

Fitts' Law

— An Overview of HCI's Most Enduring Model —

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Plan

- What is Fitts' law?
- Fitts' law as a human performance measure
- Fitts' law as a predictive model

Background

- Fitts' law is a model of human movement
- Paul Fitts, 1954^a, 1964^b (before HCI existed!)
- Derived by analogy from Shannon's work in information theory^c
- A model *by analogy*
- No basis in human neuroscience or physiology

^a Fitts, P. M., The information capacity of the human motor system in controlling the amplitude of movement, *Journal of Experimental Psychology*, 47, 1954, 381-391.

^b Fitts, P. M. and Peterson, J. R., Information capacity of discrete motor responses, *Journal of Experimental Psychology*, 67, 1964, 103-112.

^c Shannon, C. E. and Weaver, W., *The mathematical theory of communications*. Urbana, Il: University of Illinois Press, 1949.

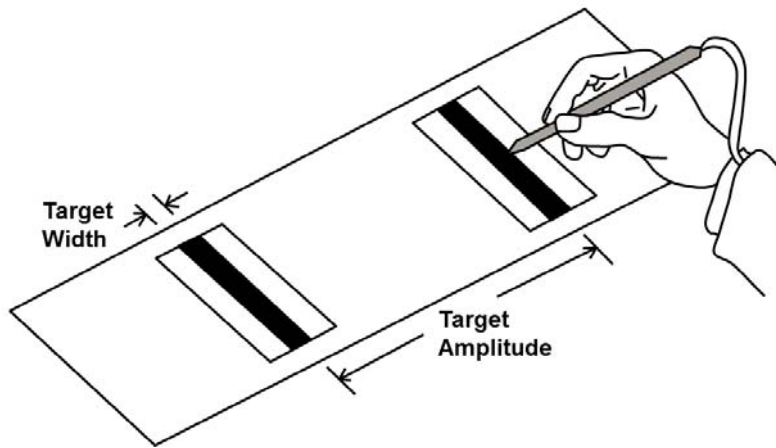
HCI Context

- First used by Card, English, and Burr (1978)^a
 - Mouse and joystick models
 - Motivated to “harden the science” of HCI
- HCI’s most widely used model
- Perennial “Fitts Session” at CHI conferences
- The de facto model for testing and evaluating devices/techniques for pointing/selecting
- Adopted in ISO 9241-9 (2000) and 9241-411 (2012)

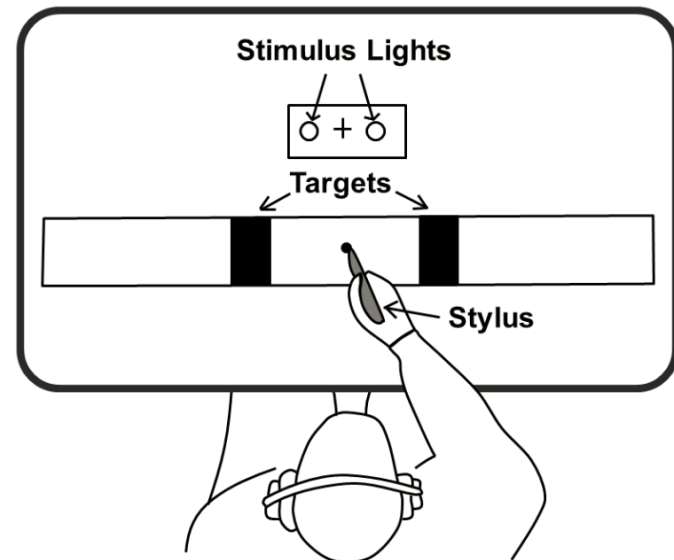
^a Card, S. K., English, W. K., and Burr, B. J., Evaluation of mouse, rate-controlled isometric joystick, step keys, and text keys for text selection on a CRT, *Ergonomics*, 21, 1978, 601-613.

Tasks

- Only for “rapid-aimed movements”
 - Target selection, point-select, touch-select ✓
 - Path following, tracing, handwriting ✗
- Two paradigms
 - Serial tasks
 - Discrete tasks

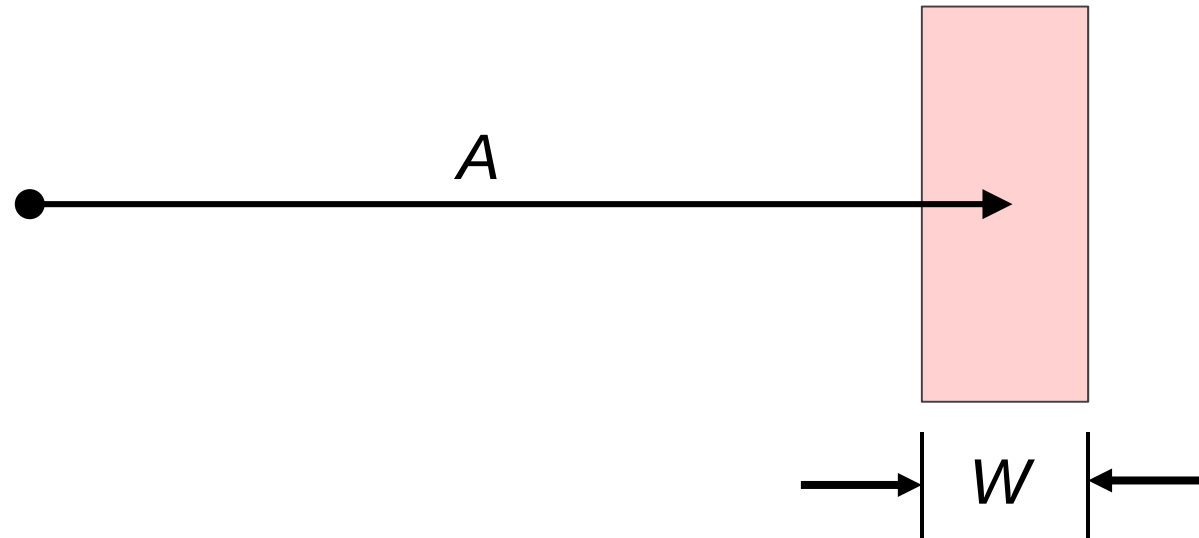


Serial task
(Fitts, 1954)



Discrete task
(Fitts & Peterson, 1964)

Fitts' Index of Difficulty (*ID*)

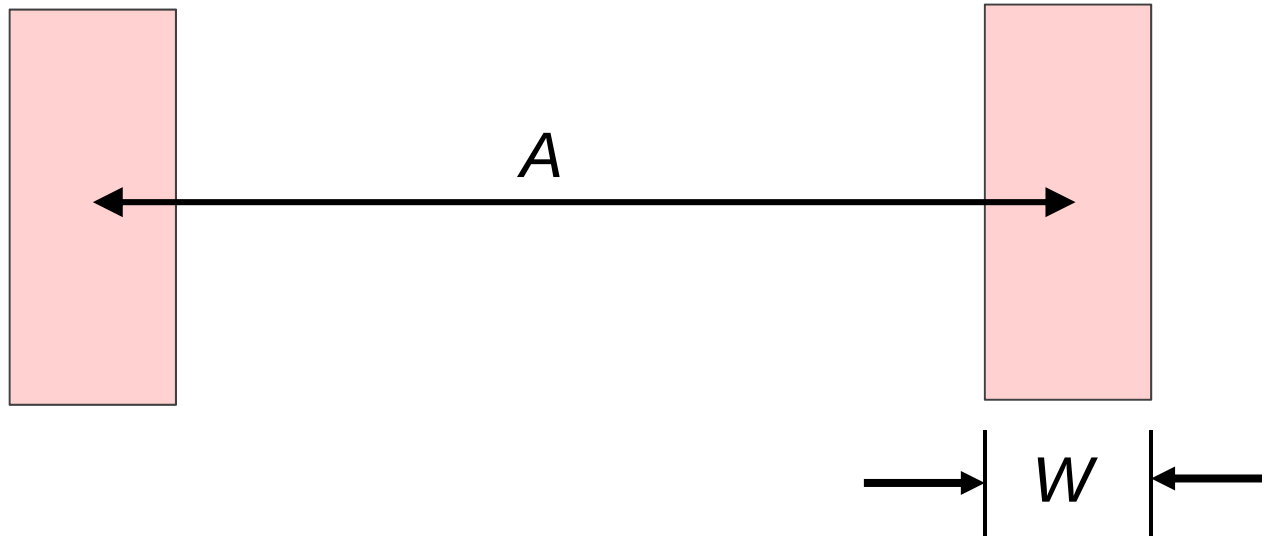


$$ID = \log_2\left(\frac{A}{W} + 1\right)$$

Units: “bits”

Fitts' Index of Difficulty (ID)

Serial Task

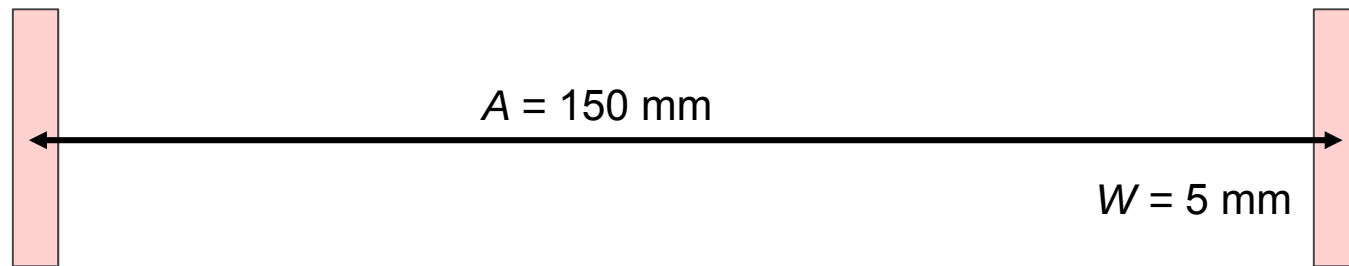


$$ID = \log_2\left(\frac{A}{W} + 1\right)$$

Units: "bits"

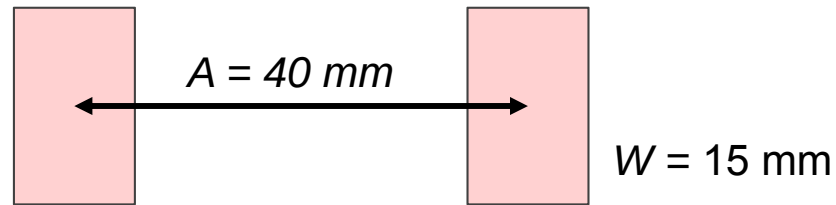
ID Examples

“Hard”



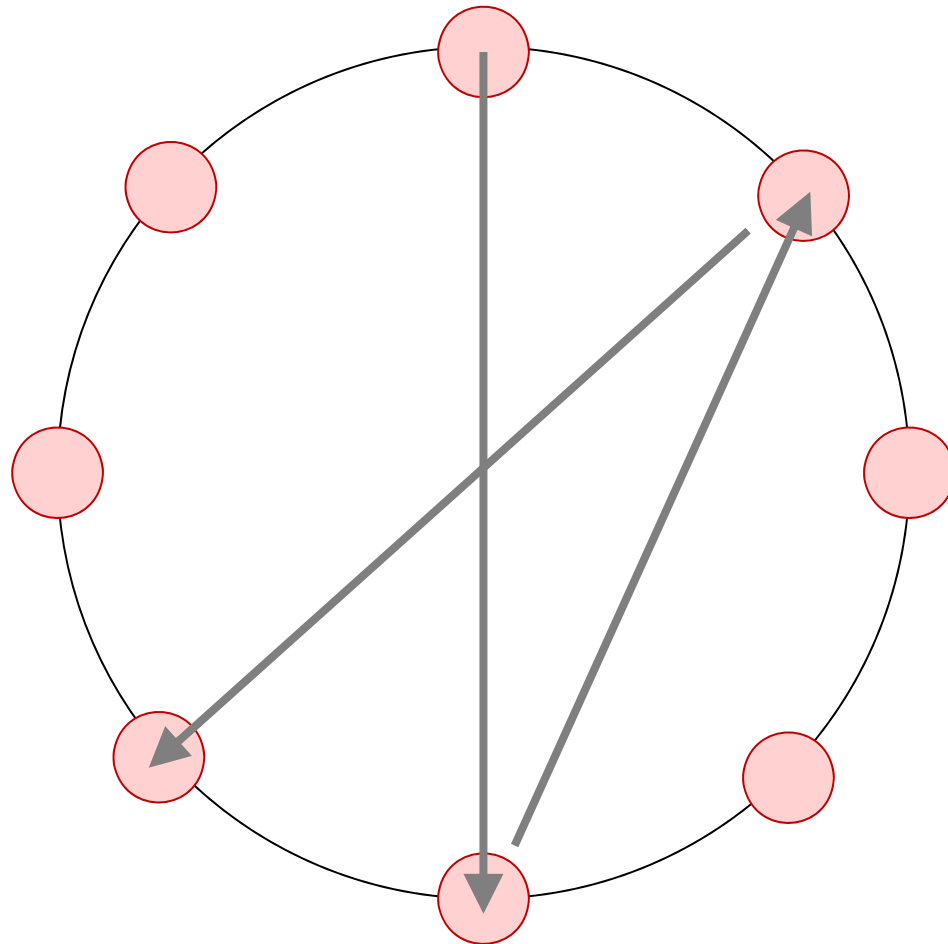
$$ID = \log_2\left(\frac{150}{5} + 1\right) = 4.9 \text{ bits}$$

“Easy”

