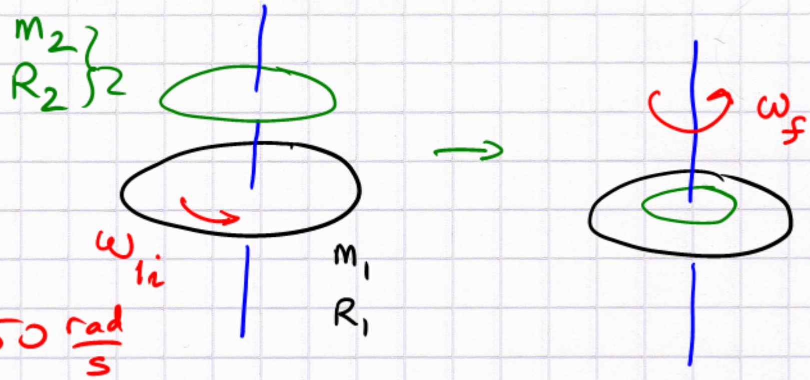
Week 11 9.46



$m_1 = 20$ kg

$R_1 = 2.0$ m

$m_2 = 7.0$ kg

$R_2 = 1.0$ m

$\omega_{1i} = 50 \frac{rad}{s}$

a) is there a torque ? → lower disks, → upper

b) external system with zero torque ? → $\vec{L}$ conservation

c,d) $\vec{L}_{in} = ?$, $\vec{L}_{fin} = ?$ e) $\omega_f = ?$

Solution.

a) disk 2 spins up (from $\omega_{2i} = 0$) $\therefore \exists \; \vec{\tau}_{on\,2}$

disk 1 slows down from ω_{1i} $\therefore \exists \; \vec{\tau}_{on\,1}$

orientation : $\vec{\omega} = (0,0,\omega_z)$ $\therefore$ $\vec{\tau} = (0,0,\tau_z)$

b) axle is fixed, but about $\hat{k}$ the combined system is free (ignored friction in axle bearings!)

$(\vec{L}_1)_z + (\vec{L}_2)_z = (\vec{L}_0)_z$ where $L_{0z} = I_1 \omega_{1i}$ (>0 as shown)

c) $L_i = I_1 \omega_{1i} = 2.0 \times 10^3 \frac{kg\,m^2}{s^2}$ d) $L_f = (I_1 + I_2)\,\omega_f = 43.5\,\omega_f$

There is a pair of torques : $\tau_{z\,on\,1} = -\tau_{z\,on\,2}$

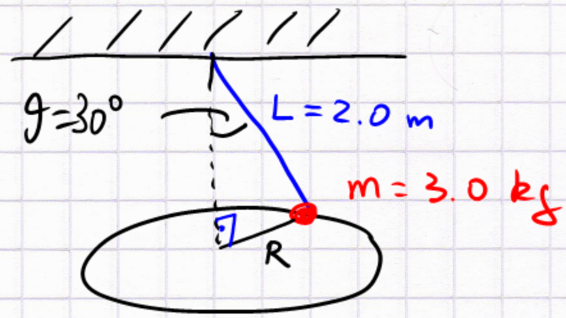
but no external torque

Analogous to totally inelastic collision

$$\omega_f = \frac{I_1}{I_1 + I_2}\,\omega_{1i} = \underbrace{\frac{m_1 R_1^2}{m_1 R_1^2 + m_2 R_2^2}}_{.9195}\,\omega_{1i} = 46.0 \;\frac{rad}{s}$$

9.56 Conical pendulum

$L_z = ?$



m executes circular motion.

Solution

The circular motion of m is perpendicular to $\hat{k}$

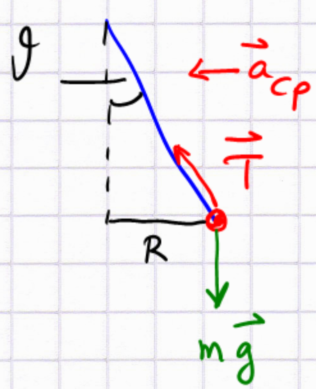
$\vec{L} = \vec{r} \times \vec{p}$ by the RH rule $\vec{L} = L_z \hat{k}$

$L_z = I\omega$ where $I = mR^2$

$\sin\vartheta = \frac{R}{L}$ $\therefore$ $L = |L_z| = (mL^2 \underbrace{\sin^2\vartheta}_{1/4})\,\omega$ as $\sin\frac{\pi}{6} = \frac{1}{2}$

$L_z = \omega_z \left(\frac{1}{4} mL^2\right)$ ⇝ can we know more?

FB diagram, side view



$|T_y| = mg$ $|T_x| = \frac{mv^2}{R} = mR\omega^2$

$T_x = T\sin\vartheta$ $T_y = T\cos\vartheta$

$= \frac{1}{2}T$ $= \frac{1}{2}\sqrt{3}\,T = .866T$

y: $.866T = mg = 3.0 \cdot 9.8$ $\therefore$ $T = 34.0\ N$

or: $T = \frac{mg}{\cos\vartheta} \longrightarrow$ x: $\frac{\cancel{m}g}{\cos\vartheta} \cdot \cancel{\sin\vartheta} = \cancel{m}R\omega^2$

$\downarrow$ $L\cancel{\sin\vartheta}\,\omega^2$

$\therefore \omega^2 = \frac{g}{L\cos\vartheta} = \frac{9.8}{2.0 \cdot .866} = 5.66 \left(\frac{rad}{s}\right)^2$ $\omega = 2.38\ \frac{rad}{s}$

$L_z = I\omega_z = 3.0\,(1.0)^2 \cdot 2.38 = 7.14 \rightarrow$ $L = 7.1\ \frac{kg\,m^2}{s}$

