed rationale	☑ Yes	☐ No

Math6622 final exam will last 3 hours. So the comprehensive exam with extra questions will last 4 hours.

Outline of topics to be covered

review of linear models; formulation of GLMs; link functions; iterative least squares algorithms; deviance and asymptotic theory; model selection; residuals and quasi-likelihood; generalized linear mixed models.

Passing threshold (% of the exam): 60%

Faculty	Signature	Date
Name Wei Liu		Jan 27, 2023

This form must be submitted electronically by email to: gradmath@yorku.ca.

graduate studies | YORK UNIVERSITÉ UNIVERSITY

Office of the Dean
York University

230 York Lanes — 4700 Keele Street
Toronto, ON, Canada M3J 1P3

Tel: (416) 736-5521
yorku.ca/gradstudies

Graduate Program in Mathematics and Statistics

Comprehensive Exam

Course information	
Title	Numerical Solutions to Differential Equations
Course number	Math 6652
Semester	Winter 2022-23

Exam is: ☑ Closed-book ☐ Open-Book

If open-book, provide a detailed rationale

Is Exam invigilated? ☑ Yes ☐ No

If No, provide a detailed rationale

Evaluation method

Three hour written examination. The exam will have 6 questions; four questions will be evaluated out of 15 marks while two questions will be evaluated out of 20 marks (total 100 marks). The exam will be written in person at the University and invigilated by the course instructor.

Office of the Dean
York University
230 York Lanes — 4700 Keele Street
Toronto, ON, Canada M3J 1P3
Tel: (416) 736-5521
yorku.ca/gradstudies

Graduate Program in Mathematics and Statistics

Comprehensive Exam

Does the duration of exam exceed 3 hours? ☐ Yes ☑ No If Yes, provide a detailed rationale

Outline of topics to be covered
Section 1: Finite Difference Formulations Section 2: Parabolic Equations in One Space Dimension Section 3: Parabolic Equations in Two and Three Space Dimensions Section 4: Stability Analysis Section 5: Linear Second Order Elliptic Equations in One and Two Space Dimensions Section 6: Hyperbolic Equations in One Space Dimension

Passing threshold (% of the exam): 60%

Faculty	Signature	Date
Name Michael Haslam	Michael Haslam Digitally signed by Michael Haslam Date: 2022.10.20 17:05:16 -04'00'	10/20/22

This form must be submitted electronically by email to: gradmath@yorku.ca.