Q1: A 250g air track glider is attached to a spring with constant $k = 4.0$ N/m. The damping constant due to air resistance is $b = 0.015$ kg/s. The glider is pulled out 20 cm from equilibrium and released

How many oscillations will it make during the time in which the amplitude decays to $1/e$ of its initial value?

(K.14.29)

Solution.

This is a damped oscillation described by Newton's 2nd:

$$m \underbrace{\frac{d^2x}{dt^2}}_{a} + \underset{\substack{\uparrow \\ \text{damping const in } \frac{kg}{s}}}{b} \underbrace{\frac{dx}{dt}}_{v} + \underbrace{kx}_{\text{Hooke}} = 0 \quad \text{for equilibrium} = 0$$

For small b the circular frequency $\omega \approx \sqrt{\frac{k}{m}}$

but in general $\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

This formula allows one to discuss critical damping when $\omega = \frac{2\pi}{T} \to 0$ (infinite period, no oscillation)

Solution for $x(t)$: $x(t) = A e^{-t/2\tau} \cos(\omega t + \varphi_0)$

$\tau \equiv m/b$ ← damping time

The phase φ_0? For $t = 0$ glider is released from $A = 20$ cm, $v \approx 0$ ∴ $\varphi_0 = 0$

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \xrightarrow[\text{in SI}]{} \sqrt{\frac{4.0}{0.25} - \frac{0.015}{4 \cdot (0.25)^2}} \to 4.0 \ \frac{\text{rad}}{s}$$

$T = \frac{2\pi}{\omega} \to 1.57$ s

(b): $\frac{1}{e} A = A e^{-t_0/(2\tau)}$ ∴ $t_0 = 2\tau = 2\frac{m}{b}$ $= 33.3$ s

Nr of oscillations : $n = t_0/T = \frac{33.3}{1.57} = 21$ oscillations.

Q2: What is the period of a 1.0 m long pendulum on

(a) earth (b) venus ?

(K. 14.24)

Solution.

Assuming small-amplitude harmonic oscillations.

$$T = 2\pi \sqrt{\frac{L}{g}}$$ (from analogy with harmonic oscillator, or spring-mass system)

On earth: $g = 9.8 \ m/s^2$ $\therefore T = \sqrt{\frac{1.0}{9.8}} \cdot 6.28 = 2.0 \ s$

On venus $$g_v = \frac{G M_{venus}}{R^2_{venus}} = \frac{6.67 \times 10^{-11} \cdot 4.88 \times 10^{24}}{(6.06 \times 10^6)^2}$$

$$= 8.86 \ m/s^2$$

$$\therefore \quad T = 2\pi \sqrt{\frac{L}{g_v}} = 2.1 \ s$$

(not so different!)

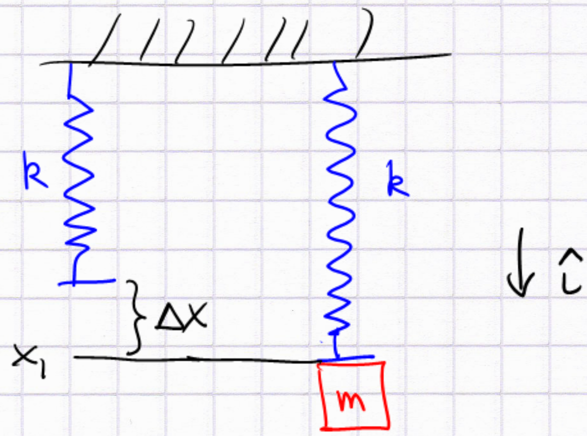
Q3: A spring with constant $k = 15.0$ N/m hangs from the ceiling. A ball is attached to the spring and allowed to come to rest. It is then pulled down 6.0 cm and released. If the ball makes 30 oscillations in 20 sec, what are (a) its mass (b) maximum speed?

(K. 14.19)

Solution.

Vertical oscillator

before attaching m $\to x_0$



step 1: figure out new equilibrium position x_1 or its displacement $\Delta x = x_1 - x_0$

$$k|\Delta x| = mg$$

gravity (mg) is balanced by spring stretch force

step 2: ignore gravity (since it is balanced by $k\Delta x$) and have simple harmonic motion about new equilibrium

Based on the info the oscillator is undamped

Determine period $T = \dfrac{20\,s}{30 \text{ oscillations}} = 0.667\ s$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.667\,s} = 9.425\ \frac{rad}{s}$$

a) Mass m: $\omega = \sqrt{\frac{k}{m}}$ $\therefore$ $m = \dfrac{k}{\omega^2} = \dfrac{15.0}{(9.425)^2} = 0.169\ kg$

b) v_{max}: $x = A\cos(\omega t + \varphi_0)$ $\therefore$ $v = \omega A(-\sin(\omega t + \varphi_0))$

$\therefore$ $v_{max} = \omega A = 9.425 \cdot 0.06 = 0.565\ \frac{m}{s}$

Q4 The velocity of an object in simple harmonic motion is given by $v_x(t) = (-0.35\ \frac{m}{s}) \sin(20t + \pi)$ where t is in sec.

a) What is the first time after t = 0 sec. at which the velocity is $-0.25\ \frac{m}{s}$?

b) What is the object's position at that time ?

(K. 14.44)

Solution.

$$v_x(t_1) = -0.25 = -0.35 \sin(20t_1 + \pi)$$

$$\therefore \sin(20t_1 + \pi) = \frac{.25}{.35} = .714$$

$$20t_1 + \pi = \sin^{-1}(.714) = .796 \text{ rad}$$

$$\therefore t_1 = \frac{.796 - 3.14}{20} = -.117 \text{ s}$$

This is BEFORE t = 0, so add 2π $\therefore t_1 = \frac{.796 + 3.14}{20} = .20\text{s}$
$= .197\text{s}$

b) Anti derivative : $x(t) = \frac{.35}{20} \cos(20t + \pi)$ (in SI, i.e., m)

$$x(t_1) = \frac{.35}{20} \cos(20 \times .197 + 3.14) = 1.2 \text{ cm}$$