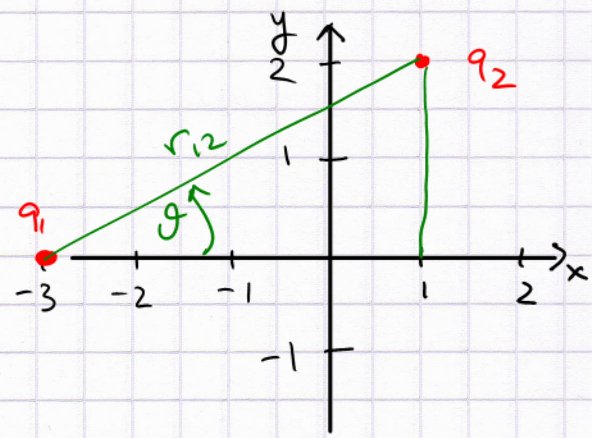
Week 13 17.14 A point charge, $q_1 = +2.5\,C$, is located at $(x_1, y_1) = (-3.0, 0.0)$ m, while $q_2 = +4.0\,C$ @ $(x_2, y_2) = (1.0, 2.0)$ m

a) Force exerted by q_1 on q_2; b) force exerted by q_2 on q_1?

Solution. Start with a figure:



The distance: $r_{12}^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

$= (-3.0 - 1.0)^2 + (0 - 2.0)^2$

$= 16.0 + 4.0 = 20.0\ m^2$

The magnitude of $F_{1\,on\,2}$ and $F_{2\,on\,1}$:

$$F = K \frac{|q_1 q_2|}{r_{12}^2} \qquad K = 9.0 \times 10^9 \frac{N m^2}{C^2}$$

$$= 9.0 \times 10^9 \frac{2.5 \times 4.0}{20.0} N = 4.5 \times 10^9 N$$

The direction: $\sin\vartheta = \frac{y_2 - y_1}{r_{12}} = \frac{2.0}{\sqrt{20.0}} = .447$

$\cos\vartheta = \frac{x_2 - x_1}{r_{12}} = \frac{4.0}{\sqrt{20.0}} = .894$

$\vartheta = 26.6°$

$F_x = F\cos\vartheta$ $\qquad F_y = F\sin\vartheta$

$= 4.02 \times 10^9$ N $\qquad = 2.0$ GN

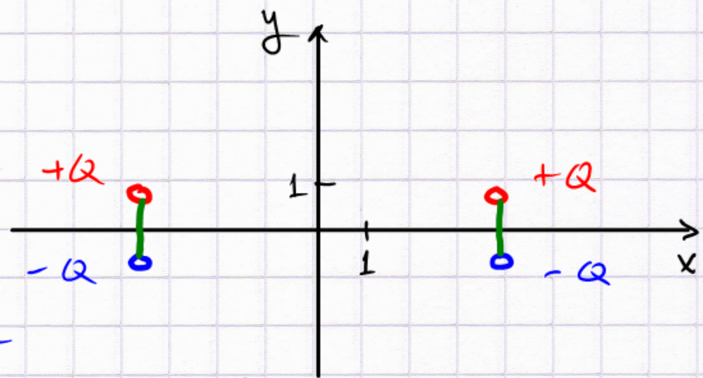
$= 4.0$ GN

or $\vec{F}_{on\,1} = (4.5 \times 10^9 N, \ \vartheta = 26.6°)$

b) This force is in the opposite direction: $(-4.0, -2.0)$ GN

or: magnitude $= 4.5 \times 10^9 N$, $\vartheta = 180 + 26.6 = 206.6°$

17. 22 Two electric dipoles $\pm Q$ with $Q = 5.0\ C$ and $d = 1.5\,m$ are parallel to each other and separated by $7.5\,m$. The dipoles themselves are fixed. Find the electric force between them.



Solution.

