Analyzing the Performance of Optimization Algorithms

Curse of Dimensionality

Have you ever noticed when doing a Google Search, that your search results start to deteriorate after you add a certain number of words to the search bar? This degradation of results with increasing search terms or dimensions is what we refer to as the Curse of Dimensionality. We are trying to show that this deterioration does not occur due only to increasing dimensions, but also with the change of "*attraction basin*" shape.

Attraction Basin Explained

A search space can have one or more peaks, and so it can have many local minimum points but only has one global minimum point. An attraction basin is all of the points around a local optimum that lead to that optimum when greedy local search is used. So one complete wave is one attraction basin.

Attraction Basin

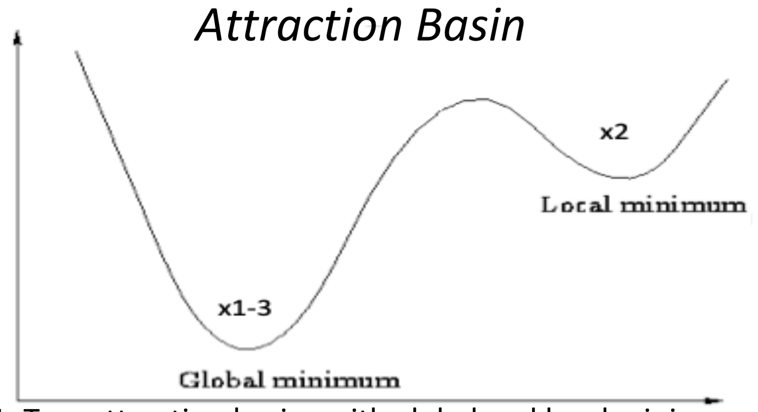


Fig. 1. Two attraction basins with global and local minimum. x1-3 creates a slope between the two attraction basins which makes one better than the other.

Change of "shape" of Attraction basins with Dimensionality

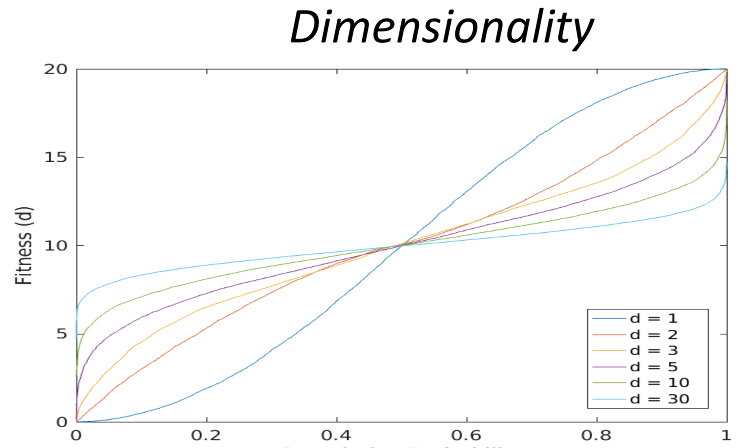
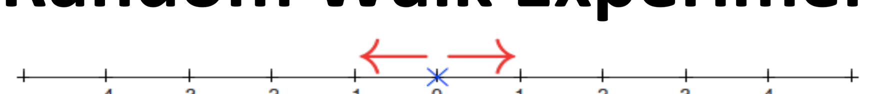


Fig. 2. With increasing dimensions, the distribution becomes more vertical at the edges that indicates that proportion of solutions with fitness close to zero reduces drastically.