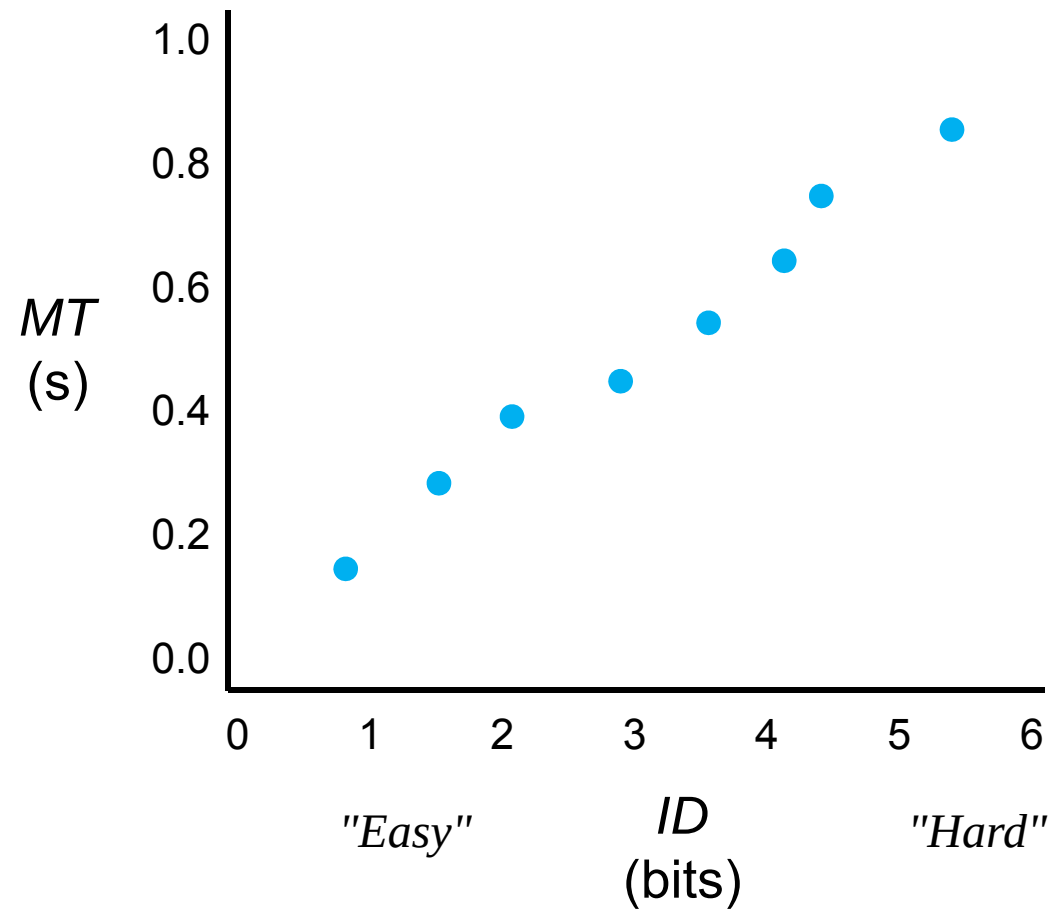


$$ID = \log_2\left(\frac{40}{15} + 1\right) = 1.9 \text{ bits}$$

2D Tasks



Movement Time (MT) vs. ID



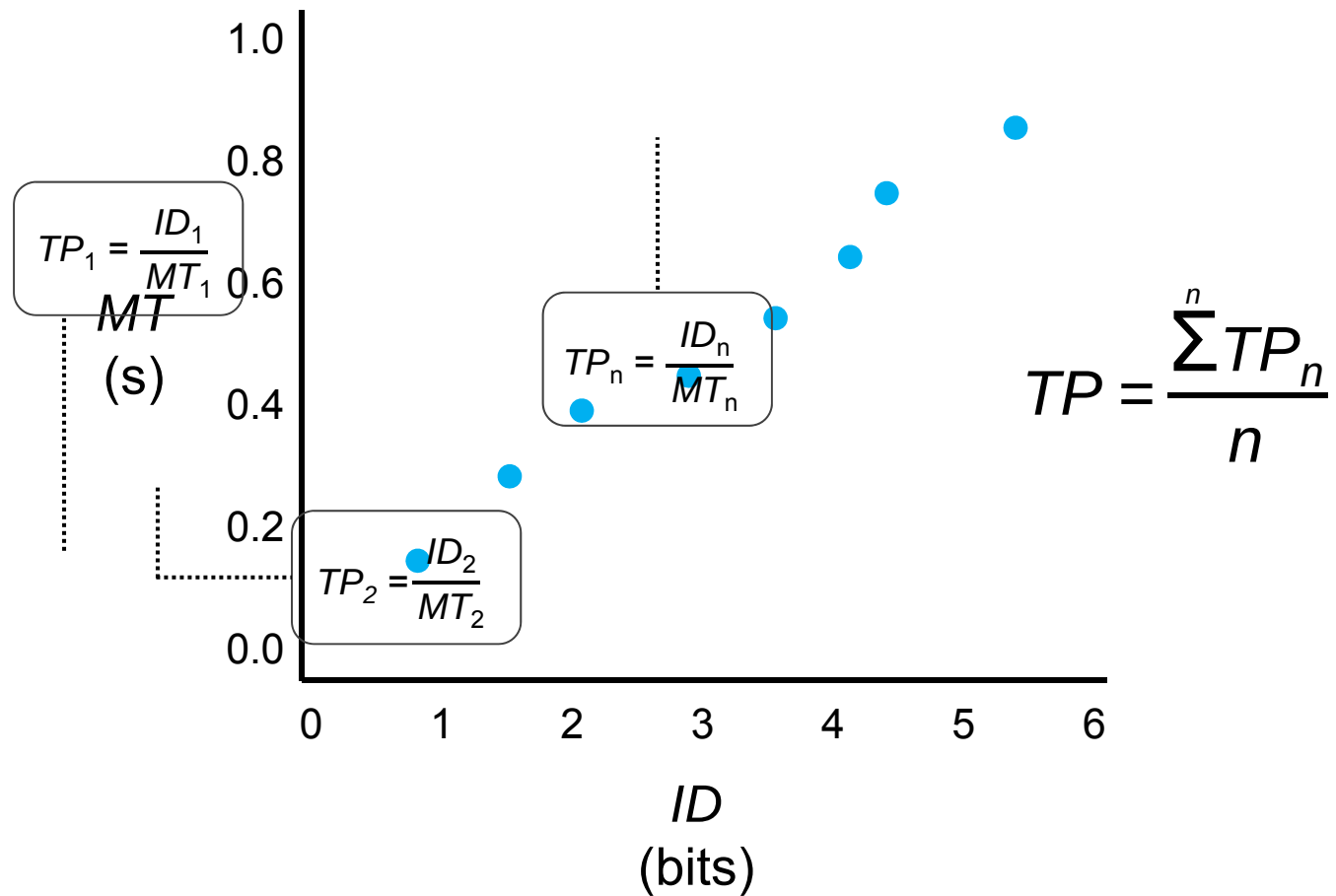
Throughput (TP)

- Fitts' proposed that the ID - MT ratio would be constant across a wide range of ID s
- The MT - ID ratio is the “human rate of information processing” in bits per second
- Fitts' terms:
 - IP – *Index of Performance* (Fitts, 1954)
 - C – *Information Capacity* (Fitts & Peterson, 1964)
- Today:
 - TP – *Throughput*

The diagram shows the formula $TP = \frac{ID}{MT}$ with units in boxes. A box labeled 'Throughput' is connected to the TP term. A box labeled 'Bits' is connected to the ID term. A box labeled 'Seconds' is connected to the MT term.

$$\text{Throughput} \quad TP = \frac{ID}{MT} \quad \begin{matrix} \text{Bits} \\ \text{Seconds} \end{matrix}$$

Calculation of Throughput

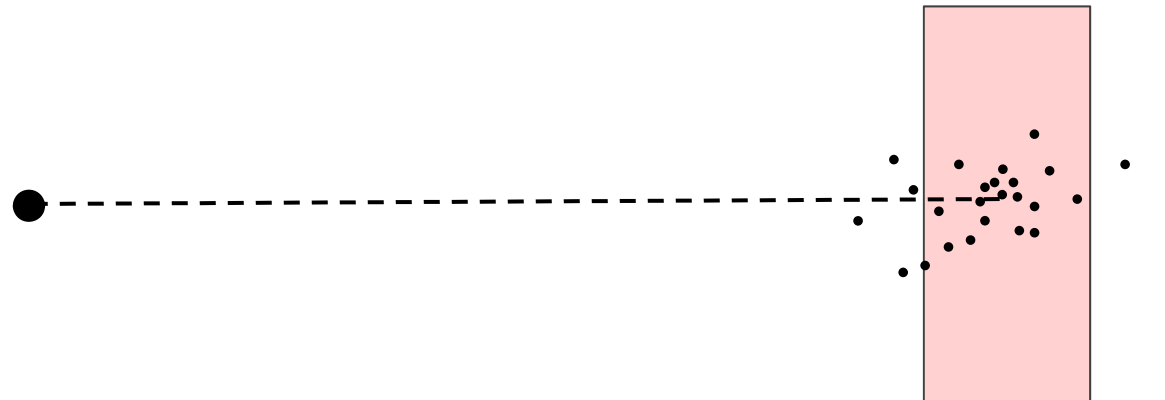


Adjustment for Accuracy

- Throughput is adjusted to include accuracy
- Conceptually:

$$\text{Speed} + \text{Accuracy} = \text{Throughput}$$

Selection Coordinates



25 trial repetitions

Including Accuracy

- Transform W into W_e
- $W_e \rightarrow$ the “effective target width”
- $W_e \rightarrow$ the width of the distribution of selections
- W is what participants were asked to do
- W_e is what participants actually did

Effective Target Width

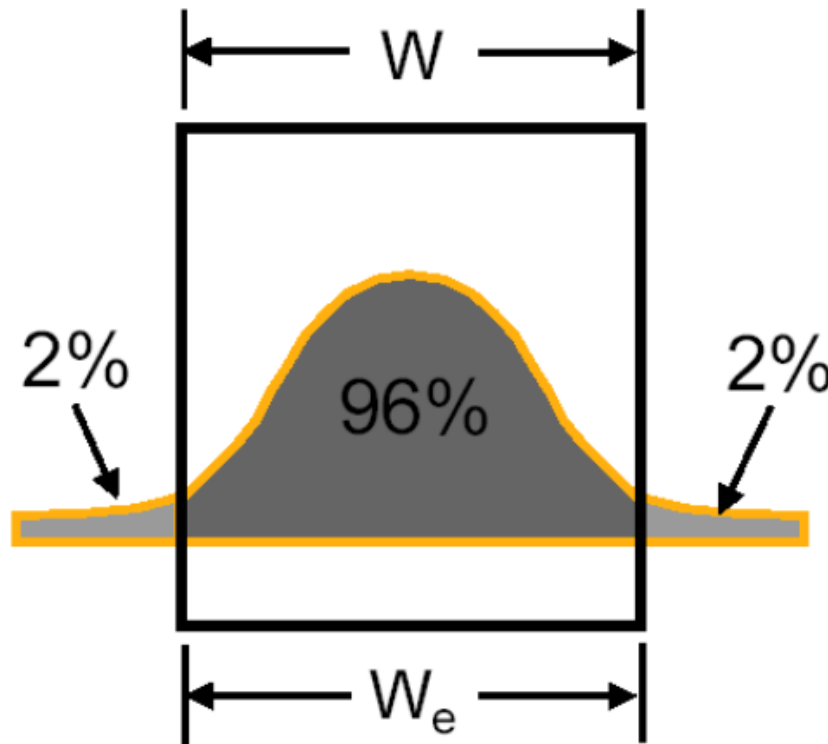
$$W_e = 4.133 SD_x$$

- SD_x – standard deviation in selection coordinates along axis of approach
- W_e is the central 96% of the spatial distribution
- Assumption: distribution is normal (generally true)
- Nominal error rate is 4%
- $H = \sqrt{2\pi e} \sigma = 4.133 \sigma$
- For full details, see MacKenzie (1992^a, 2018^b)

^a MacKenzie, I. S. (1992). Fitts' law as a research and design tool in human-computer interaction. *Human-Computer Interaction*, 7, 91-139.

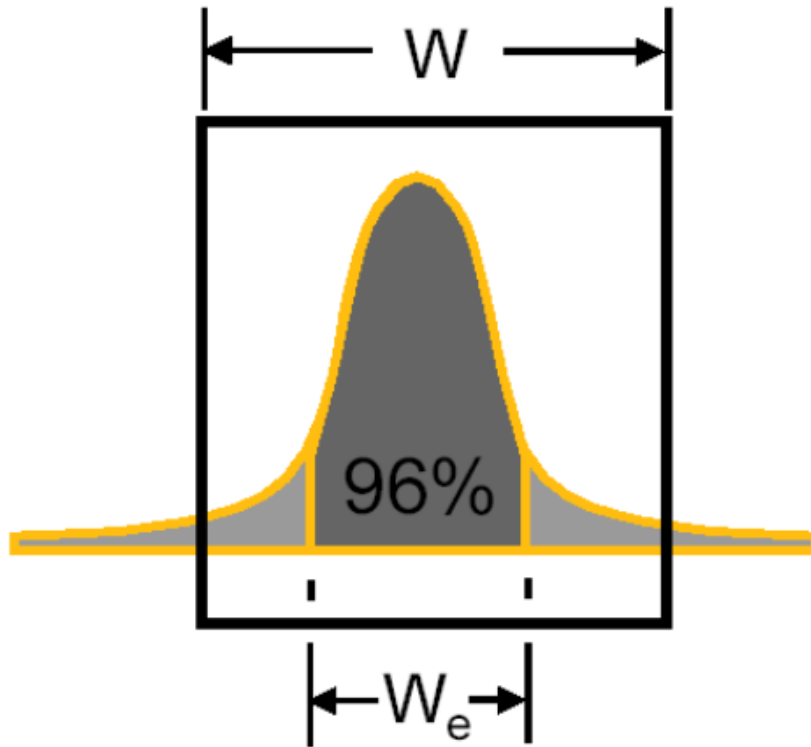
MacKenzie, I. S. (2018). Fitts' law. In K. L. Norman & J. Kirakowski (Eds.) *Handbook of human-computer interaction*, pp. 349-370. Hoboken, NJ: Wiley.

4% Errors



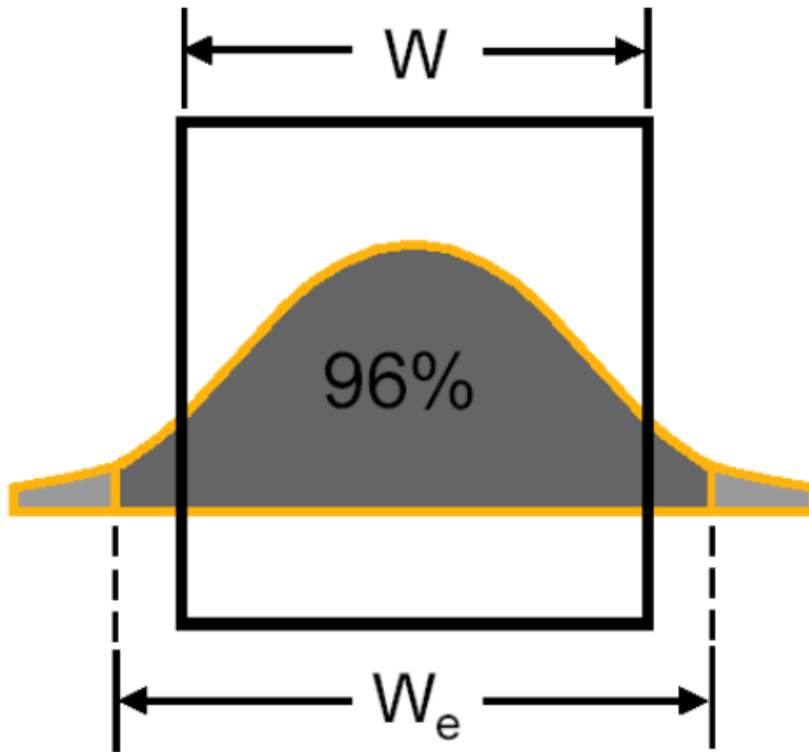
$$W_e = W$$

Less Than 4% Errors



$$W_e < W$$

Greater Than 4% Errors



$$W_e > W$$

“Effective” Index of Difficulty

$$ID_e = \log_2(A_e / W_e + 1)$$

- ID is the difficulty of the task presented to the participants
- ID_e is the difficulty of the task achieved by the participants
- ID_e reflects what participants actually did
- Should use A_e instead of A (in case participants do not “go the distance” or to avoid “cheating”)

Throughput (Revised)

$$TP = \frac{\log_2 \left(\frac{A_e}{4.133 \times SD_x} + 1 \right)}{MT}$$

The diagram illustrates the components of the revised throughput formula. Three pink ovals at the bottom are labeled 'Throughput', 'Speed', and 'Accuracy'. A red arrow points from 'Throughput' to the 'TP' variable in the numerator. Another red arrow points from 'Speed' to the 'MT' variable in the denominator. A third red arrow points from 'Accuracy' to the 'SD_x' variable in the denominator.

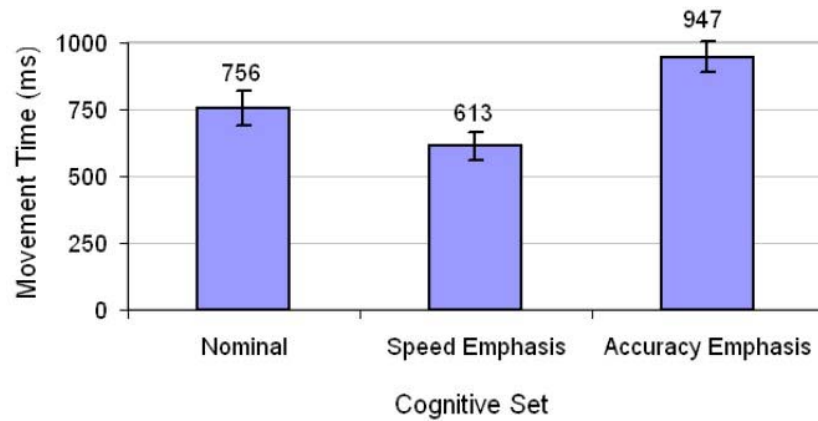
Research Question

- Since throughput includes *speed* and *accuracy*, we ask...

