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A Note on the Information-Theoretic Basis for Fitts' Law

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ABSTRACT. Fitts' law is an information-theoretic view of human motor behavior developed from Shannon's Theorem 17, a fundamental theorem of communications systems. Using data from Fitts' original experiments, we demonstrate that Fitts' choice of an equation that deviates slightly from the underlying principle is perhaps unfounded, and that the relationship is improved by using an exact adaptation of Shannon's equation.

FITTS' RESEARCH into the information capacity of the human motor system culminated in the publication of a paper (Fitts, 1954) that proposed a fundamental relationship. It expresses movement time (*MT*), the time to complete a movement task, in terms of the distance or amplitude of the move (*A*) and the width of the region within which the move must terminate (*W*). The mathematical relationship, known as Fitts' law, is

$$MT = a + b \log_2 (2A/W). \tag{1}$$

The so-called *index of difficulty* (*ID*) of a movement task is expressed as

$$ID = \log_2 (2A/W). \tag{2}$$

and allows Fitts' law to be restated as

$$MT = a + b\ ID. \tag{3}$$

Equations 1 and 3 are linear in *ID*, with empirically determined constants for the intercept (*a*) and slope (*b*). Although the base of the log term is arbitrary, practice dictates the use of base 2; thus *ID* carries the unit *bits*. The number of bits is the information content of the posi-

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tioning task, or, stated another way, the information transmitted in carrying out the task.

Fitts' paper provided data for four movement task experiments that substantially verified the model's appropriateness. Numerous other motor behavior experiments have also demonstrated a high correlation between Fitts' relationship and observed data.

This paper is not a thorough examination of Fitts' law. For that the reader is referred to Welford (1968, pp. 145–160) or to Crossman and Goodeve (1983), Kvalseth (1979, 1981), and Sheridan (1979) for more critical reviews.

Fitts' law was developed from an analogy with physical communications systems. In such systems, the amplitude of a transmitted signal is described as perturbed by noise that results in amplitude uncertainty. The effect is to limit the information capacity of a communications channel to some value less than its theoretical bandwidth. Shannon's Theorem 17 expresses the effective information capacity C (in bits $\times$ s^{-1}) of a communications channel of band B (in s^{-1}) as

$$C = B \log_2 \left(\frac{P + N}{N}\right), \tag{4}$$

where P is the signal power and N is the noise power (Shannon & Weaver, 1949, pp. 100–103).

It is the purpose of this note to suggest that Fitts' model contains an unnecessary deviation from Shannon's Theorem 17 and that a model based on an exact adaptation provides a better fit with empirical data. The variation of Fitts' law suggested by direct analogy with Shannon's Theorem 17 is

$$MT = a + b \log_2 \left(\frac{A + W}{W}\right). \tag{5}$$

It is revealing to examine the source Fitts cites in his paper at the point where he introduces his relationship (Fitts, 1954, p. 388). His derivation is based on Goldman's Equation 39 (Goldman, 1953), which is similar to Fitts' law except in its use of the terminology of communications systems:

$$C = B \log_2(P/N). \tag{6}$$

Goldman (1953) offers this equation as an "approximation" of Shannon's theorem, adding that it is useful "if the transmitted power is large in comparison with the noise" (p. 157). Indeed, this is so. When a substantial signal is transmitted along a low-noise channel, Equation 6 is an accurate and less cumbersome substitute for Equation 4. As the signal decreases or the noise increases, however, Equation 6 becomes increasingly inaccurate and Equation 4, Shannon's Theorem

17, must be used. Psychomotor experiments employing Fitts' law commonly use conditions in which the signal (movement amplitude) is very small in comparison with the noise (target width). In fact, two of Fitts' experiments used conditions extending down to an $A{:}W$ ratio of 1:1! These are precisely the conditions under which Goldman cautions that his equation (and, we might conjecture, Fitts' law) is inaccurate.

Fitts recognized that his analogy was imperfect. The "2" was added (see Equation 1) to avoid a negative *ID* when $A = W$; however, $\log_2 (2A/W)$ is zero when $A = (W/2)$ and negative when $A < (W/2)$. These conditions could never occur in the experiments that Fitts devised; other researchers, however, have reported experimental conditions with *ID* less than 1 bit (Drury, 1975), or with a negative *ID* (Crossman & Goodeve, 1983; Ware & Mikaelin, 1987). It is noteworthy that, in the model based on Shannon's theorem (see Equation 5), *ID* cannot be negative.