1) We ignore the forces between $\pm Q$ within each dipole, they are balanced by reaction forces from the rods holding $\pm Q$ in place

2) $+Q$ on the left is repelled by $+Q$ on the right, and attracted by $-Q$ on the right: $\vec{F}_{on\,+Q,L} = -\hat{\imath}\left(\frac{KQ^2}{7.5^2}\right) + \vec{F}_{on\,+Q,L\ from\,-Q_R}$

3) Likewise $\vec{F}_{on\,-Q,L} = -\hat{\imath}\left(\frac{KQ^2}{7.5^2}\right) + \vec{F}_{on\,-Q,L\ from\,+Q_R}$

4) The y-component of $\vec{F}_{on\,+Q,L\ from\,-Q_R}$ cancels that of $\vec{F}_{on\,-Q,L\ from\,+Q_R}$ but the x-components of these forces add to yield an attraction, which weakens the overall repulsion.

5) $\left(\vec{F}_{on\,+Q,L\ from\,-Q,R}\right)_x = |\vec{F}|\cdot\cos\vartheta$ where $|\vec{F}| = \frac{KQ^2}{r_{12}^2}$

and $r_{12}^2 = 7.5^2 + 1.5^2 = 58.5 \quad \therefore |\vec{F}| = 3.85\times10^9\ N$

$\cos\vartheta = \frac{7.5}{\sqrt{58.5}} = 0.981 \ \therefore \quad F_x = 3.775\times10^9\ N$

6) Thus, the left dipole $\vec{F}_{on\,left} = -\hat{\imath}\,(2.4)\ GN$

$+\hat{\imath}\ (2\times3.775)\ GN$

$= -\hat{\imath}\ (0.45)\times10^9\ N$

and the force on the right dipole: $+\hat{\imath}\cdot4.5\times10^8\ N$

17.32 Dust particle $m = 1.0\,\mu g$ goes through $\vec{E}$, where $E = 500\ \frac{N}{C}$. Electric force equals weight. $q_{dust} = ?$, how many excess electrons does q_{dust} correspond to?

Solution

$$F = mg = 1.0 \times 10^{-6} \times 10^{-3} \times 9.8\ N$$
$$= 9.8 \times 10^{-9}\ N$$

$$F_E = qE = q \cdot 500 = 9.8 \times 10^{-9} \quad \text{(in SI)}$$

$$\therefore \quad q = \frac{9.8}{500}\ nC = 0.020 \times 10^{-9}\ C$$
$$= 2.0 \times 10^{-11}\ C$$

$$n = \frac{q}{e} = \frac{2.0 \times 10^{-11}\ C}{1.6 \times 10^{-19}\ C} = 1.25 \times 10^{8}$$

= 125 million excess electrons

17.40 An infinitely massive proton is orbited by an electron with a circular orbit of $r = 0.10\ \text{nm} = 1.0 \times 10^{-10}\ \text{m}$.
The centripetal acceleration of the e^- is provided by the Coulomb force from the proton. What is the electron's speed?

Solution.

In uniform circular motion

$$m\, a_{cp} = m \frac{v^2}{r} = F_{net} = \frac{K e^2}{r^2}$$

$$\therefore \quad v^2 = \frac{K e^2}{m\, r}$$

$$v^2 = \frac{9.0 \times 10^{9}\ (1.60 \times 10^{-19})^2}{9.11 \times 10^{-31}\ \ 10^{-10}} \qquad \text{SI}$$

$$v^2 = 2.53 \times 10^{12}\ \frac{m^2}{s^2}$$

$$v = 1.6 \times 10^{6}\ m/s$$

$$= 0.0053\ c \qquad c = 3.0 \times 10^{8}\ \frac{m}{s}$$

½ % of the speed of light

Bohr model of H atom : $r = 0.53 \times 10^{-10}\ m$

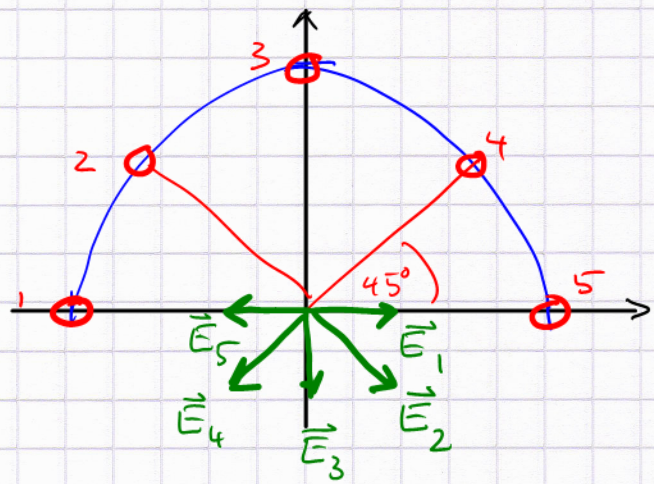
$\rightarrow \sim 1\%$ of c is the speed.

go to a $_{92}U$ nucleus $\rightarrow$ get 92 times the speed

K-shell electrons in heavy atoms are relativistic!