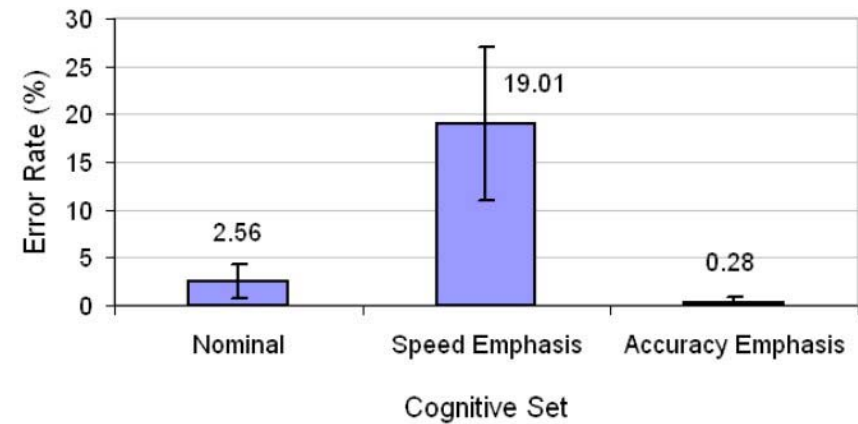
Is throughput independent of the speed-accuracy tradeoff? (all else being equal)

- If the answer is “yes”, this strengthens throughput as a performance measure because changes in speed and accuracy tend to *tradeoff* (i.e., cancel out).

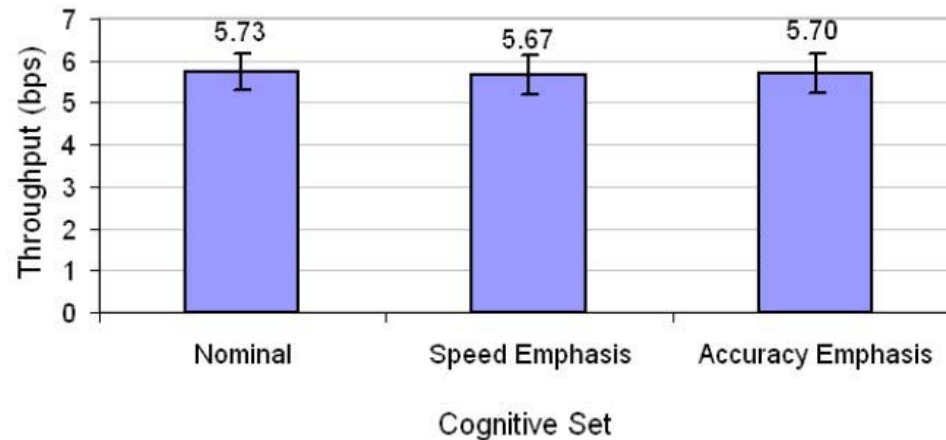
Speed ^a



Accuracy ^a



Throughput ^a



($F_{2,34} = 0.149$, ns)

^a MacKenzie, I. S., and Isokoski, P. (2008). Fitts' throughput and the speed-accuracy tradeoff. *Proc CHI 2008*, pp. 1633-1636. New York: ACM.

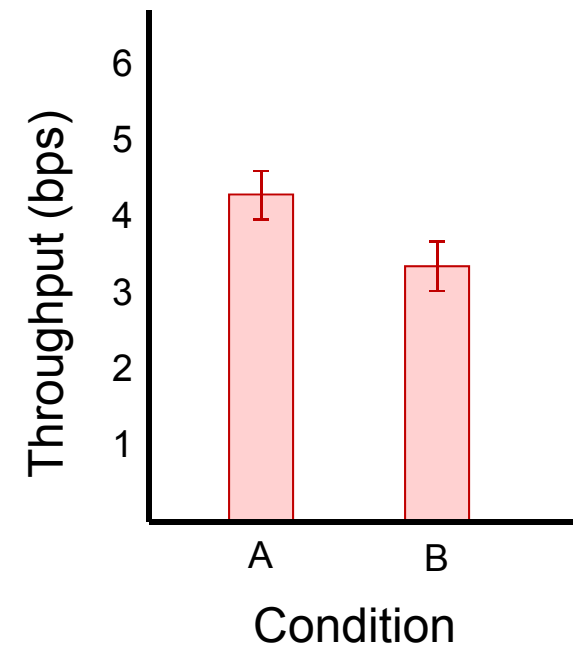
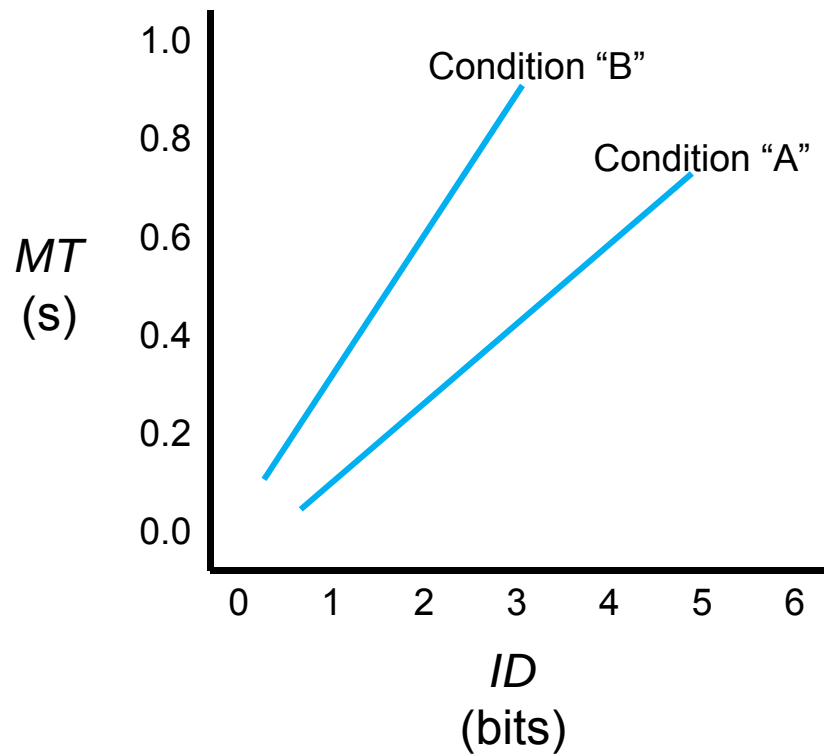
Plan

- What is Fitts' law
- Fitts' law as a human performance measure
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Throughput

- Throughput (see previous slides) quantifies the performance of a human using a device to perform target selection tasks in a particular context and environment
- Best used for “comparative evaluations” (next slide)

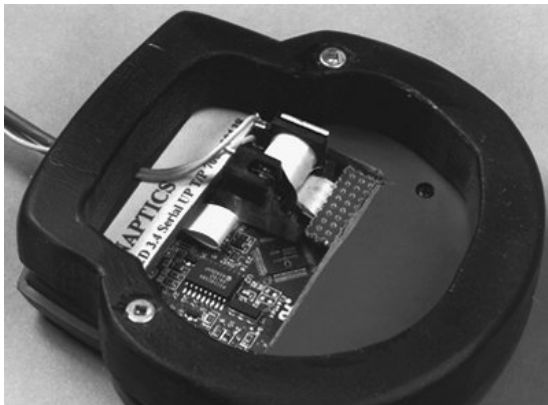
Comparative Evaluations



Three Examples

1. Touchpad with tactile feedback
2. Eye tracking
3. Touch selection

Example #1: The Tactile Touchpad^a



- Selection techniques
 - Button
 - Position cursor, then press and release physical button above touchpad
 - Lift-and-tap
 - Position cursor, then raise finger and quickly tap touchpad
 - Tactile
 - Position cursor, then press harder
 - Relay activates when pressure crosses a threshold
 - Relay provides critical tactile feedback

^a MacKenzie, I. S., & Oniszczak, A. (1998). A comparison of three selection techniques for touchpads. *Proc CHI '98*, pp. 336-343. New York: ACM.



<https://www.youtube.com/watch?v=fxfu-Yo6yEk>

Experiment Details

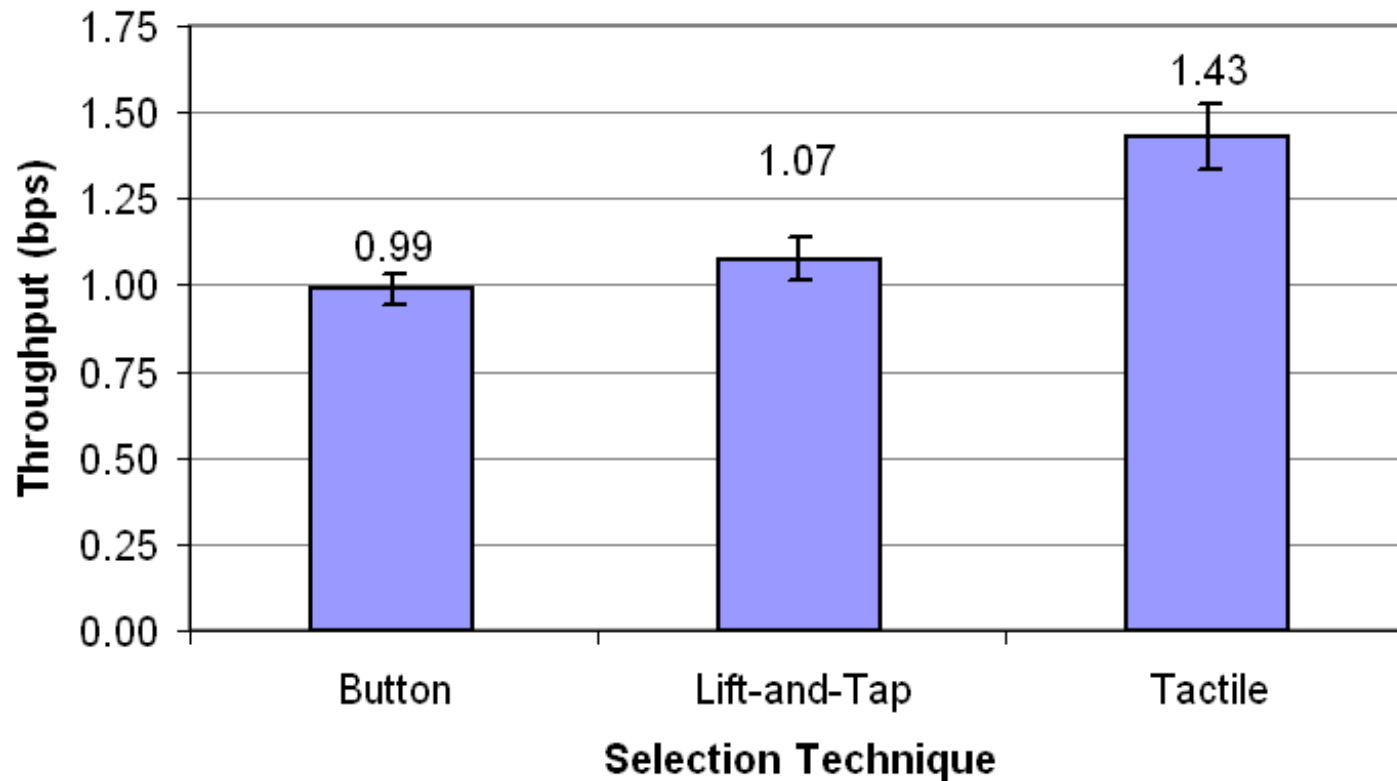
- 12 participants
- Independent variables
 - Selection technique, Amplitude, Width, Block
- Dependent variables
 - Throughput, etc.
- Task
 - Fitts 1D, serial (20 trials per block)
- Trials
 - $12 \times 3 \times 3 \times 3 \times 20 = 19,440$

Experiment Details

- 12 participants
- Independent variables
 - Selection technique, Amplitude, Width, Block
- Dependent variables
 - Throughput, error
- Task
 - Fitts 1D, serial (20 trials per block)
- Trials
 - $12 \times 3 \times 3 \times 3 \times 20 = 19,440$

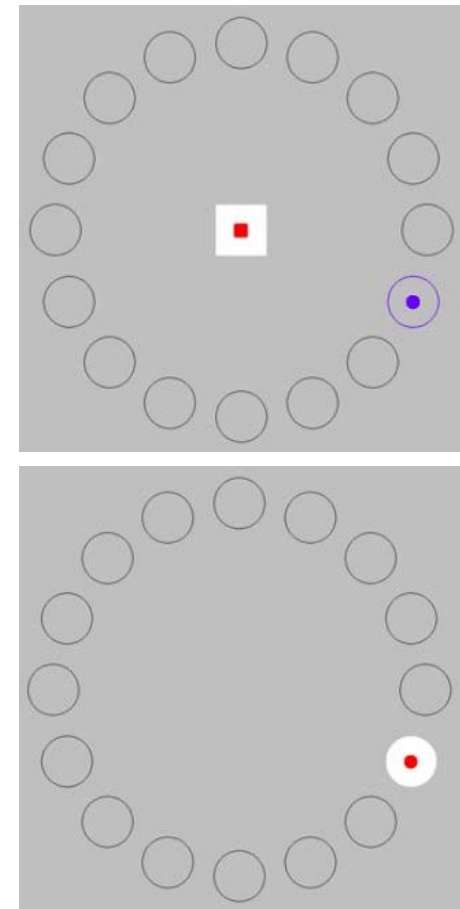
- 
- 
- Button
 - Lift-and-tap
 - Tactile

Results - Throughput



$(F_{2,18} = 18.0, p < .0001)$

Example #2: Eye Tracking^a



^a Zhang, X., and MacKenzie, I. S. (2007). Evaluating eye tracking with ISO 9241 – Part 9. *Proc HCII 2007*, pp. 779-788. Heidelberg: Springer.