The reason Fitts did not use Shannon's original equation was not stated; it may lie, however, in the greater facility in working with Equation 1. Fitts' law may be recast as

$$MT = a + b_1 \log_2 (2A) - b_2 \log_2(W). \quad (7)$$

This appealing and accurate transformation separates *A* and *W* and offers an extra degree of fine tuning for the prediction model by using three empirically determined constants. This approach has been studied by Kerr (1978), Sheridan (1979), and Welford (1968, p. 153); it may, however, be inappropriate from an information-theoretic perspective because similar recasting is not possible with Equation 5, which mimics Shannon's original equation. Indeed the difference between Equations 1 and 5 is most apparent for small values of *ID* (i.e., as the ratio $A{:}W$ decreases); and it is for small values of *ID* that Fitts' law has been demonstrated to fail (Buck, 1986; Crossman & Goodeve, 1983; Klapp, 1975; Langolf, Chaffin, & Folke, 1976; Welford, 1968, pp. 145–146).

It has not been proposed in the literature subsequent to the appearance of Fitts' original paper that an exact adaptation of Shannon's equation may be more appropriate; numerous other variations of Fitts' law have been proposed over the years, however, including those by Welford (1960); Beggs, Graham, Monk, Shaw, and Howarth (1972); Kvalseth (1980); Kantowitz and Knight (1978, pp. 222–223); and Jagacinski, Repperger, Ward, and Moran (1980). Welford's proposal has been the most favorably received, and usually takes the form

$$MT = a + b \log_2\left(\frac{A}{W} + 0.5\right). \quad (8)$$

Equation 8 goes "half way" to the proposed variation and can be restated as

$$MT = a + b \log_2\left(\frac{A + 0.5\,W}{W}\right) \quad (9)$$

to more closely illustrate the similarity with Shannon's original theme.

Fitts recognized the improvement brought by Welford's equation and preferred it in subsequent research (Fitts & Peterson, 1964). Others have also used Welford's model, noting the data-to-model fit to be as good as, or better than, the data derived using Fitts' model (Beggs et al., 1972; Card, English, & Burr, 1978; Drury, 1975; Glencross & Barrett, 1983; Kerr & Langolf, 1977; Kerr, 1978; Knight & Dagnall, 1967).

Nevertheless, many researchers still use Fitts' law in its original form in current motor behavior experiments (Buck, 1986; Epps, 1986; Georgopoulos & Massey, 1987; Kantowitz & Elvers, 1988; Zelaznik, Mone, McCabe, & Thamam, 1988).

Table 1 contains a comparison of the correlation coefficients and regression line intercepts resulting from a least squares regression analysis using the Fitts, Welford, and Shannon models. The comparison employed the data from Fitts' reciprocal tapping experiments (1 oz stylus and 1 lb stylus), disc transfer experiment, and pin transfer experiment. The data provided in Fitts' original experiments are thought to be particularly valid since 16 subjects were used and were tested over 16 *ID*s. Each movement time recorded was the average of more than 600 observations (Fitts, 1954).

Examining first the tapping experiment using a 1 oz stylus, a high correlation existed using Fitts' relationship (r = .9831); it was improved, however, by using Welford's equation (r = .9900), and improved still further using Equation 5 (r = .9936), which was based directly on Shannon's Theorem 17. The trend was similar with the other three experiments.

Ideally, the intercept should have been (0,0), predicting 0 ms to complete a task of zero difficulty. As evident in Table 1, Fitts' relationship yielded the intercept closest to the origin in each experiment, with the Shannon and Welford models ranked second and third in each experiment. A possible explanation for the nonzero intercepts stems from Crossman's observation (Crossman & Goodeve, 1983, p. 253) that movement time appears to approach a constant as *ID* gets small. As for the one negative intercept in Table 1, sampling error was perhaps the cause.

A proper information analysis of a Fitts' law experiment must investigate the extent of noise in the subjects' execution of the positioning tasks. A standard method was provided by Crossman and given by Welford (1968, pp. 147–148) to convert target width (W) to "effective" target width (W_e) in experiments such as Fitts' that record errors. In the long run, the subjects' dispersion of hits forms a Gaussian or normal distribution (Fitts, 1954; Crossman & Goodeve, 1983). The effective taraget width, analogous to "noise," is the width corresponding to the central 96% of the distribution.

Since Fitts reported percentage errors (for the tapping experiments), a simple transformation of W to W_e was obtained by multiplying W by a ratio of z scores. Table 2 shows the results of a reanalysis of Fitts' tapping experiments in which W_e was substituted for W. The correlations were in the same order as earlier, with the Shannon model providing the highest correlation for both experiments. The intercepts were generally negative, with the Welford model providing the intercepts closest to the origin in each case, and the Fitts model the one farthest from the origin.