11.7

$$y = 9.4 \sin(15t) \quad \text{in SI}$$

$$v_{max} = ?$$

Solution.

$$v_y = 9.4 \cdot 15 \cos(15t)$$

This is maximum when $\cos(15t) = +1$

$$v_y^{max} = 9.4 \cdot 15 = 141 \ \frac{m}{s}$$

$$v_y^{max} = 140 \ m/s$$

Note: When $\cos(15t) = -1$

we get the minimum velocity

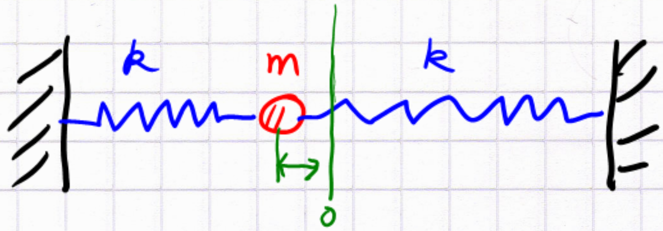
$$v_y^{min} = -140 \ m/s$$

11.17

$m = 2.4\ kg$ $\quad k = 400\ \frac{N}{m}$

equal springs!

ignore gravity (air track)



$f = ?$

Solution.

Displacing m from equilibrium by ΔX results in the left spring providing $-k\,\Delta X$

and the right " " $-k\,\Delta X$

Thus, $F_{net} = -2k\,\Delta x \quad \therefore \quad m x'' = -2kx$

(as we measure using $x_0 = 0$ as a coordinate)

Compare this to $\quad m x'' = -k_N x \rightarrow \omega_N = \sqrt{\frac{k_N}{m}}$

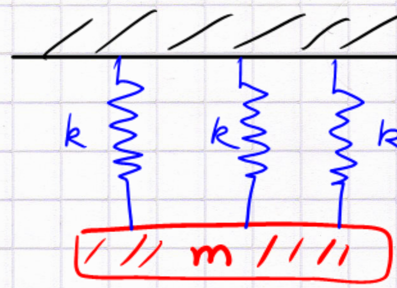
$\therefore \quad \omega = \sqrt{\frac{2k}{m}}$ $\qquad f = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} = \frac{1}{6.28}\sqrt{\frac{800}{2.4}}\ \text{Hz}$

$= 2.91\ \text{Hz}$

$f = 2.9\ \text{Hz}$

11.20

$k = 40 \frac{N}{m}$ $m = 20\ kg$



$f = ?$

Solution

Idea: the three springs with constants k each act together as a single spring with k_{eff}

Q: what is k_{eff} ?

For a given displacement Δy, mass m is stretching / compressing three springs at the same time by the same amount. The three forces add up:

$$F_{net} = -3\,k\,\Delta y$$ (Δy = displacement from vertical equilibrium)

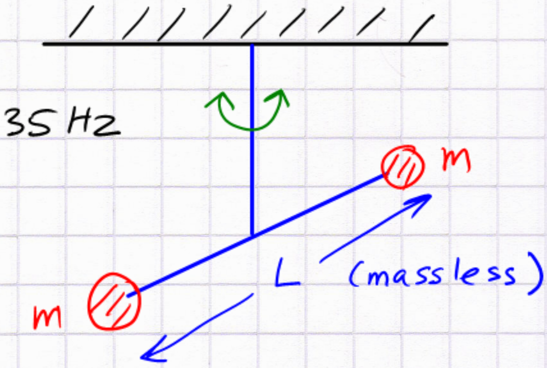
$\therefore$ $k_{eff} = 3k$ $\Rightarrow$ n parallel springs: $k_{eff} = \sum_{i=1}^{n} k_i$

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{eff}}{m}} = \frac{1}{2\pi}\sqrt{\frac{3k}{m}} = \frac{1}{6.28}\sqrt{\frac{3 \cdot 40}{20\,kg}\frac{N}{m}}$$

$f = 0.39\ Hz$

Note: springs in series are "tricky" $\rightarrow$ the displacement is made up of $\Delta y = \sum \Delta y_i$

11.30 Torsional oscillator



$L = 1.5\,m$ $\quad m = 2.0\ kg$ $\quad f = 0.035\,Hz$

torsional constant $\kappa = ?$

Solution

Torsional oscillator: $I\alpha = \tau$ (about z-axis)

$I = \sum m_i r_i^2$

$$2m\left(\frac{L}{2}\right)^2 \frac{d^2\vartheta}{dt^2} = \underbrace{-\kappa\vartheta}_{\text{restoring torque towards } \vartheta_0 = 0}$$

equilibrium: $\vartheta_0 = 0$

$$= m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 = 2m\frac{L^2}{4} = \frac{mL^2}{2}$$

$$\vartheta'' = -\frac{\kappa}{I}\vartheta$$

is solved by $\vartheta = A\cos\sqrt{\frac{\kappa}{I}}\,t + B\sin\sqrt{\frac{\kappa}{I}}\,t$

$$\therefore\ \omega^2 = (2\pi f)^2 = \frac{\kappa}{I} \qquad \therefore\ \kappa = I\,(2\pi f)^2 = \frac{mL^2}{2}(2\pi f)^2$$

$$\kappa = m L^2\, 2\pi^2 f^2 = 2.0 \cdot 2.25 \cdot 2 \cdot (3.14)^2\,(3.5\times10^{-2})^2\ SI$$

$$\kappa = 0.109\ \frac{kg\ m^2}{s^2} = 0.11\ \frac{Nm}{rad}$$

$\frac{1}{rad}$ reminds us that $-\kappa\vartheta$ is a torque with ϑ supplied in rads, not degrees!