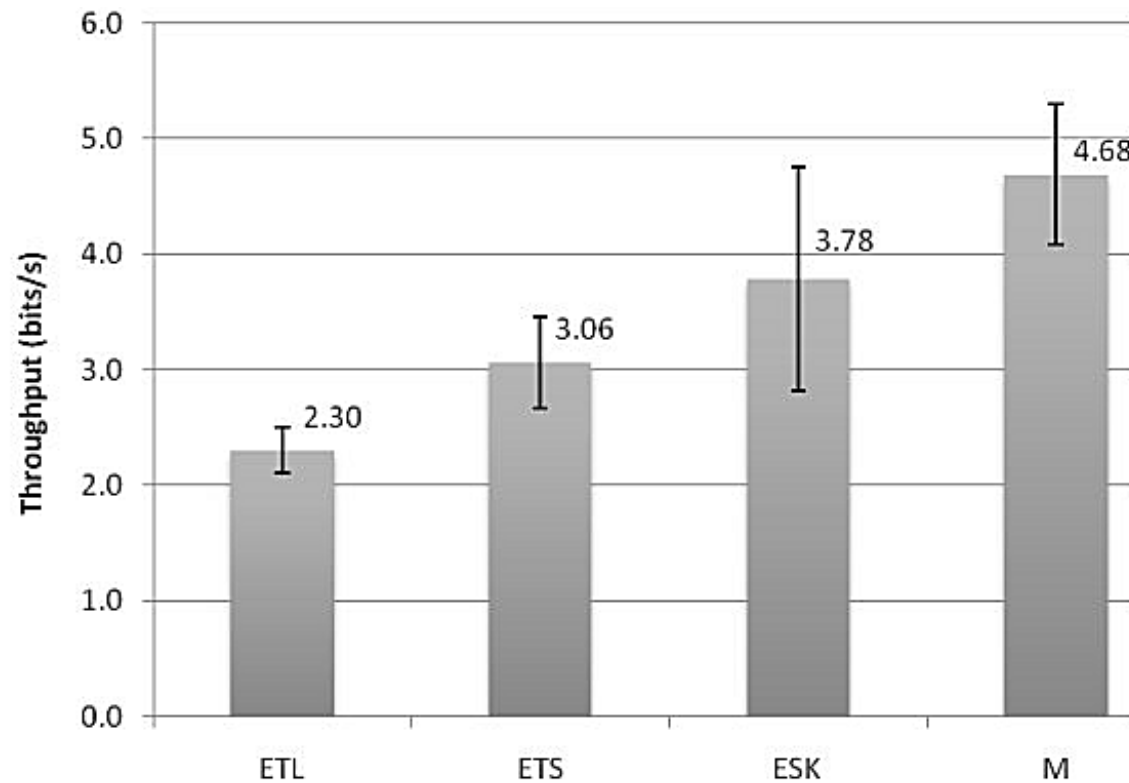
Experiment Details

- 16 participants
- Independent variables
 - Interaction technique, Amplitude, Width, Block

- 
- ETL - Eye tracker, long dwell time select (750 ms)
 - ETS - Eye tracker, short dwell time select (500 ms)
 - ETK - Eye tracker, key press to select
 - M - Mouse

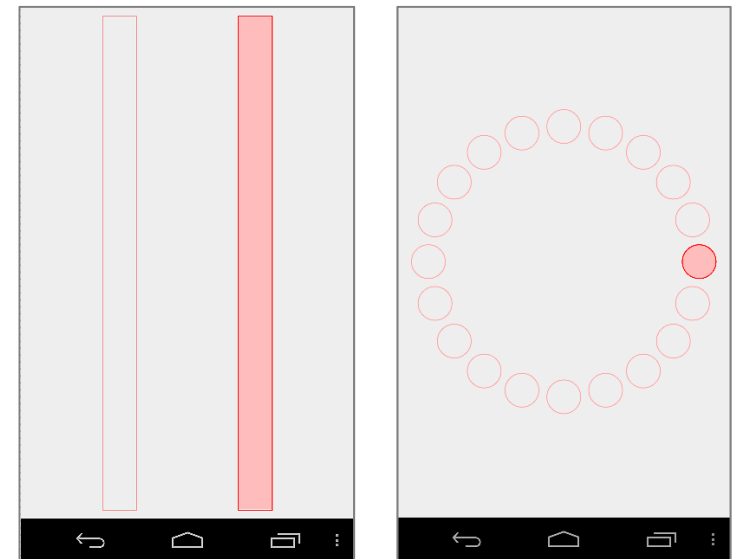
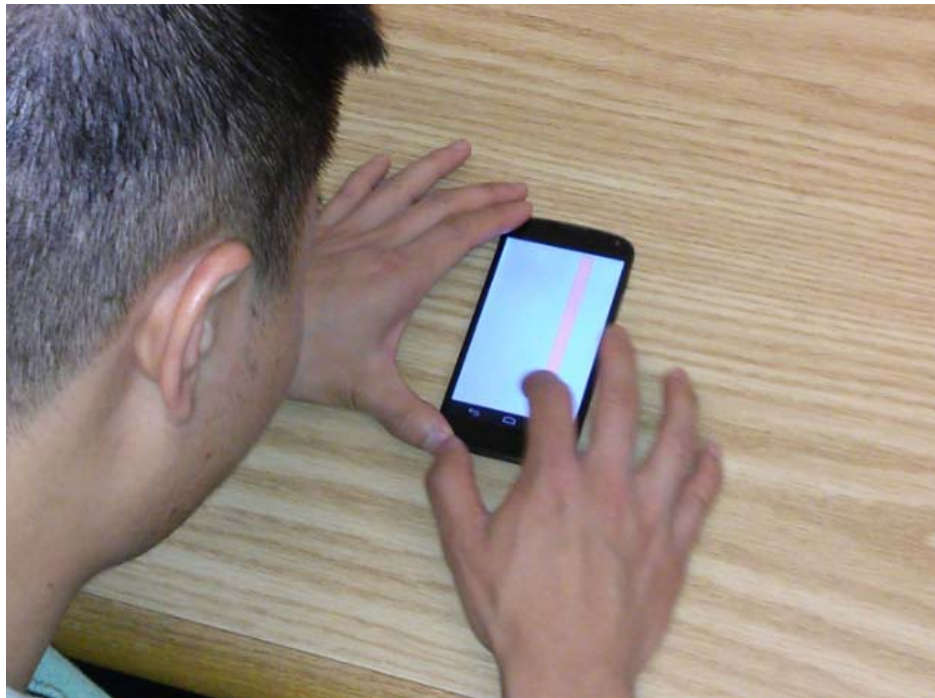
- Dependent variables
 - Throughput, etc.
- Task
 - Fitts 2D, discrete (16 trials per block)
- Trials
 - $16 \times 4 \times 2 \times 2 \times 16 = 4,096$

Results - Throughput



$$(F_{3,45} = 47.46, p < .0001)$$

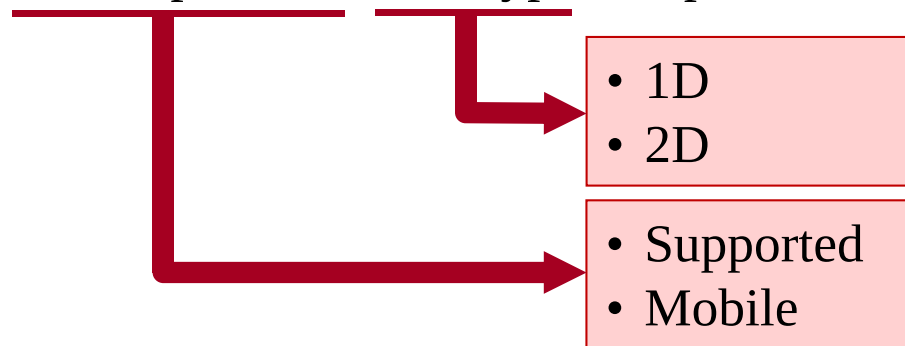
Example #3: Touch Selection^a



^a MacKenzie, I. S. (2015). Fitts' throughput and the remarkable case of touch-based target selection. *Proceedings of HCII 2015* (LNCS 9170), pp. 238-249. Switzerland: Springer.

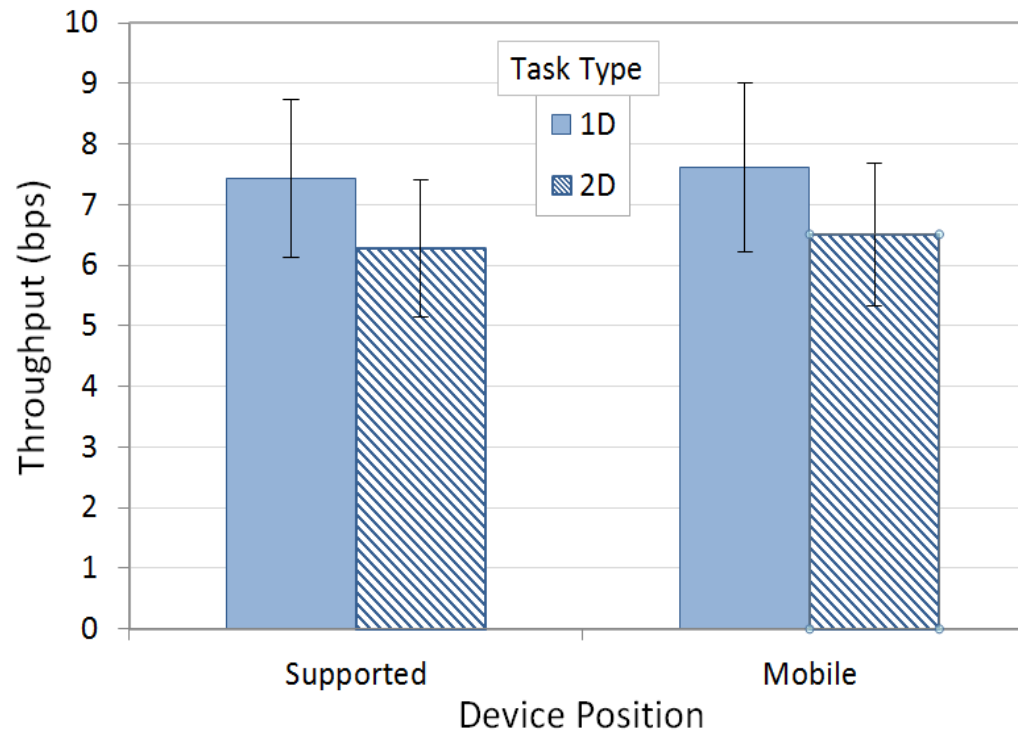
Experiment Details

- 16 Participants
- Apparatus: Google *Nexus 4*
- Independent variables
 - Device position, Task type, Amplitude, Width, Block



- Dependent variables
 - Throughput, etc.
- Trials
 - $16 \times 2 \times 2 \times 5 \times 3 \times 2 \times 20 = 38,400$

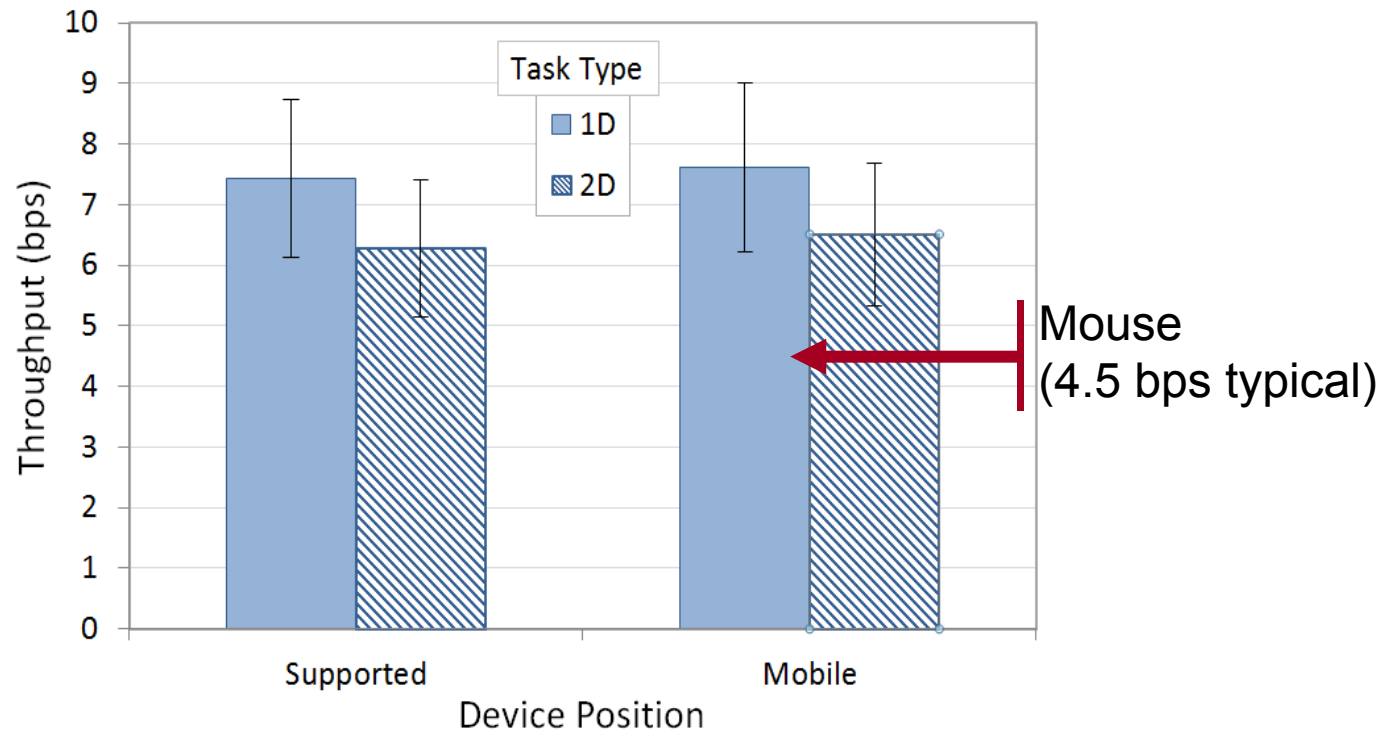
Results - Throughput



(Device position: $F_{1,12} = 2.80$, $p > .05$)

(Task type: $F_{1,12} = 29.6$, $p < .0001$)

Wow! (Touch >> Mouse)



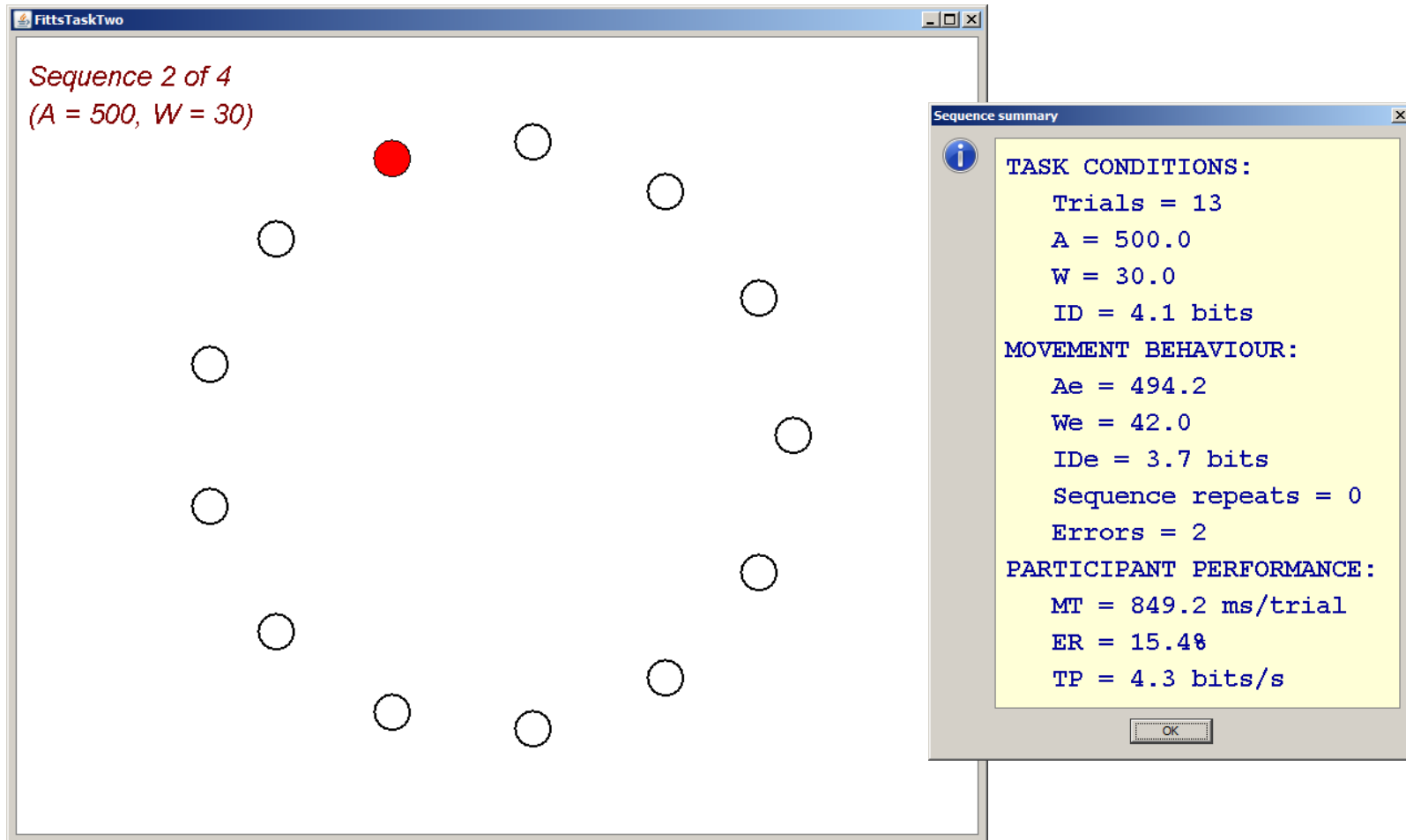
(Device position: $F_{1,12} = 2.80, p > .05$)

(Task type: $F_{1,12} = 29.6, p < .0001$)

Summary (Part I)

- Many, many studies similar to these
- Since 2000, the methodology is enshrined in an ISO standard (but, still, the details are sketchy!)
- ISO 9241-9:
 - *Ergonomic design of visual display terminals (VDTs) used for office work: Requirements for non-keyboard input devices*
- Annex B
 - *Performance testing*
- Updated in 2012 as ISO/TC 9241-411

Demo



Free Download – FittsTaskTwo



Human-Computer Interaction: An Empirical Research Perspective

[I. Scott MacKenzie](#)

York University, Toronto, Canada

Last update: November 27, 2017



Web resources for the book will be posted here. To see the complete Table of Contents, click the cover-image above. You can purchase the book through [Amazon](#) or the publisher, [Elsevier](#).