Random Walk Experiment

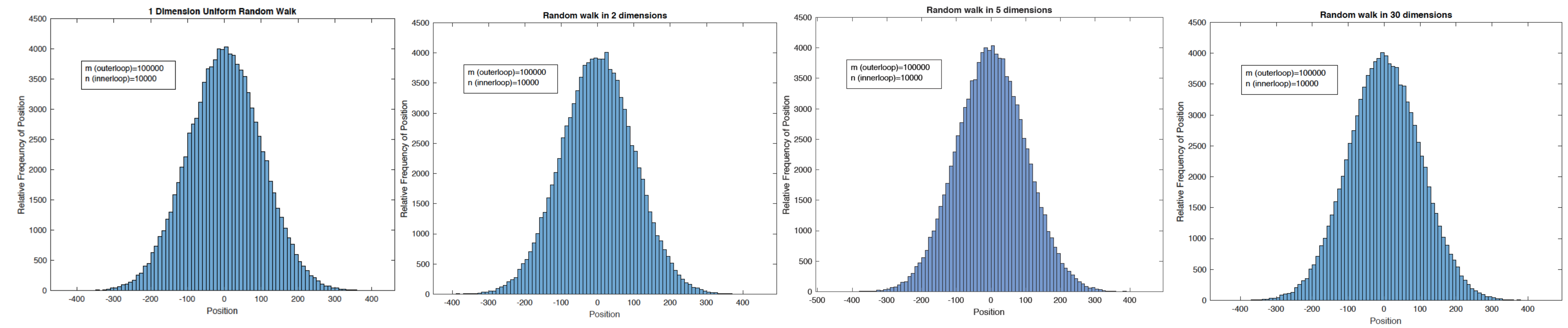


Our goal is to show that optimization algorithms behave like a "*Random Walk*", and that certain Random Walks in higher dimensions lead to better optimization performance. So what is a Random Walk?

Let X be a particle randomly moving along an axis. The initial position of X is 0, and it can only take a forward step (+1) or a backward step (-1). Two random numbers are generated namely x1 and x2. The random numbers are drawn from uniform random and sinusoidal random number generator (aka attraction basins) in 1,2,3,5,10,30 dimensions.

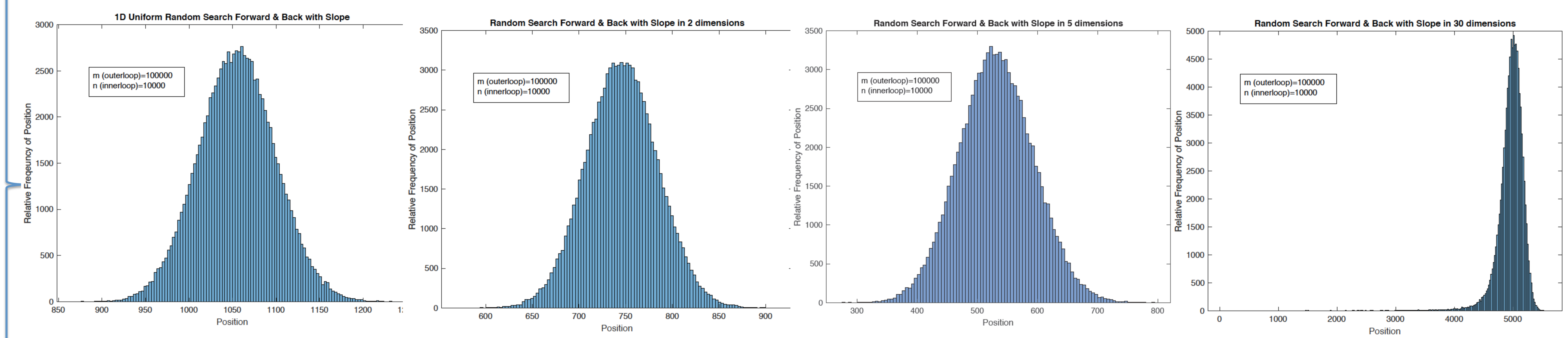
First set of experiments

- Takes a forward step if x2 < x1, otherwise takes a backward step.
- We see that all the histograms are the same. This shows that the behavior of random walk stays the same even with increasing dimensions.