An important question to raise is whether the difference between the correlations was statistically significant: Was the higher correlation demonstrated with Shannon's Theorem 17 due to chance? This was tested with Hotelling's t test for the difference between coefficients of correlation for correlated samples (e.g., Guilford & Fruchter, 1978, p. 164; Bruning & Kintz, 1977, pp. 215–217). The t statistic was calculated using the Fitts and Shannon correlations of movement time with ID (r_{12} and r_{13}), the intercorrelation between the Fitts and Shannon IDs (r_{23}), and the sample size n. The data in Table 3 indicated a statistically significant difference between the Fitts and Shannon correlations for the tapping experiments when using W ($p < .001$) or W_e ($p < .05$, $p < .02$) but no significant difference between the correlations in the disc and pin transfer experiments.

The statistical insignificance in the difference between the rs in the later cases was perhaps due to the nature of the tasks. The tapping

TABLE 1
Correlation Coefficients and Regression Line Intercepts (ms) for Three Variations of Fitts' Law Based on Fitts' (1954) Experiments

		Tapping (1 oz)		Tapping (1 lb)		Disc transfer		Pin transfer	
Model	Equation	r^a	Intercept	r^a	Intercpet	r^a	Intercept	r^a	Intercept
Fitts	1	.9831	+12.8	.9796	−6.2	.9186	+150.0	.9432	+22.3
Welford	8	.9900	+65.3	.9874	+51.7	.9191	+231.8	.9443	+96.1
Shannon	5	.9936	+27.7	.9916	+9.7	.9195	+223.4	.9452	+84.4

$^a p < .001$.

TABLE 2
Correlations and Intercepts (ms) for Fitts' Tapping Experiments Using Effective Target Width

		Tapping (1 oz)		Tapping (1 lb)	
Model	Equation	r^a	Intercept	r^a	Intercept
Fitts	1	.9907	−73.2	.9885	−118.0
Welford	8	.9927	+1.2	.9911	−33.0
Shannon	5	.9938	−31.4	.9927	−69.8

$^a p < .001$.

TABLE 3
Test of Statistical Significance of Difference Between Correlations for the Fitts and Shannon Models

Correlation coefficients[a]			Range for	Hotelling's difference test[b]			
Fitts r_{12}	Shannon r_{13}	Inter-corr r_{23}	Fitts' *ID* (bits)	*t*	*n*	*p*	Experiment
.9831	.9936	.9966	1– 7	6.47	16	.001	Tapping, 1 oz stylus, using *W*
.9796	.9916	.9966	1– 7	6.94	16	.001	Tapping, 1 lb stylus, using *W*
.9185	.9195	.9999	4–10	.64	16	—	Disc transfer
.9432	.9452	.9997	3–10	1.07	20	—	Pin transfer
.9904	.9937	.9987	1– 7	2.20	16	.05	Tapping, 1 oz stylus, using W_e
.9882	.9925	.9987	1– 7	2.83	16	.02	Tapping, 1 lb stylus, using W_e

[a] $p < .001$.

[b] $df = n - 3$, two-tailed test.

tasks were highly ballistic, whereas the disc and pin transfer tasks were more regulated by feedback mechanisms. The variations in the terminating positions for the tapping tasks were analogous to noise and were recorded as percentage errors when the subjects missed the targets. Errors, however, could not occur in the disc or pin transfer task; the subjects simply took the time necessary to guide the disc or pin until it was secured in place. The extra time spent in the final placement of the disc or pin may also explain the generally higher intercepts for those experiments (see Table 1).

Another possible explanation, which is particularly important in establishing the viability of the Shannon model over the Fitts model, stems from the range of experimental conditions employed. The tapping experiments had an *ID* range of 1–7 bits, whereas the disc transfer experiment had an *ID* range of 4–10 bits and the pin transfer experiment 3–10 bits (see Table 3). As mentioned earlier, the difference between the two models is likely to surface only when experimental conditions include small values of *ID*.

These results are thought to be significant because the proposed variation of Fitts' law is one that simplifies the law, returning it to the underlying theory. By maintaining the premise that human motor behavior can be modeled from an information-theoretic perspective, Shannon's original and unaltered theorem, as expressed in Equation 5, provides an appealing and perhaps appropriate model. A reanalysis of data from Fitts' original experiments appears to support this suggestion.

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