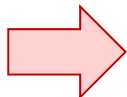
Page numbers appear below in parentheses. These refer to the location in the book where the software or other information in the file is discussed.

The zip files below contain Java applications, including source files and supporting files for the APIs. Instructions for running the applications, etc., are found in the APIs. Refer as well to the page in the book where the application is presented.

Experiment Software

Note: The latest version of the software below is on the web site for my experiment software: [click here](#)

- [LetterGuessingExperiment.zip](#) (p. 68) ([API](#))
- [GraffitiExperiment.zip](#) (p. 69) ([API](#))
- [ReactionTimeExperiment.zip](#) (p. 70) ([API](#))
- [PhoneKeypadExperiment.zip](#) (p. 270) ([API](#))
- [FittsTaskTwo](#) (p. 291) ([API](#))
- [TypingTestExperiment.zip](#) (p. 317) ([API](#))
- [SoftKeyboardExperiment.zip](#) (p. 317) ([API](#))



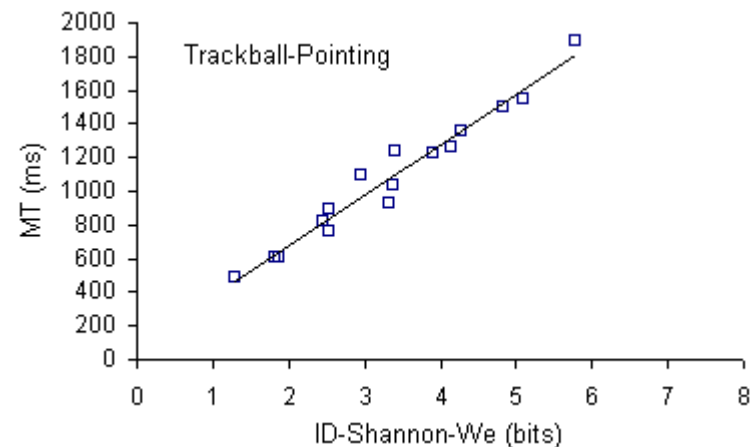
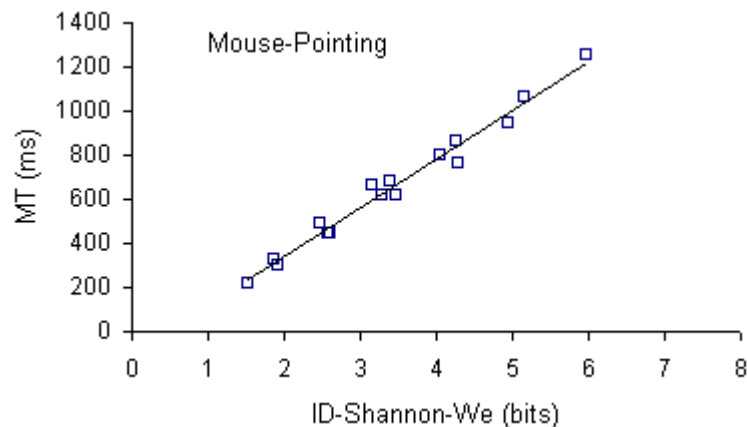
<http://www.yorku.ca/mack/HCIbook/>

Plan

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 - Fitts' law as a human performance measure
 - Fitts' law as a predictive model
- 

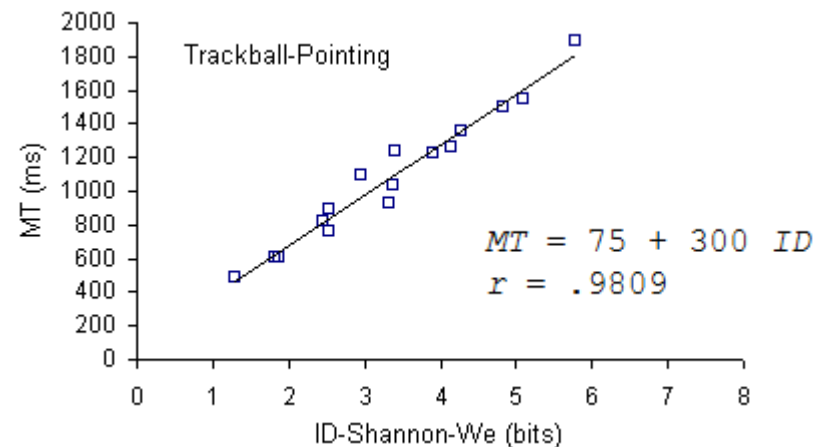
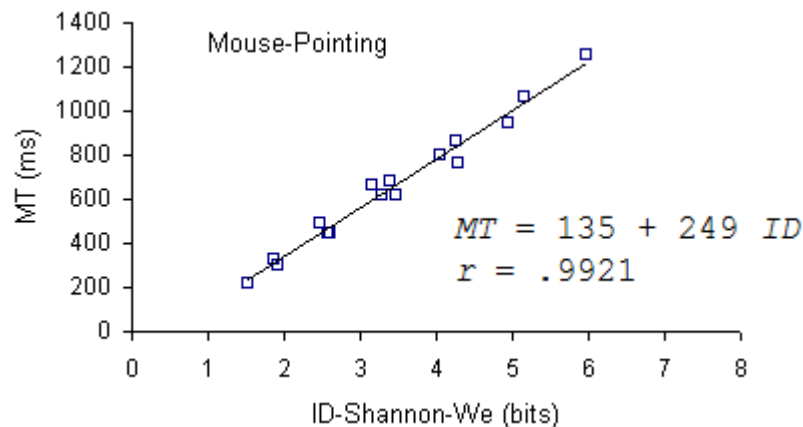
Fitts' Law as a Predictive Model

- Step 1:
 - Conduct an experiment to collect data
 - Record *MT* and selection coordinates over multiple *IDs*, over multiple test conditions (e.g., finger vs. stylus), etc.
 - Illustrate in scatter plots



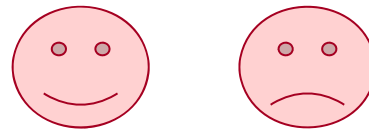
Fitts' Law as a Predictive Model

- Step 2:
 - Build prediction equations using least squares linear regression
 - Check r or R^2 for “goodness of fit”



Fitts' Law as a Predictive Model

- Step 3:
 - Pat self on back
 - Write it up
 - Submit to conference
 - Wait for notification
 - Read referee comments, scratch head, utter expletives
 - Etc.



Example #1: Remote Pointing

- Comparison of four pointing devices, including two for remote pointing^a



Interlink RemotePoint



Microsoft Mouse 2.0

^a MacKenzie, I. S., & Jusoh, S. (2001). An evaluation of two input devices for remote pointing. *Proc EHCI 2000*, 235-249, Heidelberg, Berlin: Springer.

Observations and Data

Conditions			Mouse Observations				RemotePoint Observations			
<i>A</i> (pixels)	<i>W</i> (pixels)	<i>ID</i> (bits)	Mouse				RemotePoint			
			<i>W_e</i> (pixels)	<i>ID_e</i> (bits)	<i>MT</i> (ms)	<i>TP</i> (bits/s)	<i>W_e</i> (pixels)	<i>ID_e</i> (bits)	<i>MT</i> (ms)	<i>TP</i> (bits/s)
40	10	2.32	11.23	2.19	665	3.29	13.59	1.98	1587	1.25
40	20	1.58	19.46	1.61	501	3.21	21.66	1.51	1293	1.17
40	40	1.00	40.20	1.00	361	2.76	37.92	1.04	1001	1.04
80	10	3.17	10.28	3.13	762	4.11	10.08	3.16	1874	1.69
80	20	2.32	18.72	2.40	604	3.97	25.21	2.06	1442	1.43
80	40	1.58	35.67	1.70	481	3.53	37.75	1.64	1175	1.40
160	10	4.09	10.71	3.99	979	4.08	10.33	4.04	2353	1.72
160	20	3.17	21.04	3.11	823	3.77	19.09	3.23	1788	1.81
160	40	2.32	41.96	2.27	615	3.69	35.97	2.45	1480	1.65
<i>Mean</i>			23.25	2.38	644	3.60	23.51	2.35	1555	1.46

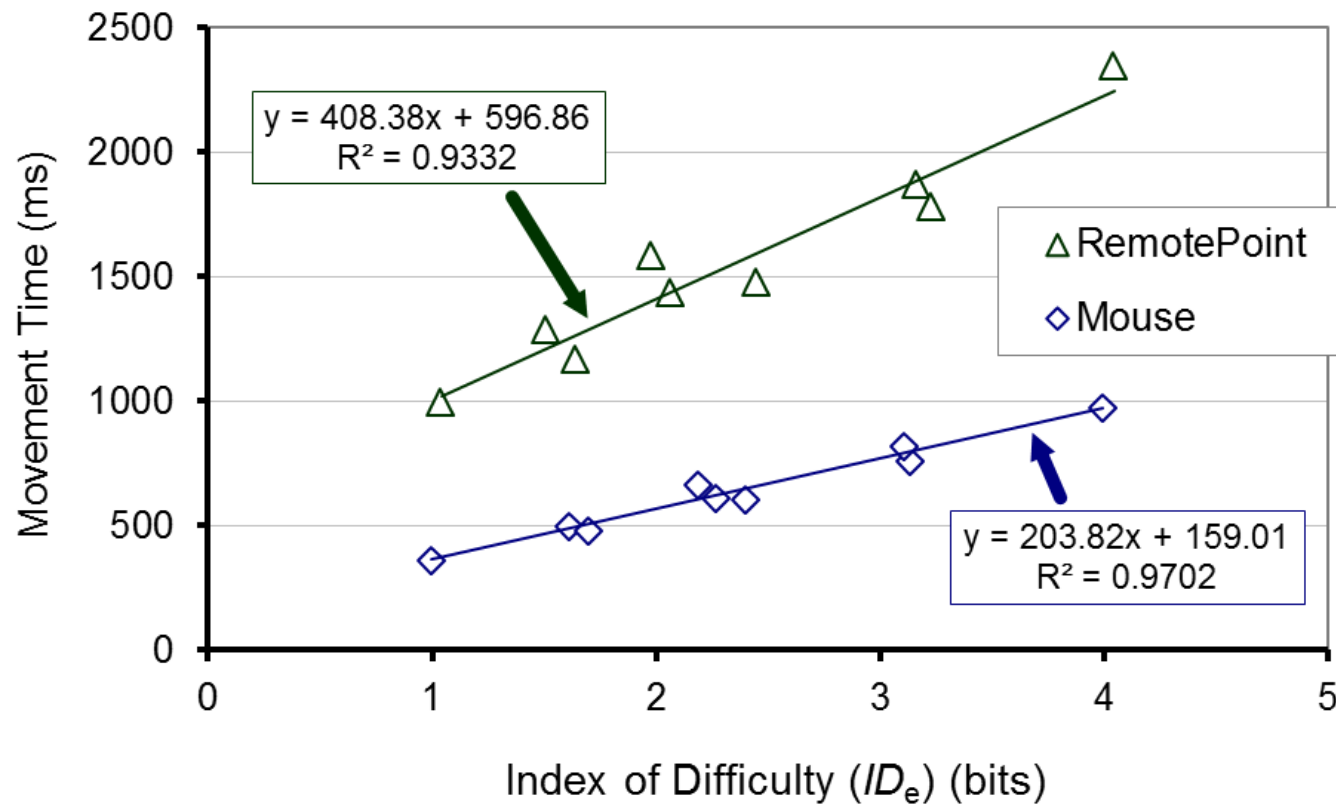
For model
building...



x sample points
y sample points

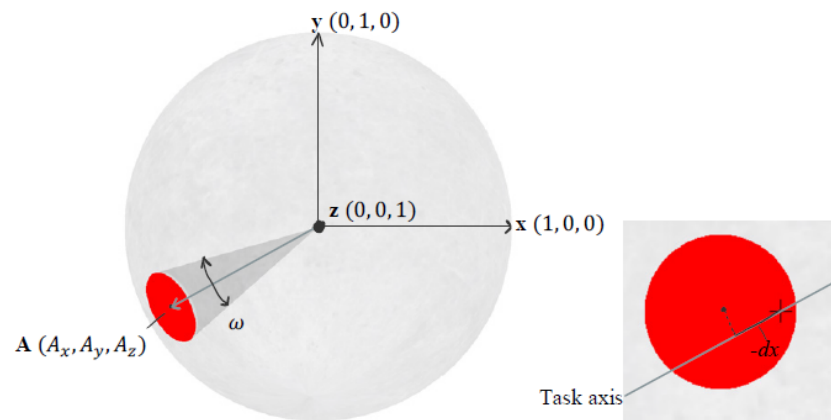
x sample points
y sample points

Fitts' Law Prediction Equations

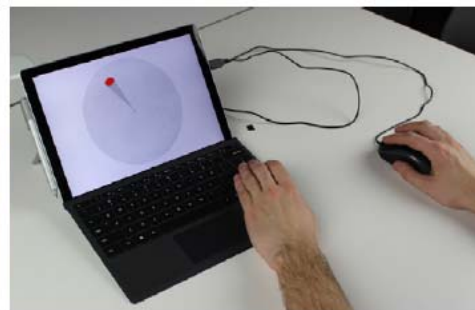


Example #2: 3D Rotation

- Extension of Fitts' law to 3D rotation tasks^a



Mobile phone accelerometer



Mouse



Joytick (gamepad)

^a DeLong, S., and MacKenzie, I. S. (in press). Evaluating devices for object rotation in 3D. *Proceedings of HCII 2018*. Berlin, Springer.

