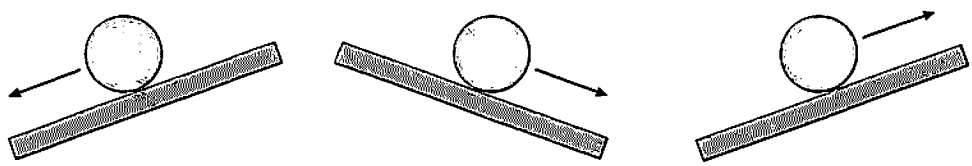
12 Rotation of a Rigid Body

12.1 Rotational Motion

1. The following figures show a wheel rolling on a ramp. Determine the signs (+ or −) of the wheel's angular velocity and angular acceleration.



ω _____ ω _____ ω _____

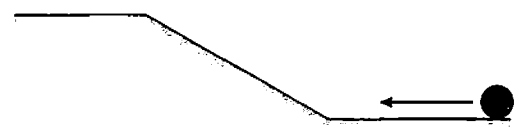
α _____ α _____ α _____

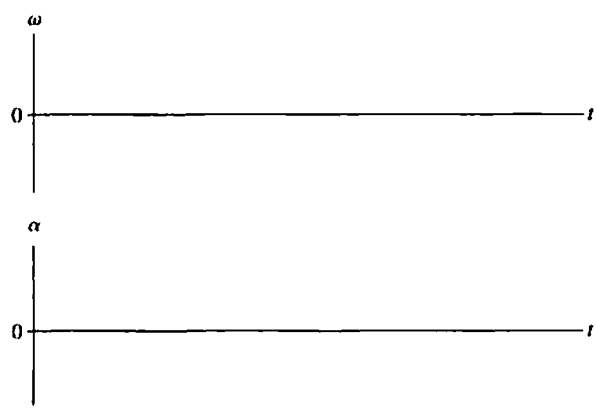
2. A ball is rolling back and forth inside a bowl. The figure shows the ball at extreme left edge of the ball's motion as it changes direction.

a. At this point, is ω positive, negative, or zero? _____

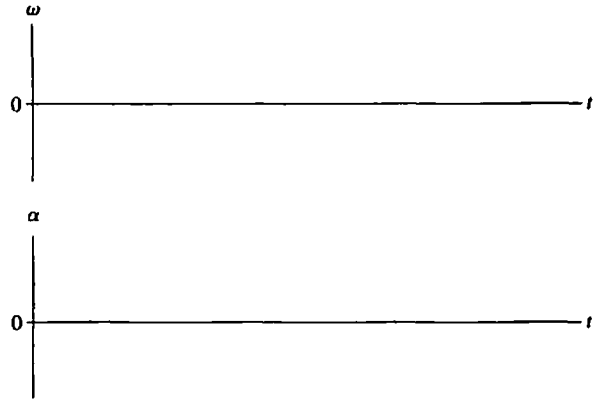
b. At this point, is α positive, negative, or zero? _____

3. A wheel rolls to the left along a horizontal surface, up a ramp, then continues along the upper horizontal surface. Draw graphs for the wheel's angular velocity ω and angular acceleration α as a function of time.



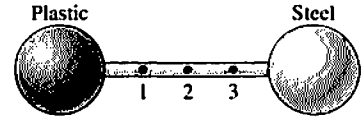


4. A wheel rolls to the right along the surface shown. Draw graphs for the wheel's angular velocity ω and angular acceleration α until the wheel reaches its highest point on the right side.

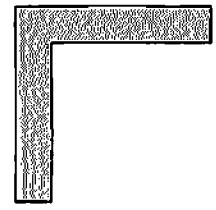


12.2 Rotation about the Center of Mass

5. Is the center of mass of this dumbbell at point 1, 2, or 3? Explain.



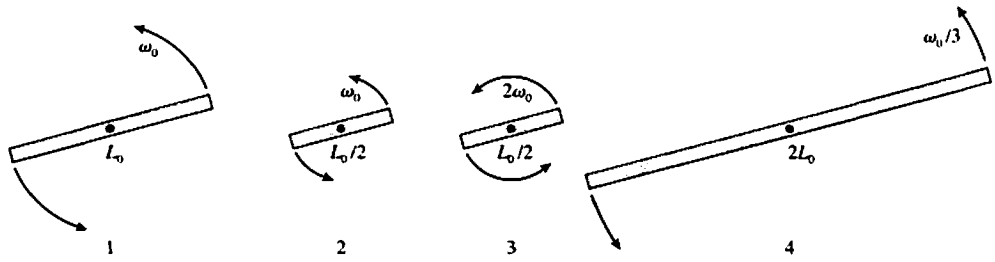
6. Mark the center of mass of this object with an ×.