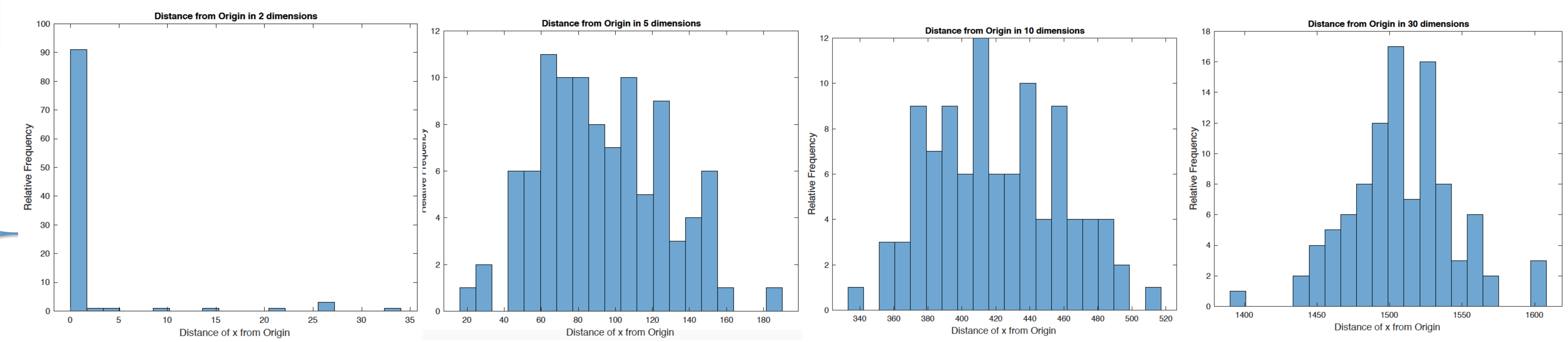
Second set of experiments

- Takes a forward step if (x1-3) < x2.
- Note that this condition subtracts 3 from x1 which creates a bias making x1 a better attraction basin than x2 (thus adding a slope to the attraction basin).
- A forward move represents a successful move because a point in the better attraction basin (x1) is selected and a backward move represents a deceptive move because even though the point selected is smaller, it is in a worse attraction basin (x2).
- We see that the number of successful steps taken decrease from 1 to 5 dimension and increase for 10 and 30 dimensions.
- Therefore, random walks with a bias to move forwards predict that optimization algorithms should be able to perform better in higher dimensions..



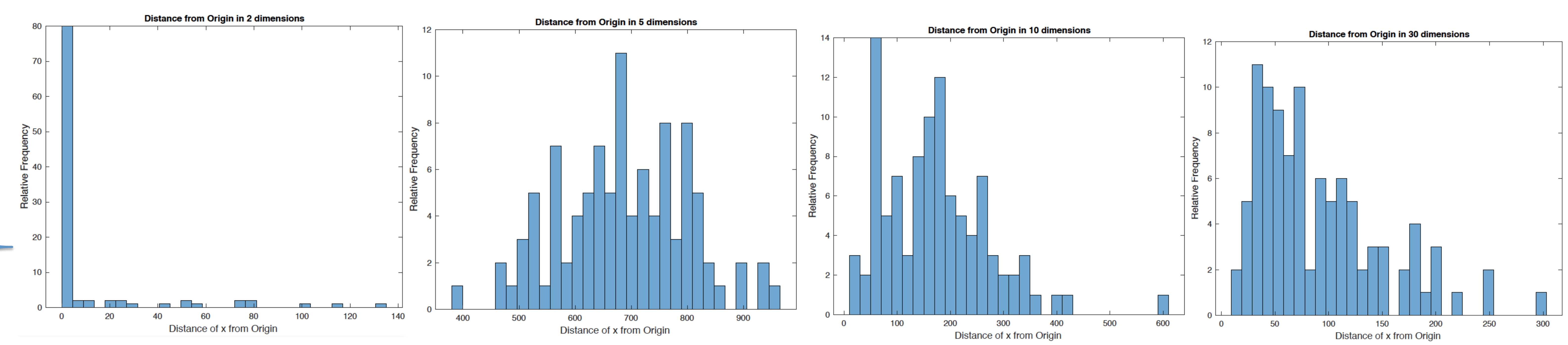
Simulated Annealing Curse of Dimensionality

- Simulated Annealing is an optimization algorithm. Since it is a minimization problem the algorithm is trying to look for the minimum number which is 0 for the function we are using which is Rastrigin.
- We can clearly see that as the dimensions increase the final solution gets further and further away from 0 (our global minimum). This is what we call performance degradation or curse of dimensionality.



Simulated Annealing Improving with Higher Dimensions like Random Walk Experiment

- We don't let the algorithm locally optimize by not allowing their step size to go below 1 (the size of an attraction basin), so that it takes steps like the Random walk experiment .
- We see that the solution gets further from 0, for 1 to 5 dimensions, and gets closer to 0, for 10 and 30 dimensions.
- Simulated Annealing mimic the behavior of Random Walk, improving with increasing dimensions.



Current Status

We have extended our research by taking ITEC4000, where we will be working with more optimization algorithms (e.g. Particle Swarm Optimization) and we have shared results with our international colleagues to work on a paper for the 2021 IEEE Congress on Evolutionary Computation.

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