Observations and Data

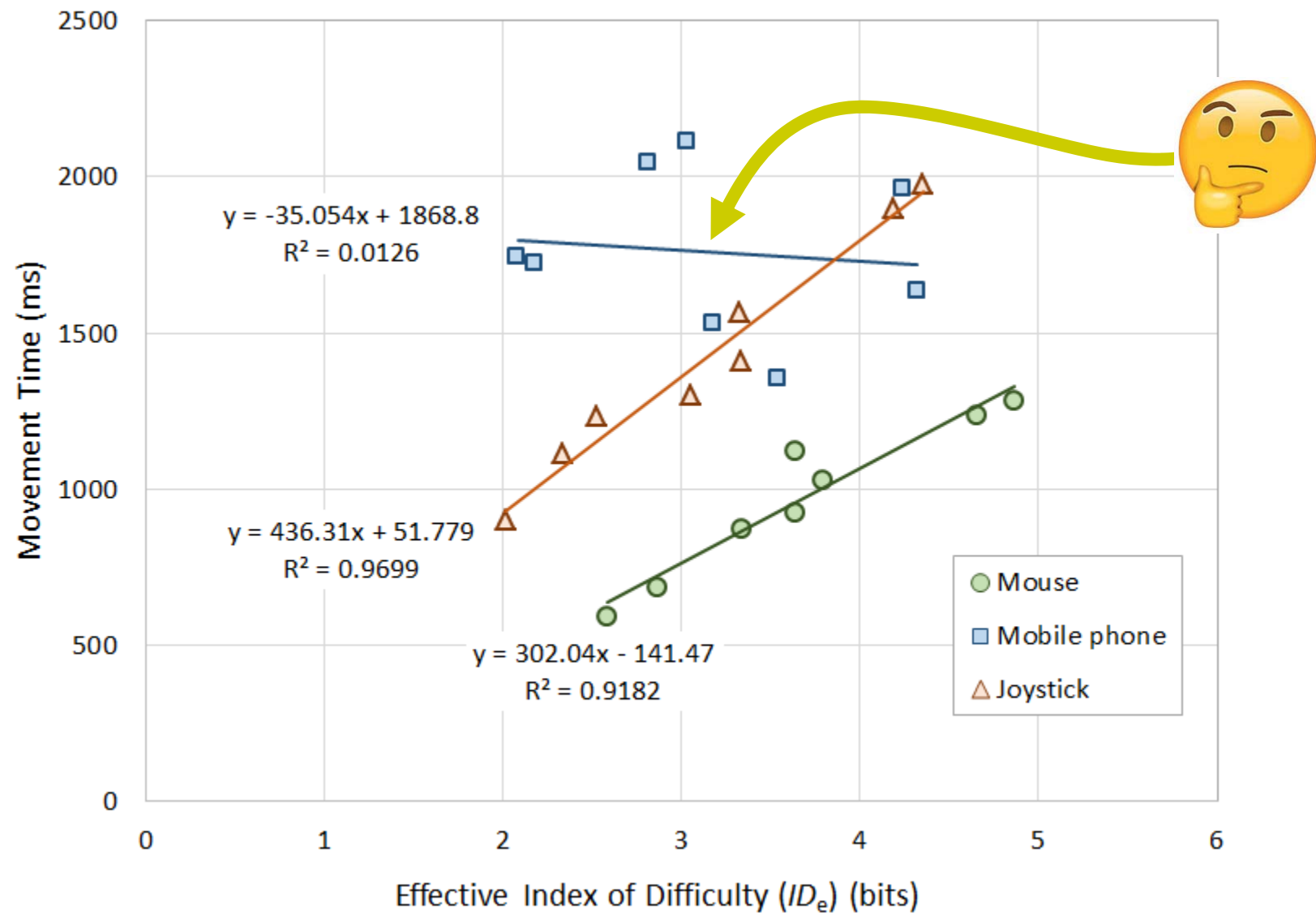
A (rads)	W (rads)	ID (bits)	Mouse				Mobile Phone				Joystick			
			W_e (rads)	ID_e (bits)	MT (ms)	TP (bit/s)	W_e (rads)	ID_e (bits)	MT (ms)	TP (bit/s)	W_e (rads)	ID_e (bits)	MT (ms)	TP (bit/s)
0.785	0.393	1.585	0.168	2.584	590	4.57	0.256	2.081	1737	1.39	0.269	2.014	899	2.58
1.047	0.393	1.874	0.185	2.857	683	4.38	0.302	2.181	1717	1.43	0.302	2.335	1110	2.41
0.785	0.196	2.322	0.098	3.336	870	4.16	0.133	2.815	2040	1.60	0.200	2.526	1231	2.29
1.047	0.196	2.663	0.106	3.631	923	4.22	0.154	3.038	2105	1.50	0.147	3.049	1297	2.63
2.356	0.393	2.807	0.205	3.788	1027	3.94	0.443	3.181	1528	2.30	0.264	3.332	1408	2.54
2.618	0.393	2.939	0.240	3.632	1120	3.44	0.248	3.539	1348	2.87	0.433	3.324	1560	2.33
2.356	0.196	3.700	0.104	4.647	1233	4.04	0.141	4.239	1954	2.45	0.141	4.192	1897	2.31
2.618	0.196	3.841	0.100	4.864	1284	4.00	0.145	4.321	1631	2.84	0.138	4.352	1973	2.30

x sample points
y sample points



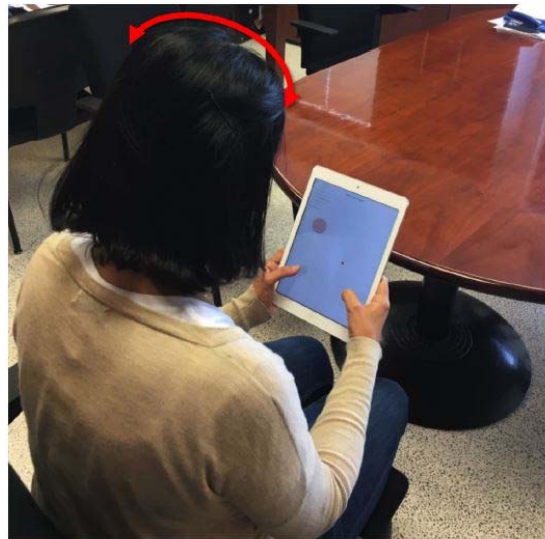
For model
building...

Fitts' Law Prediction Equations

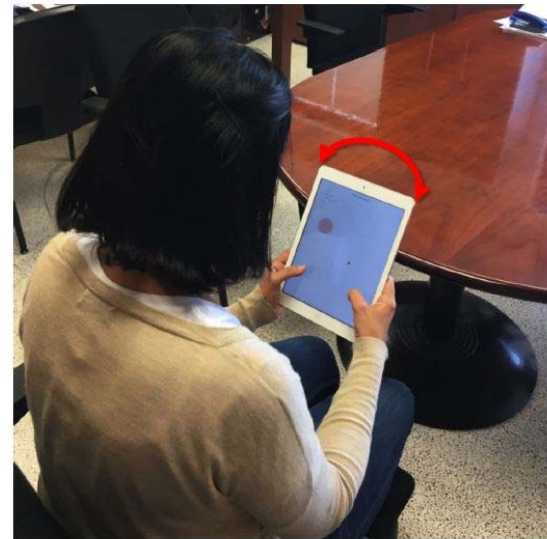


Example #3: Camera Input

- Head tracking with finger selection^a
- Moving the head vs. moving the device



(a) Head interaction



(b) Device interaction

^a Roig-Maimó, M. F., MacKenzie, I. S., Manresa-Yee, C., & Varona, J. (2018). Head-tracking interfaces on mobile devices: Evaluation using Fitts' law and a new multi-directional corner task for small displays. *International Journal of Human-Computer Studies*, 112, 1-15. [[video](#)]

Observations and Data

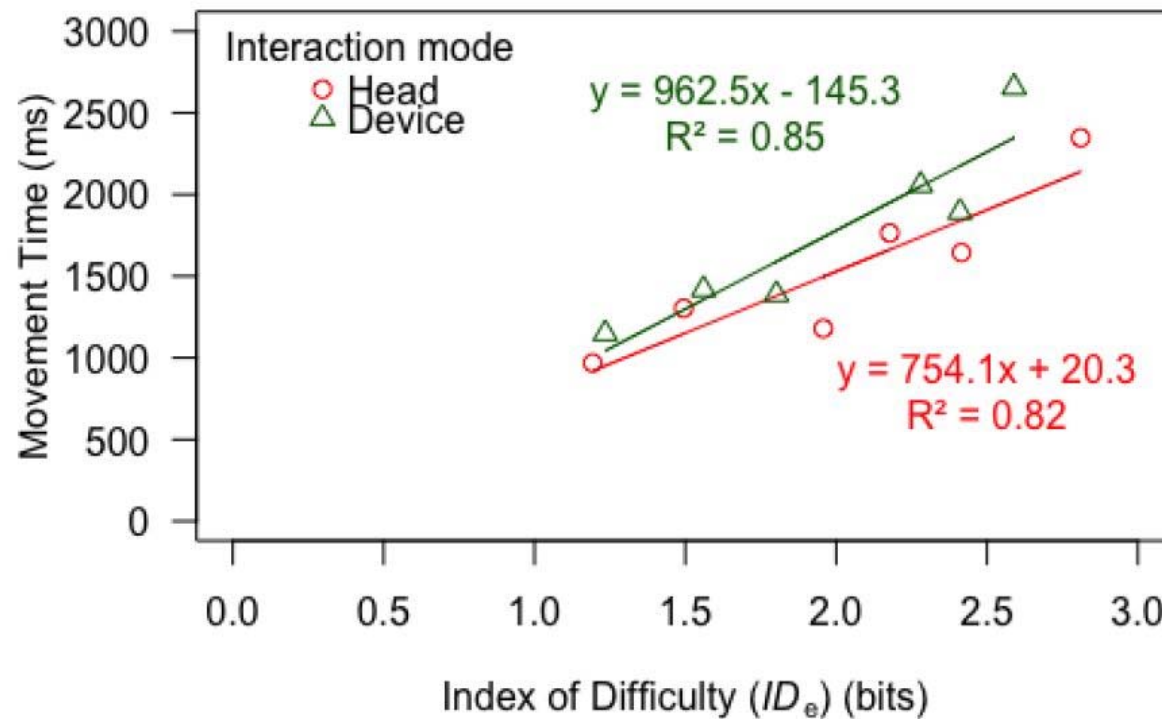
A (px)	W (px)	ID (bits)	Head			Device		
			W_e (px)	ID_e (bits)	MT (ms)	W_e (px)	ID_e (bit)	MT (ms)
260	130	1.58	143.0	1.49	1304	133.4	1.56	1418
260	260	1.00	202.2	1.19	968	192.2	1.23	1149
520	130	2.32	147.6	2.18	1765	134.8	2.28	2057
520	260	1.58	180.4	1.96	1180	209.2	1.80	1388
1040	130	3.17	172.8	2.81	2347	207.2	2.59	2656
1040	260	2.32	240.0	2.42	1645	241.0	2.41	1892

x sample points
y sample points



For model
building...

Fitts' Law Prediction Equations



Applications

- Very few examples in HCI where Fitts' law actually used as a predictive model
- Most examples are to “establish conformance” or explore the underlying theory
- Exception: design and analysis of soft keyboards
- Method: Fitts-digram model^a
- Unlike touch typing (two hands, ten fingers!), stylus or finger tapping on a soft keyboard is well modeled by Fitts' law (next slide)

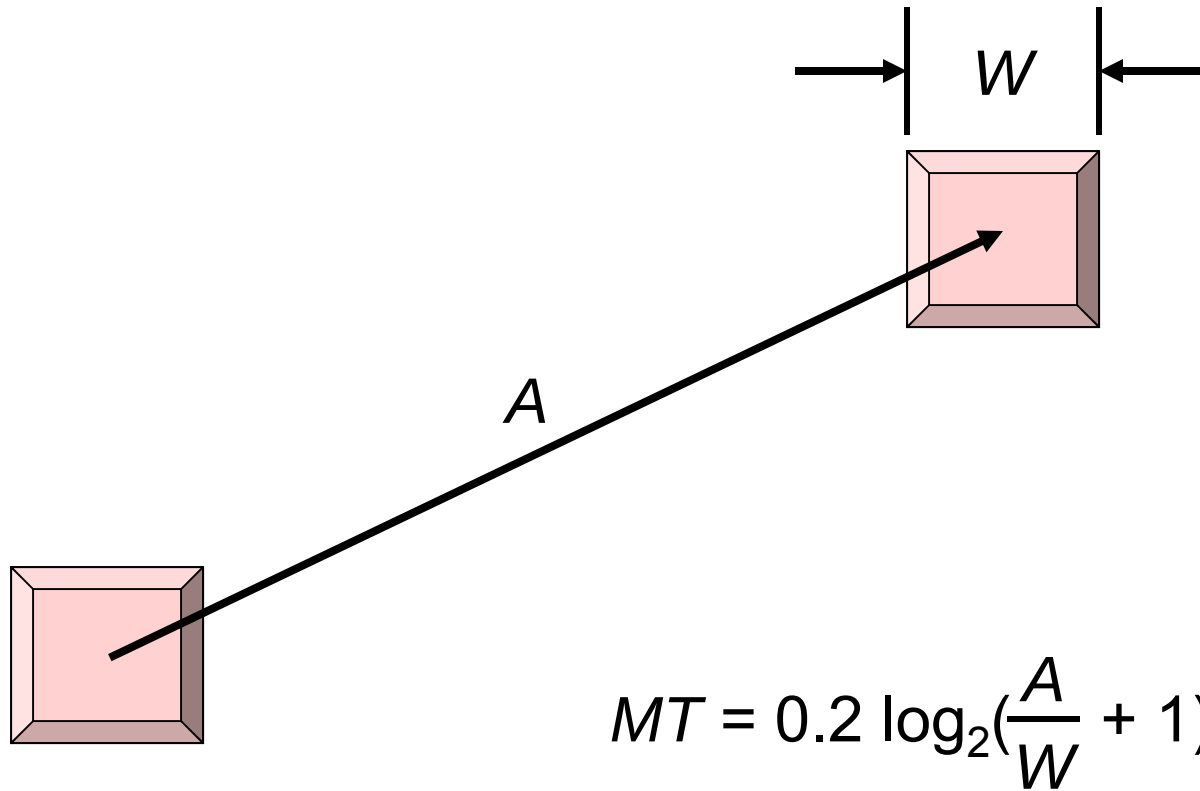
^a Soukoreff, R. W. & MacKenzie, I. S. (1995). Theoretical upper and lower bounds on typing speed using a stylus and soft keyboard. *Behaviour & Information Technology*, 14, 370-379.

Tapping on a Soft Keyboard

- Each tap is a Fitts' law task



Stylus/Finger Tapping



Analysis

- For text entry on a soft keyboard, each tap is from one letter key to the next
- To predict the time for a tap, we need the amplitude moved (A) and the width of the key tapped (W)
- To predict the mean time “for English”, we need a digram model of the language
- Include SPACE, since this constitutes about 18% of English text entry

Digram Frequencies for English^a

<i>Digram</i>	<i>Count</i>
e_	14,788,053
_t	11,565,380
th	10,301,807
he	9,436,372
_a	8,454,634
...	
zs	3

$n = 368,832,032$

File: d1-digramfreq.xls
([click to view](#))

^a Silfverberg, M., MacKenzie, I. S., & Korhonen, P. (2000). Predicting text entry speeds on mobile phones. *Proc CHI 2000*, pp. 9-16. New York: ACM.

Qwerty Analysis

the quick brown fox jumps over the lazy dog



etc.