12.3 Rotational Energy

12.4 Calculating the Moment of Inertia

7. The figure shows four equal-mass bars rotating about their center. Rank in order, from largest to smallest, their rotational kinetic energies K_1 to K_4.



Order:

Explanation:

8. Two rotating solid spheres have the same angular velocity and the same mass. Sphere B has twice the rotational kinetic energy of sphere A.
   a. What is the ratio R_B/R_A of their radii?

   b. Would your answer differ if one sphere were solid and the other an equal-mass spherical shell? Explain.

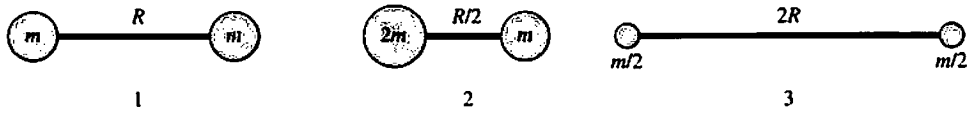
9. Which has more kinetic energy: a particle of mass M rotating with angular velocity ω in a circle of radius R, or a sphere of mass M and radius R spinning at angular velocity ω? Explain.

10. The moment of inertia of a uniform rod about an axis through its center is $\frac{1}{12}ML^2$. The moment of inertia about an axis at one end is $\frac{1}{3}ML^2$. Explain *why* the moment of inertia is larger about the end than about the center.

11. You have two steel spheres. Sphere 2 has three times the radius of sphere 1. By what *factor* does the moment of inertia I_2 of sphere 2 exceed the moment of inertia I_1 of sphere 1?

12. The professor hands you two spheres. They have the same mass, the same radius, and the same exterior surface. The professor claims that one is a solid sphere and that the other is hollow. Can you determine which is which without cutting them open? If so, how? If not, why not?

13. Rank in order, from largest to smallest, the moments of inertia I_1, I_2, and I_3 about the midpoint of the rod.



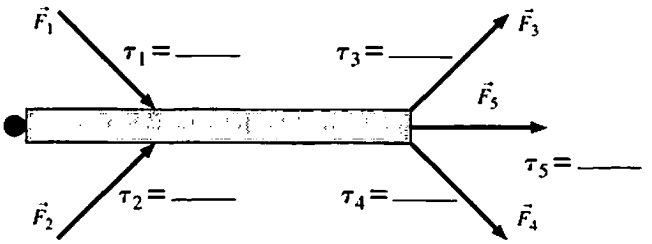
1 2 3

Order:

Explanation:

12.5 Torque

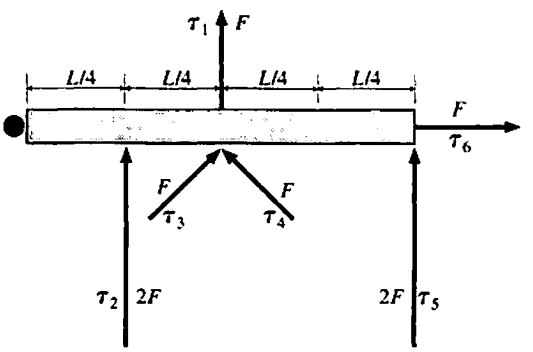
14. Five forces are applied to a door. For each, determine if the torque about the hinge is positive (+), negative (–), or zero (0).



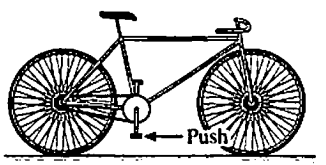
15. Six forces, each of magnitude either F or $2F$, are applied to a door. Rank in order, from largest to smallest, the six torques τ_1 to τ_6 about the hinge.

Order:

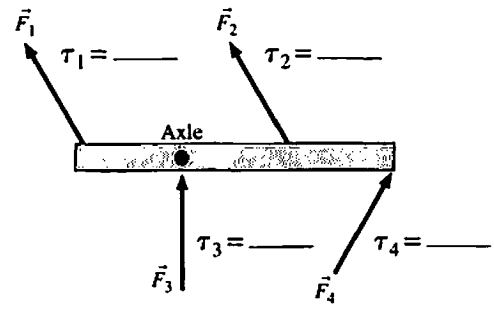
Explanation:



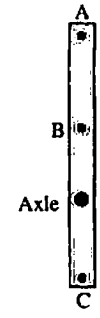
16. A bicycle is at rest on a smooth surface. A force is applied to the bottom pedal as shown. Does the bicycle roll forward (to the right), backward (to the left), or not at all? Explain.