17.46 Five point charges on semi-circle, equispaced

$q = 7.5\ C$, $r = 2.3\ m$; $\vec{E}$ at the origin?



Solution

1) $E_i = \dfrac{Kq}{r_i^2}$

points away from q

2) Magnitudes of all $\vec{E}_i$ are the same, $r_i = 2.3\,m$

3) $\vec{E}_1$ and $\vec{E}_5$ cancel; $E_{4,x}$ and $E_{2,x}$ cancel

$\therefore\ \vec{E}_{tot} \sim -\hat{y}$ $\qquad E_{tot} = E_3 + 2\,E_{2,y}$

$$E_3 = \frac{Kq}{r^2} = \frac{9.0 \times 10^9 \times 7.5}{2.3^2} = 12.8 \times 10^9\ \frac{N}{C}$$

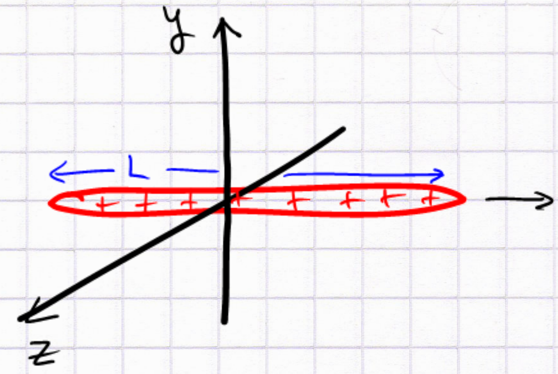
$$|E_{2y}| = E_3 \sin 45^\circ = 12.8 \times 10^9 \left(\tfrac{1}{2}\sqrt{2}\right) \frac{N}{C}$$

$$\therefore\ E_{tot} = (1 + 1.41) \cdot 12.8 \times 10^9\ \frac{N}{C} = 3.1 \times 10^{10}\ \frac{N}{C}$$

$$E_{tot} = -\,3.1 \times 10^{10}\ \hat{y}\ \frac{N}{C}$$

17.56 $\quad Q = 1.0\,C$

$L = 1.0 \times 10^6\,m$



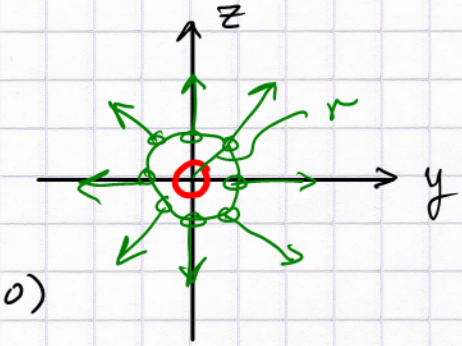
linear charge density $\lambda = \frac{Q}{L}$

a) $\vec{E} = ?$ 10 cm away from rod

b) Gaussian surface ?

Solution.

a) $\vec{E}$ - field points radially away from rod :



b) Gaussian surface :

a cylinder of radius R and length X centered on $(y,z) = (0,0)$

c) The surface contains $q = \lambda \cdot X = \frac{Q}{L} X$ charge,

we don't look at the circular end pieces

d) Gauss : $\quad \Phi_E = \frac{q}{\varepsilon_0} \quad$ electric flux

$$\Phi_E = E \cdot A = E \cdot X \cdot 2\pi r$$

$$\therefore \quad E \cdot \cancel{X}\, 2\pi r = \frac{1}{\varepsilon_0} \frac{Q}{L} \cancel{X}$$

$$E = \frac{Q}{2\pi\varepsilon_0 L r} = \frac{\lambda}{2\pi\varepsilon_0 r}$$ cf eq. (17.15)

The $\vec{E}$ field points radially away from the line of charge, and falls off $\sim \frac{1}{r}$

insert value $r = 10\,cm = 0.1\,m$: $\quad E = 1.8 \times 10^5 \frac{N}{C}$