And So...

$$\overline{MT} = \sum_i \sum_j (P_{ij} \times MT_{ij})$$

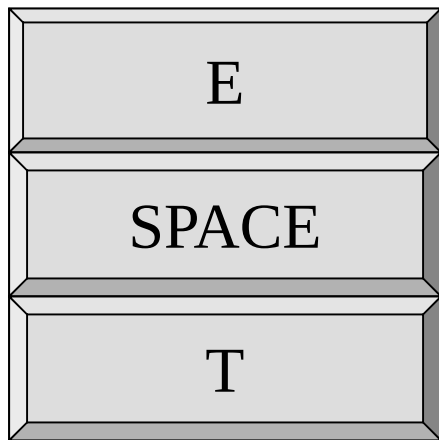
Seconds per
character

$$\text{Entry speed} = \frac{1}{\overline{MT}} \times \frac{60}{5}$$

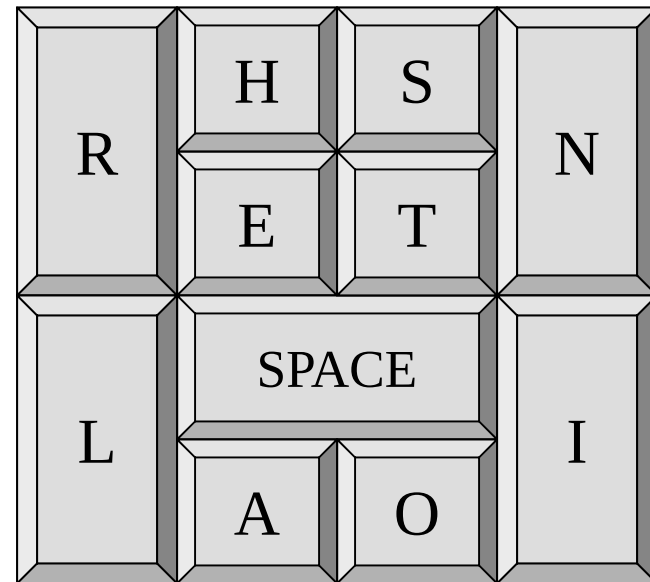
Words per
minute

= 44.2 wpm (for Qwerty)

What is the “Optimal” Keyboard?



36.5%

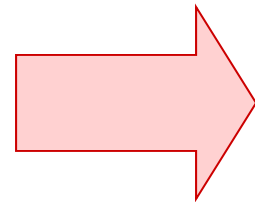


79.2%

Many, many
design issues

Quest for Optimal Soft Keyboard

Fitts' law coefficients
Keyboard geometry
Language model
Interaction technique



Opti^a



Prediction: 58.4 wpm!

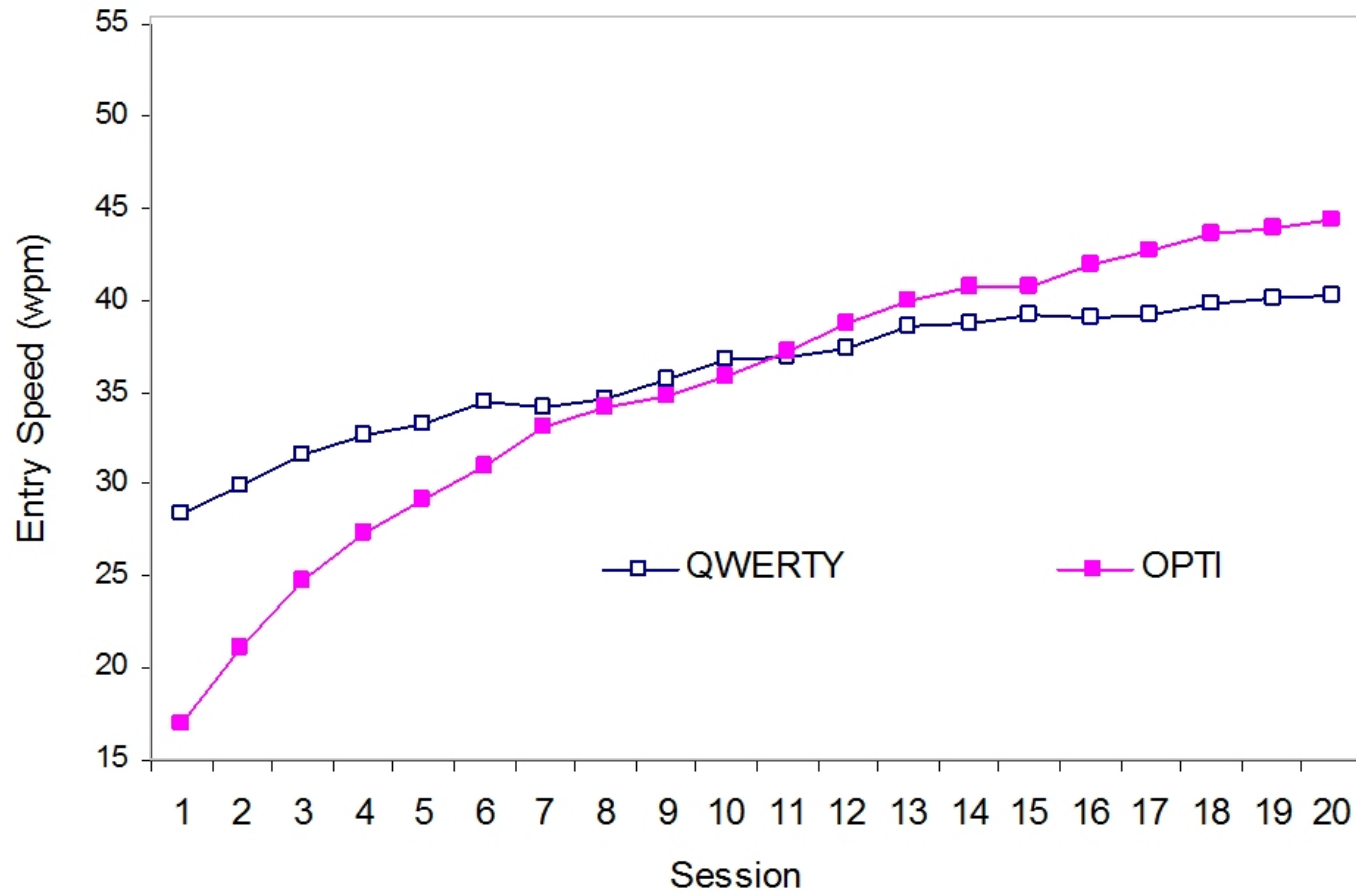
(30% faster than Qwerty)

^a MacKenzie, I. S., & Zhang, S. X. (1999) The design and evaluation of a high-performance soft keyboard. *Proc CHI '99*, pp. 25-31. New York: ACM.

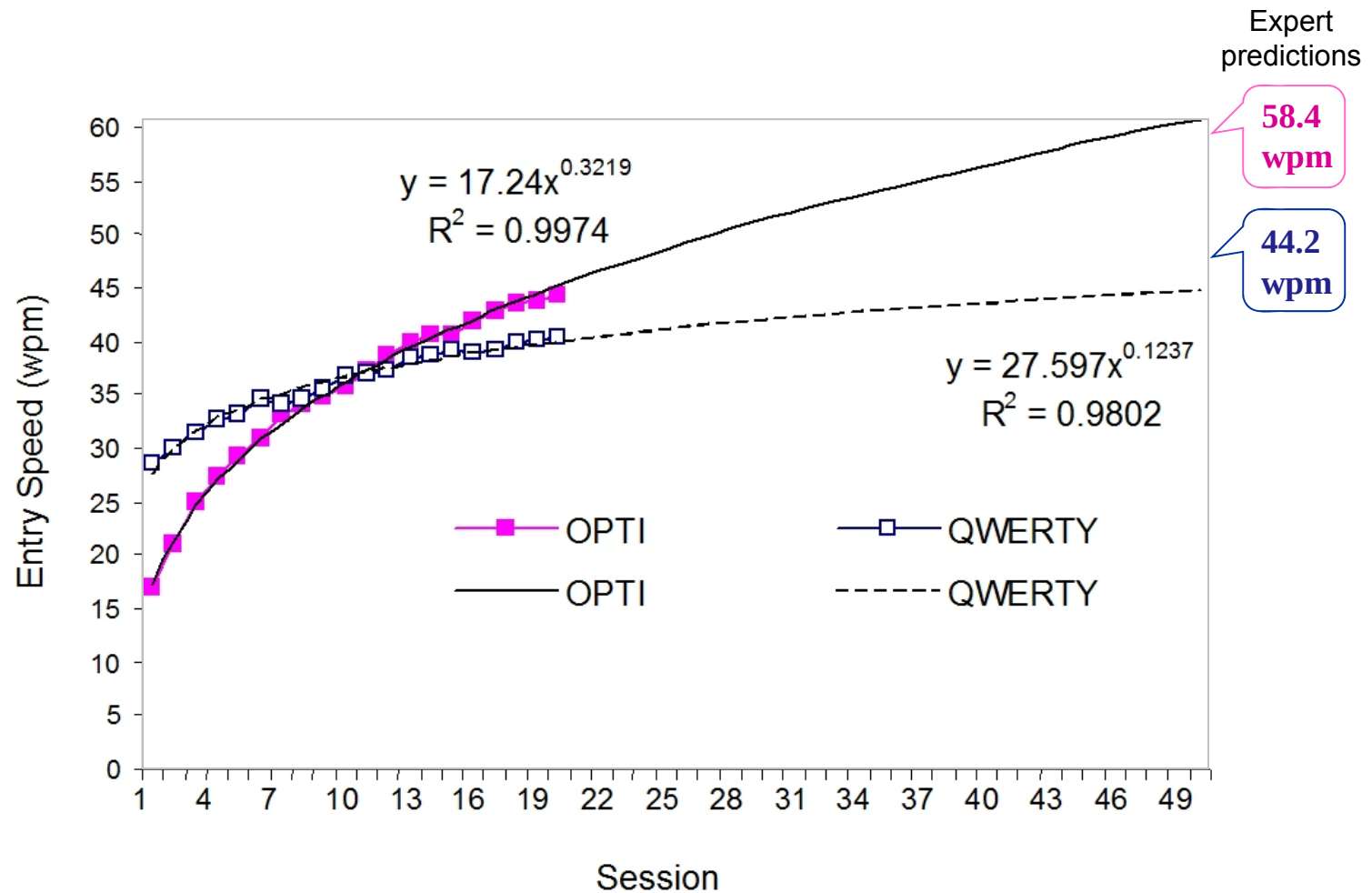
Empirical Test

- 5 participants
- Wacom *PL100V*
- Text entry task
- 20 sessions $\times$ 50 minutes
- Each session:
 - 25 minutes with Qwerty
 - 25 minutes with Opti
- Counterbalanced, etc.

Results

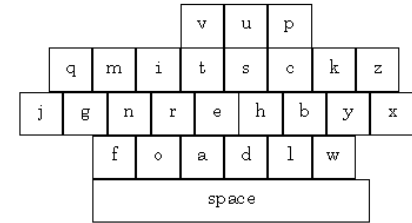
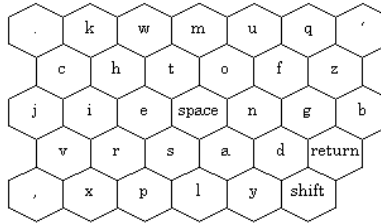
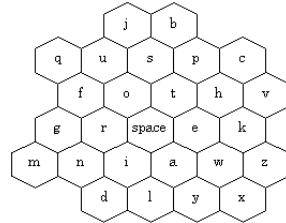
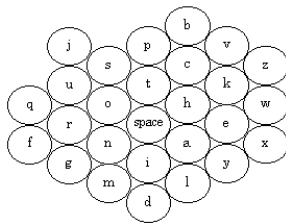


Results (2)



Summary (Part II)

- Lots and lots of follow-on work ^a
- Research on improving soft keyboards



a	b	c	d	e	f
g	h	i	j	k	l
m	n	o	p	q	r
s	t	u	v	w	x
z	y	space			

z	v	c	h	w	k
f	i	t	a	l	y
space		n	e	space	
g	d	o	r	s	b
q	j	u	m	p	x

q	k	c	g	v	j
space	s	i	n	d	space
w	t	h	e	a	m
space	u	o	r	l	space
z	b	f	y	p	x

q	r	w	x	y	
l	u	a	o	f	
t	h	e	n	g	
v	d	i	s	p	
b	c	m	j	k	z
space					

					←
A	C	D	E		↑
G	H	I	L	123	
M	N	O	P	A↔Q	
R	S	T	U	Space	

^a MacKenzie, I. S., & Soukoreff, R. W. (2002). Text entry for mobile computing: Models and methods, theory and practice. *Human-Computer Interaction*, 17, 147-198. [PDF] 70

Conclusion

- Fitts' law is a useful model in human-computer interaction
- Useful as
 - A performance measure (*Throughput*)
 - A prediction model

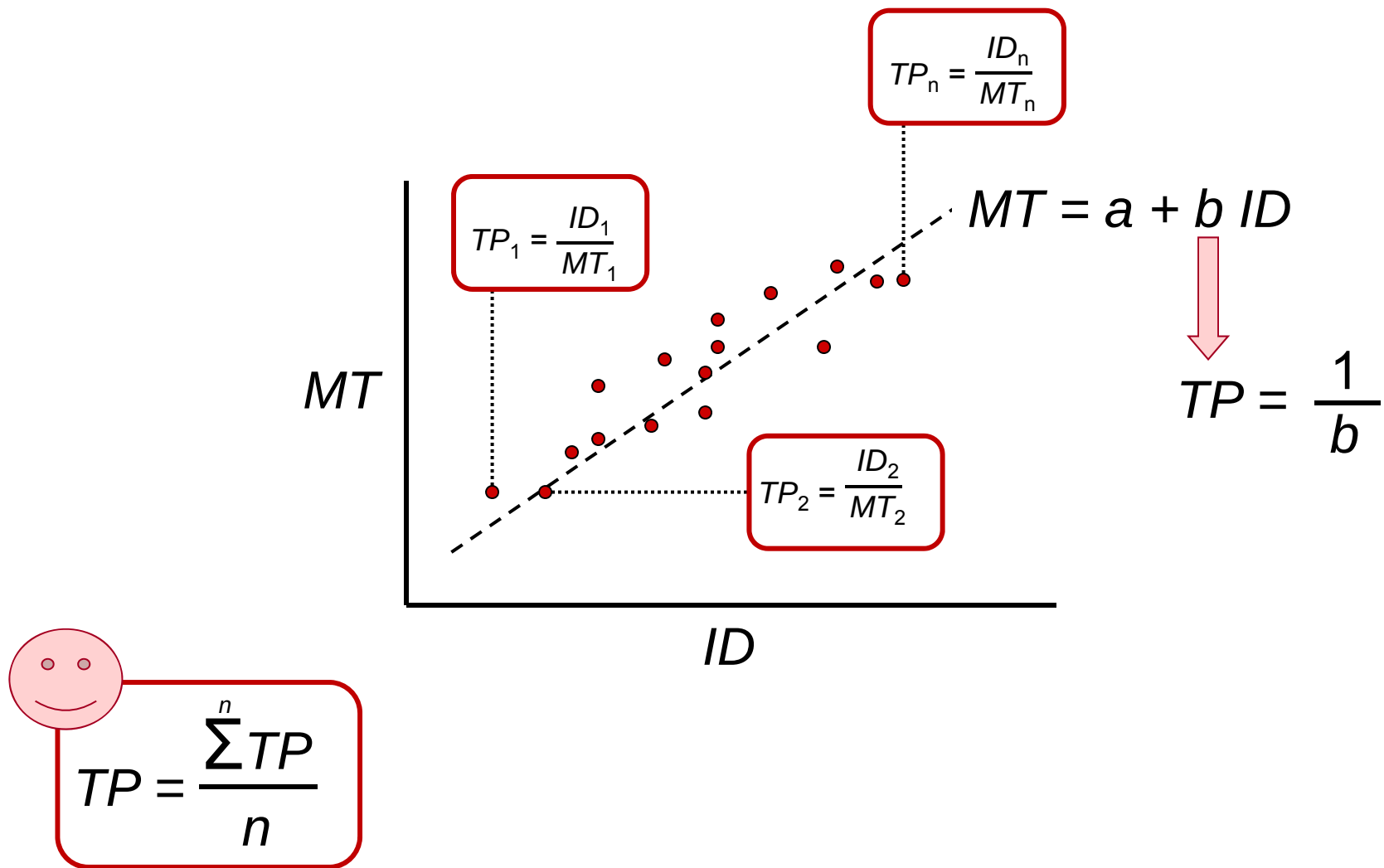
A few final thoughts

Out of time (Done)

Post Script – Problems / Challenges

- Literature “all over the map”
- Comparisons between studies difficult, e.g.,
 - What is the throughput for a mouse in point-select tasks?
 - Expect somewhat uniform answer to this question, but no
 - *TPs* range from about 1 bits/s to 60 bit/s !!!
- Main issues are in the methodology (not the theory, as some have suggested)
- Let’s consider the calculation of throughput...