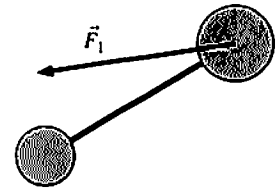
17. Four forces are applied to a rod that can pivot on an axle. For each force,
    a. Use a **black** pen or pencil to draw the line of action.
    b. Use a **red** pen or pencil to draw and label the moment arm, or state that $d = 0$.
    c. Determine if the torque about the axle is positive (+), negative (–), or zero (0). Write your answer in the blank.



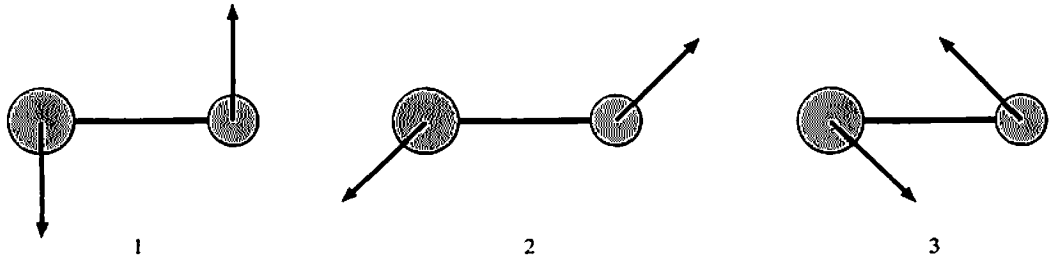
18. a. Draw a force vector at A whose torque about the axle is negative.
    b. Draw a force vector at B whose torque about the axle is zero.
    c. Draw a force vector at C whose torque about the axle is positive.



19. a. Draw a second force $\vec{F}_2$ that forms a couple with $\vec{F}_1$.
    b. Draw and label the distance l between their lines of action.
    c. Is the torque positive, negative, or zero? Explain.



20. The dumbbells below are all the same size, and the forces all have the same magnitude. Rank in order, from largest to smallest, the torques τ_1, τ_2, and τ_3.

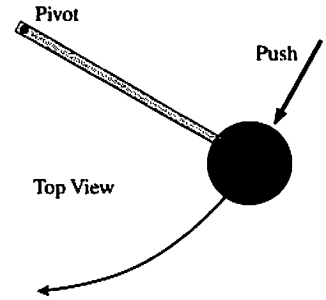


Order:

Explanation:

12.6 Rotational Dynamics

21. A student gives a quick push to a ball at the end of a massless, rigid rod, causing the ball to rotate clockwise in a *horizontal* circle. The rod's pivot is frictionless.



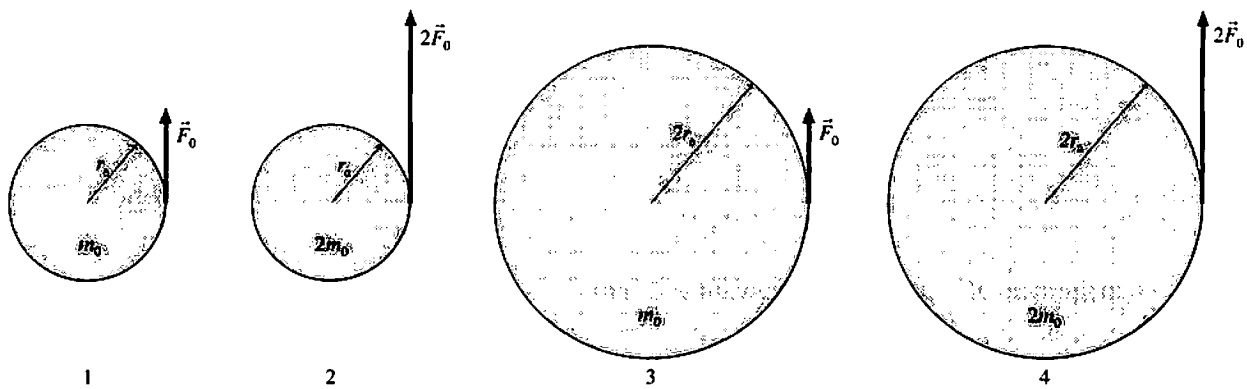
a. As the student is pushing, is the torque about the pivot positive, negative, or zero?

b. After the push has ended, does the ball's angular velocity
   i. Steadily increase?
   ii. Increase for awhile, then hold steady?
   iii. Hold steady?
   iv. Decrease for awhile, then hold steady?
   v. Steadily decrease?

