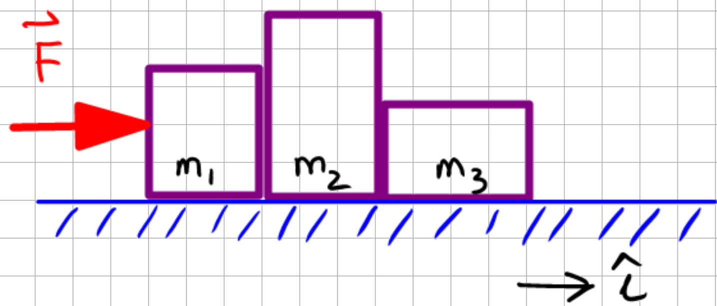
# Pushing crates: normal forces; no friction




the combined system accelerates due to $\vec{F} = F\hat{\imath}$

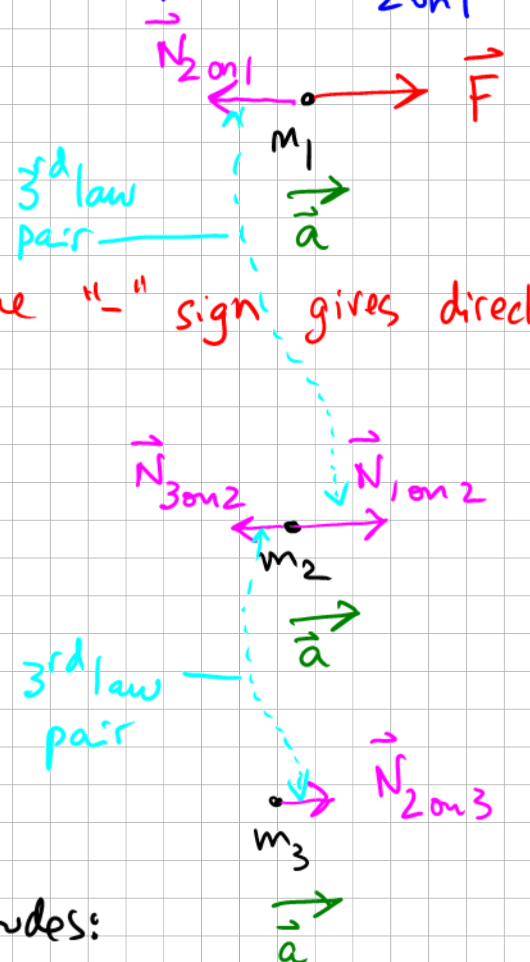
① $F = (m_1 + m_2 + m_3)\, a$

The vertical forces cancel, e.g., $m_1 g = N_{\text{floor on } 1}$

Look at $m_1$:
- it also accelerates with $a$
- the net force acting on it has to be less than $F$!

Since $m_1$ exerts a normal force to the right, on $m_2$, $\vec{N}_{1\,on\,2}$ there is an action-reaction partner force: $\vec{N}_{2\,on\,1}$.

The free-body diagram for $m_1$:




② $$m_1 a = F - N_{2\,on\,1}$$

a magnitude, the "−" sign gives direction

Now the FB diagram for $m_2$:

③ $$m_2 a = N_{1\,on\,2} - N_{3\,on\,2}$$

and for $m_3$:

④ $$m_3 a = N_{2\,on\,3}$$

The third law states for the magnitudes:

$$N_{1\,on\,2} = N_{2\,on\,1} \quad (= N_{12})$$

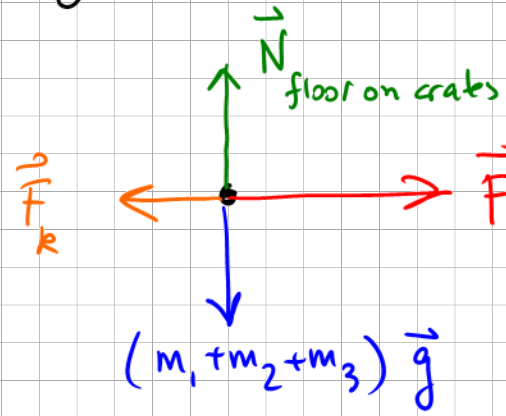
$$N_{2\,on\,3} = N_{3\,on\,2} \quad (= N_{23})$$

The three FB diagrams are consistent with acceleration $\vec{a}$ for each mass!

How does kinetic friction modify matters?

Combined system:

$\bullet = m_1 + m_2 + m_3$




$$F_k = \mu_k N_{\text{floor on crates}}$$

$$= \mu_k (m_1+m_2+m_3) g$$

Q: is all that happens a reduction of $\vec{F}$ $\left(\rightarrow (F - F_k)\hat{i}\right)$?

It would imply that each crate contributes $-\mu_k m_i g \hat{i}$ towards friction; $i = 1, 2, 3$.


(2') $\quad m_1 a = F - N_{12} - \mu_k m_1 g$

Likewise: (3') $\quad m_2 a = N_{12} - N_{23} - \mu_k m_2 g$

and: (4') $\quad m_3 a = N_{23} - \mu_k m_3 g$

To confirm: add up (2') (3') (4'):

$$(m_1 + m_2 + m_3)\, a = F - \mu_k (m_1 + m_2 + m_3) g$$

Note how the normal forces between the crates cancel out when the equations are added up.

$(m_1 + m_2 + m_3)$ - motion $\hat{=}$ center-of-mass motion