Formula for Throughput ?



$$TP = ID / MT$$

For the naysayers...

"The average rate of information generated by a series of movements is the average information per movement divided by the time per movement" (Fitts, 1954, p. 268).

Data Aggregation

- Throughput cannot be calculated on a single trial
- Variability is needed to gauge accuracy
- Throughput is calculated on the aggregate data...
 - For each participant, over a single sequence of trials for a particular *A-W* condition
 - Or...
- Throughput is calculated on the aggregate data...
 - Across participants, perhaps using the data for multiple sequences of trials
 - Or...



What is *ID* ?

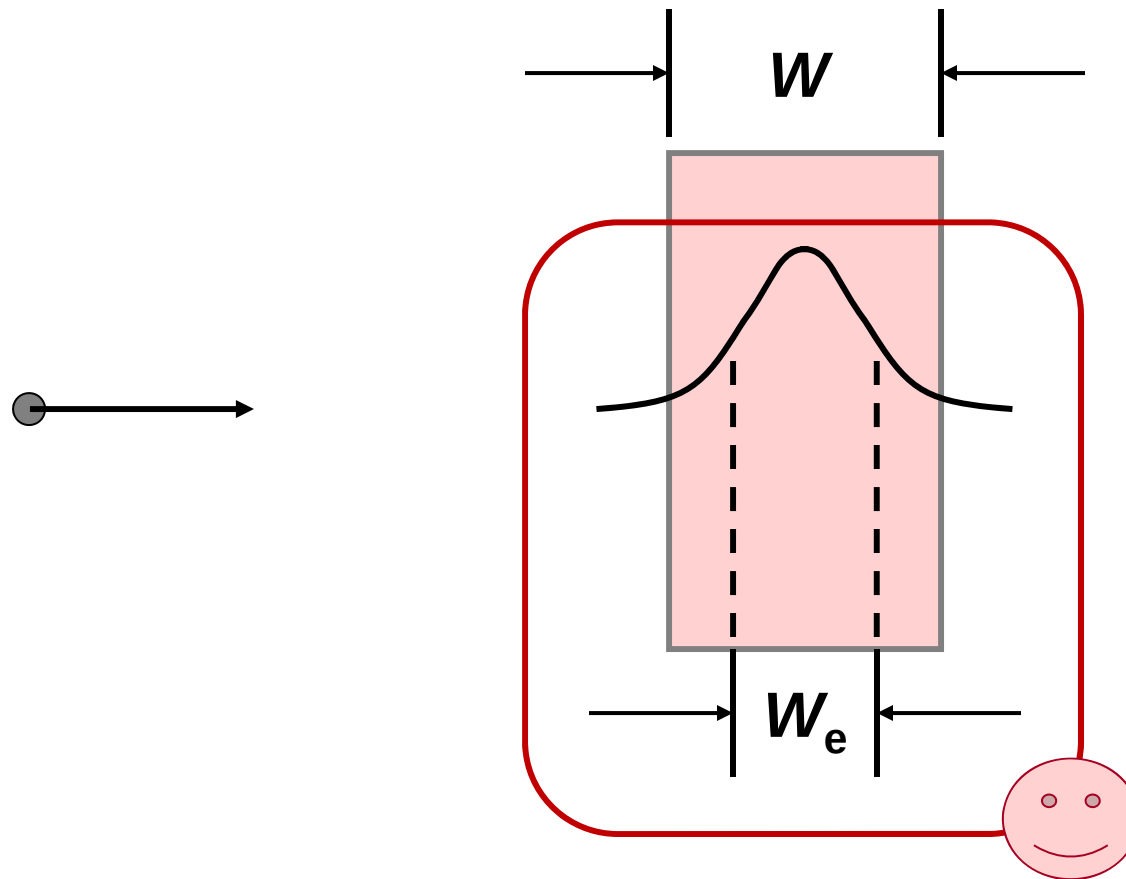

$$ID = \log_2(A / W + 1)$$

$$ID = \log_2(2A / W)$$

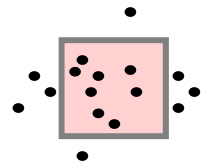
$$ID = \log_2(A / W + 0.5)$$

$$ID = A / W$$

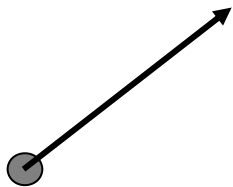
W or W_e ?



Calculation of W_e ?



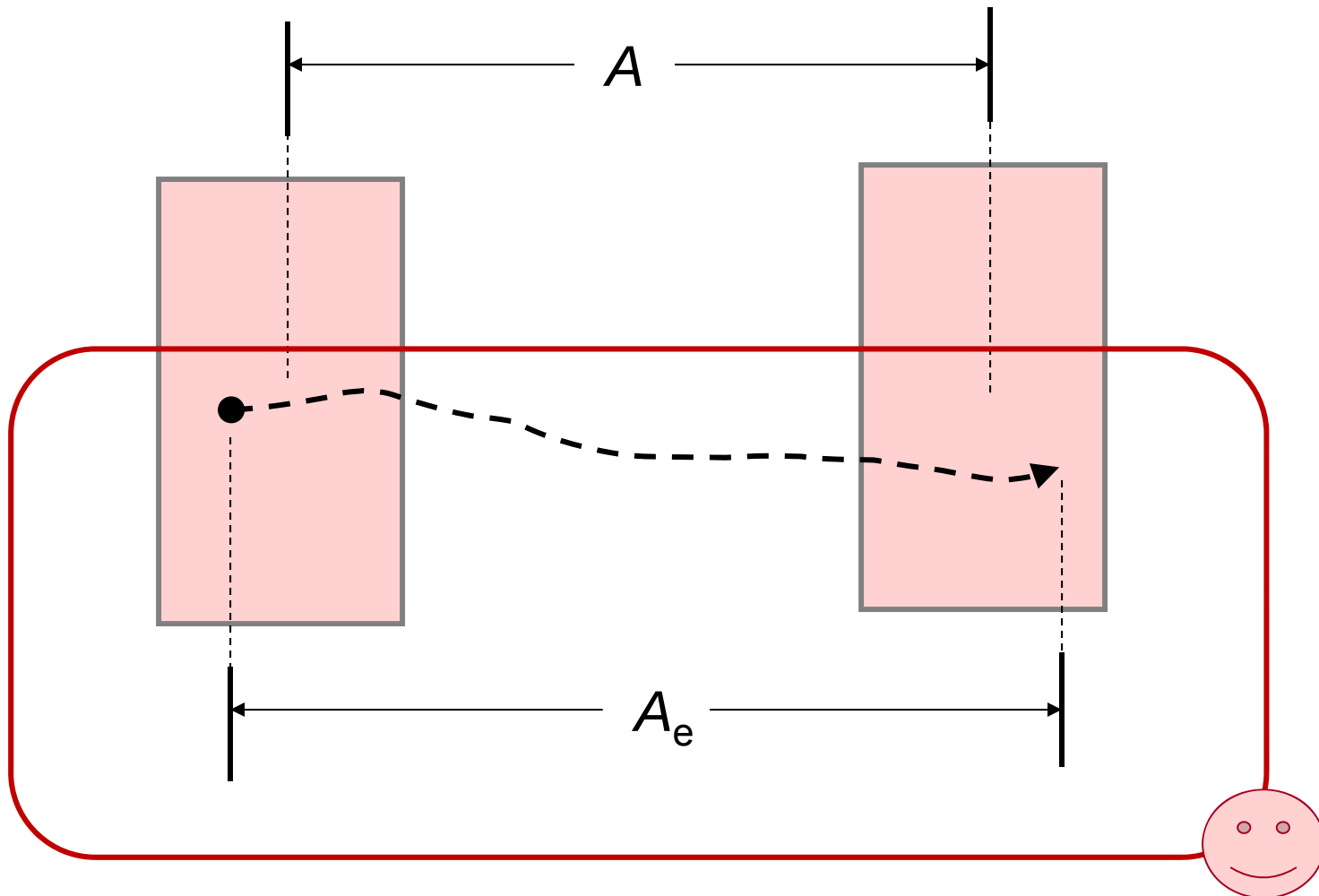
$$W_e = 4.133 \times SD_x$$



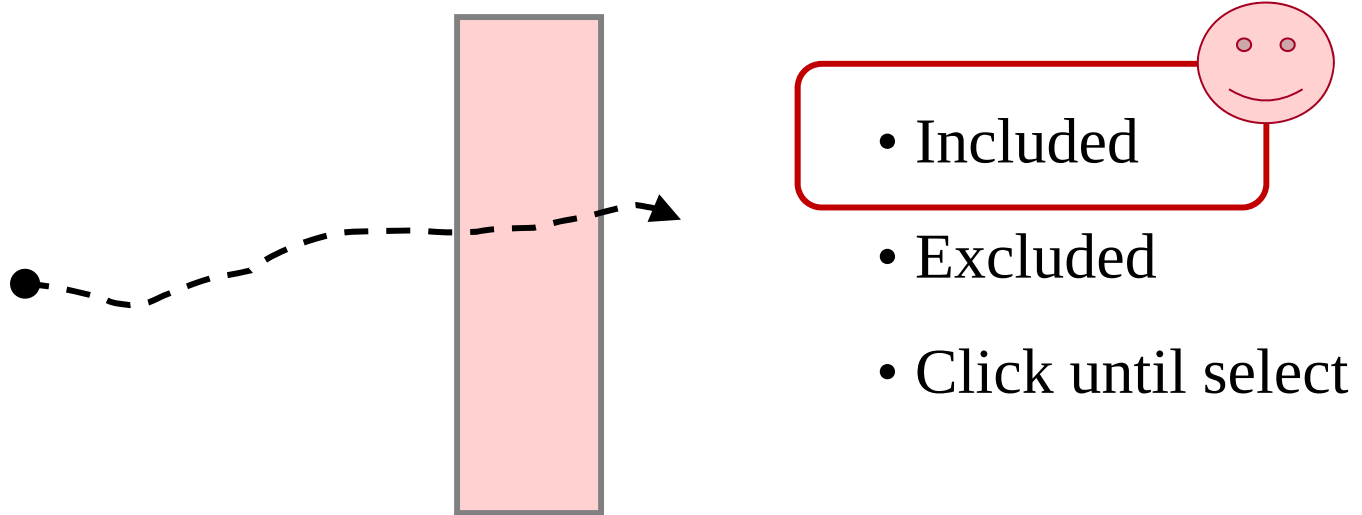
- Calculation of SD_x :
- Use a 1D transformation to compute the selection coordinates along the approach-axis, then compute SD_x
- Use an ad hoc method that makes sense to you (the method is not clearly stated in the write-up)



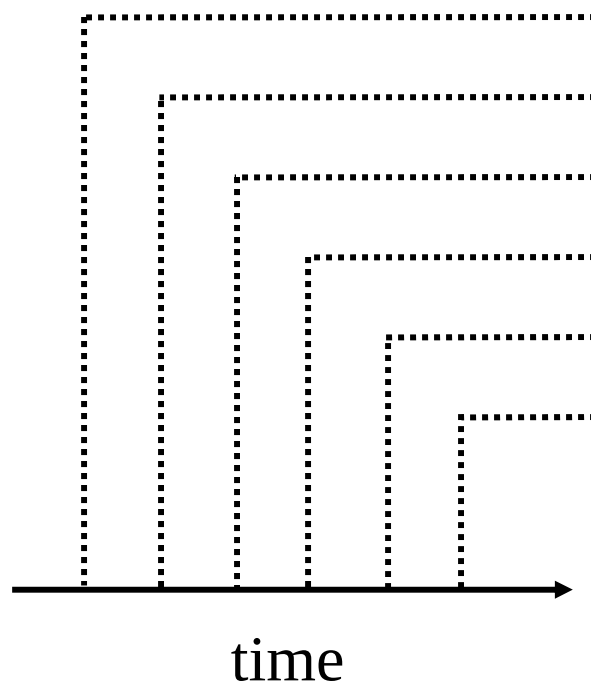
A or A_e ?



Errors ?



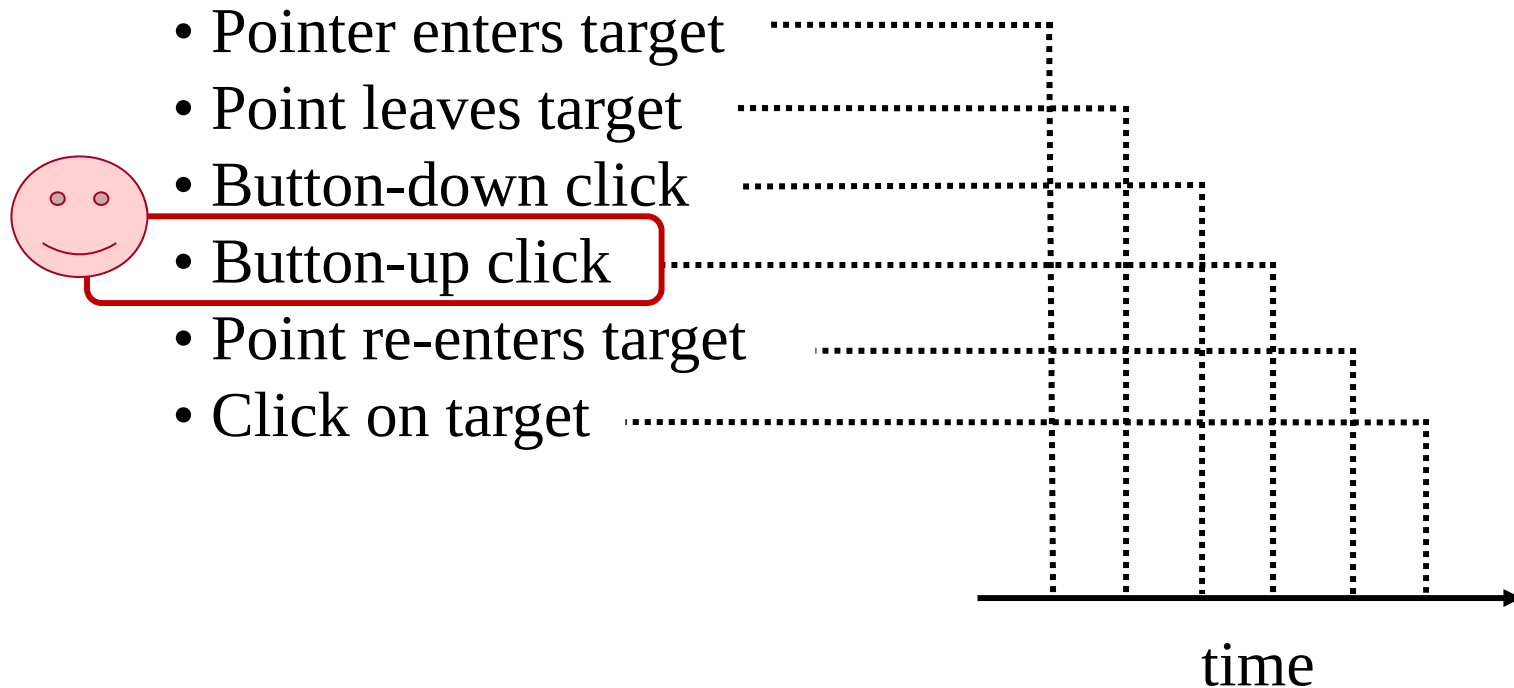
When Does a Trial Begin ?



- Stimulus appears (reaction time)
- Pointer inside start box
- Button-down click
- Button-up click **Serial**
- First movement **Discrete**
- First movement outside start box



When Does a Trial End ?



- There is still work to be done!

Thank You