   Explain the reason for your choice.

c. Right after the push has ended, is the torque positive, negative, or zero? _____

22. a. Rank in order, from largest to smallest, the torques τ_1 to τ_4.

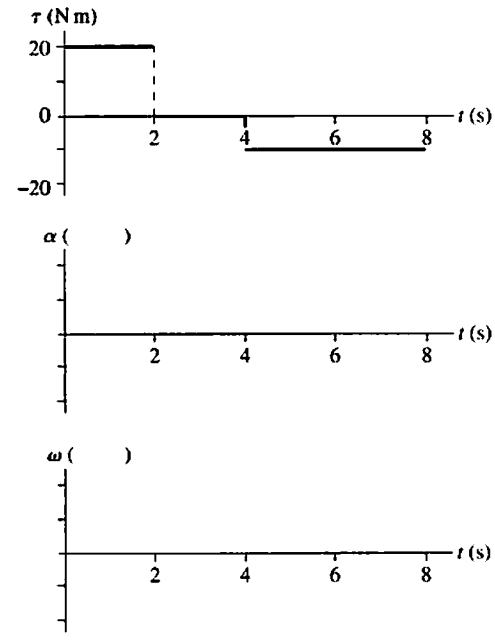


Order:

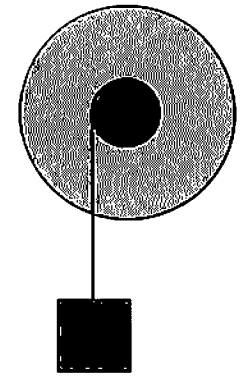
Explanation:

b. Rank in order, from largest to smallest, the angular accelerations α_1 to α_4.

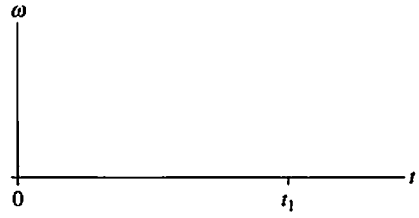
23. The top graph shows the torque on a rotating wheel as a function of time. The wheel's moment of inertia is 10 kg m^2. Draw graphs of α-versus-t and ω-versus-t, assuming $\omega_0 = 0$. Provide units and appropriate scales on the vertical axes.



24. The wheel turns on a frictionless axle. A string wrapped around the smaller diameter shaft is tied to a block. The block is released at $t = 0$ s and hits the ground at $t = t_1$.



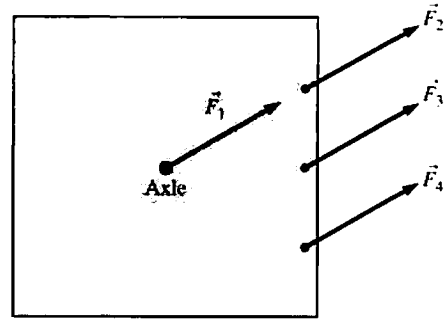
a. Draw a graph of ω-versus-t for the wheel, starting at $t = 0$ s and continuing to some time $t > t_1$.



b. Is the magnitude of the block's downward acceleration greater than g, less than g, or equal to g? Explain.

12.7 Rotation about a Fixed Axis

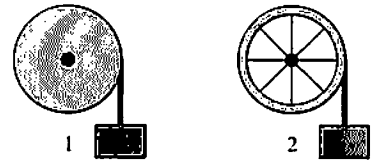
25. A square plate can rotate about an axle through its center. Four forces of equal magnitude are applied to different points on the plate. The forces turn as the plate rotates, maintaining the same orientation with respect to the plate. Rank in order, from largest to smallest, the angular accelerations α_1 to α_4.



Order:

Explanation:

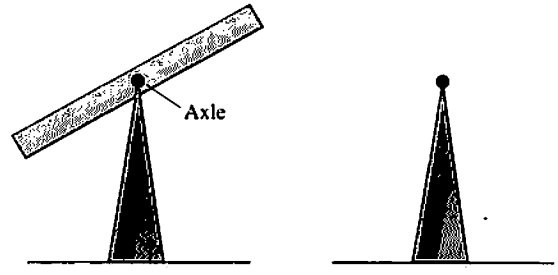
26. A solid cylinder and a cylindrical shell have the same mass, same radius, and turn on frictionless, horizontal axles. (The cylindrical shell has light-weight spokes connecting the shell to the axle.) A rope is wrapped around each cylinder and tied to a block. The blocks have the same mass and are held the same height above the ground. Both blocks are released simultaneously. The ropes do not slip.



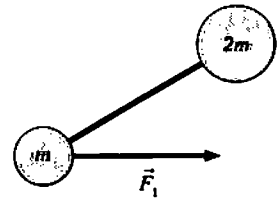
Which block hits the ground first? Or is it a tie? Explain.

12.8 Static Equilibrium

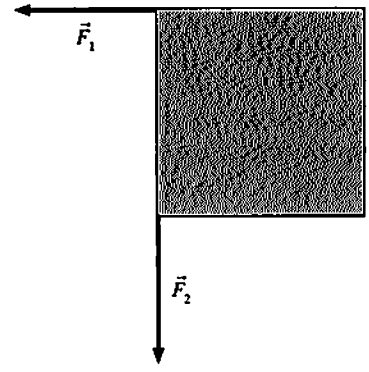
27. A uniform rod pivots about a frictionless, horizontal axle through its center. It is placed on a stand, held motionless in the position shown, then gently released. On the right side of the figure, draw the final, equilibrium position of the rod. Explain your reasoning.



28. The dumbbell has masses m and $2m$. Force $\vec{F}_1$ acts on mass m in the direction shown. Is there a force $\vec{F}_2$ that can act on mass $2m$ such that the dumbbell moves with pure translational motion, without any rotation? If so, draw $\vec{F}_2$, making sure that its length shows the magnitude of $\vec{F}_2$ relative to $\vec{F}_1$. If not, explain why not.

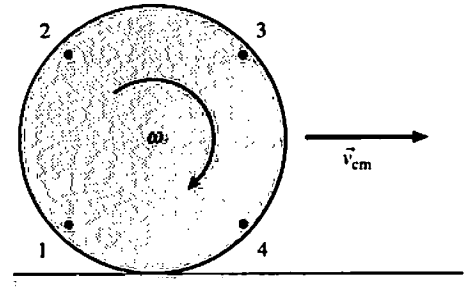


29. Forces $\vec{F}_1$ and $\vec{F}_2$ have the same magnitude and are applied to the corners of a square plate. Is there a *single* force $\vec{F}_3$ that, if applied to the appropriate point on the plate, will cause the plate to be in total equilibrium? If so, draw it, making sure it has the right position, orientation, and length. If not, explain why not.



12.9 Rolling Motion

30. A wheel is rolling along a horizontal surface with the center-of-mass velocity shown. Draw the velocity vector $\vec{v}$ at points 1 to 4 on the rim of the wheel.



12.10 The Vector Description of Rotational Motion

12.11 Angular Momentum of a Rigid Body

31. For each vector pair $\vec{A}$ and $\vec{B}$ shown below, determine if $\vec{A} \times \vec{B}$ points into the page, out of the page, or is zero.

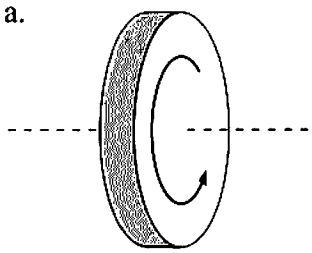
$\vec{A} \times \vec{B}$ _____ $\vec{A} \times \vec{B}$ _____ $\vec{A} \times \vec{B}$ _____ $\vec{A} \times \vec{B}$ _____

32. Each figure below shows $\vec{A}$ and $\vec{A} \times \vec{B}$. Determine if $\vec{B}$ is in the plane of the page or perpendicular to the page. If $\vec{B}$ is in the plane of the page, draw it. If $\vec{B}$ is perpendicular to the page, state whether $\vec{B}$ points into the page or out of the page.

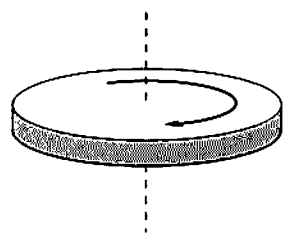
$\vec{B}$ _____ $\vec{B}$ _____ $\vec{B}$ _____

33. Draw the angular velocity vector on each of the rotating wheels.

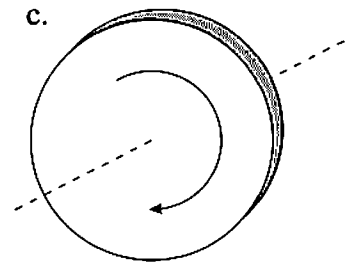
a.



b.

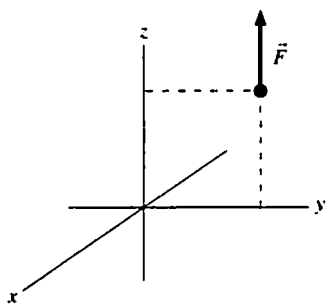


c.

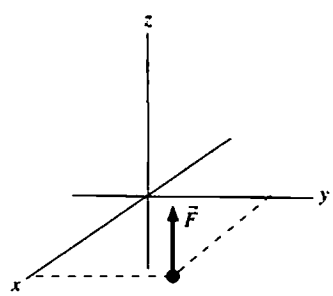


34. The figures below show a force acting on a particle. For each, draw the torque vector for the torque about the origin.
- Place the tail of the torque vector at the origin.
- Draw the vector large and straight (use a ruler!) so that its direction is clear. Use dotted lines from the tip of the vector to the axes to show the plane in which the vector lies.

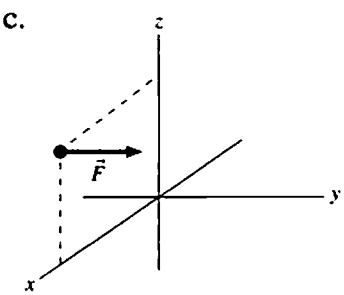
a.



b.

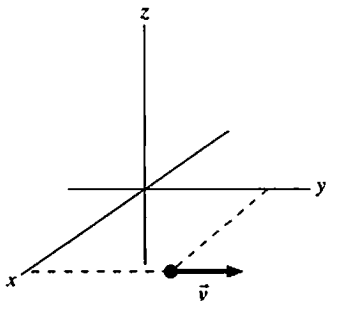


c.

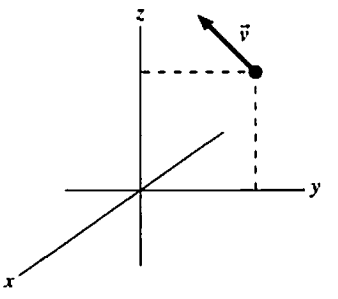


35. The figures below show a particle with velocity $\vec{v}$. For each, draw the angular momentum vector $\vec{L}$ for the angular momentum relative to the origin. Place the tail of the angular momentum vector at the origin.

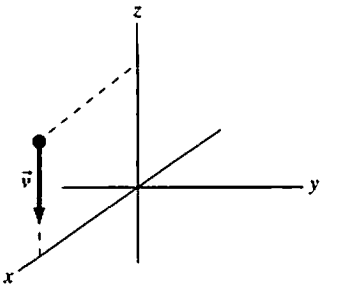
a.



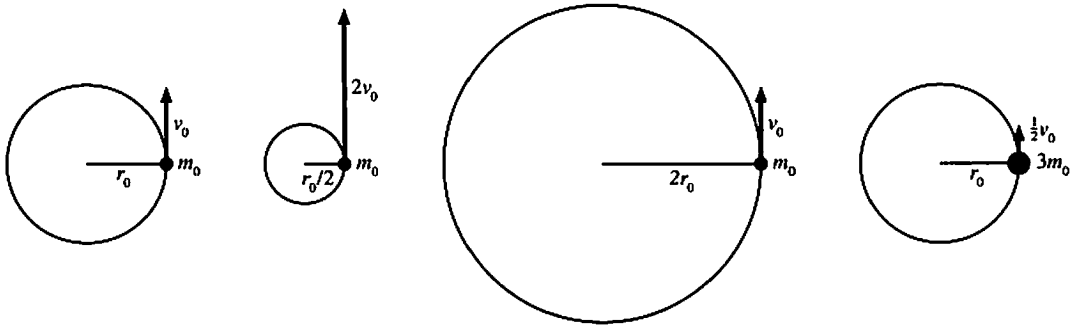
b.



c.



36. Rank in order, from largest to smallest, the angular momenta L_1 to L_4.



1 2 3 4

Order:

Explanation:

37. Disks 1 and 2 have equal mass. Is the angular momentum of disk 2 larger than, smaller than, or equal to the angular momentum of disk 1? Explain.