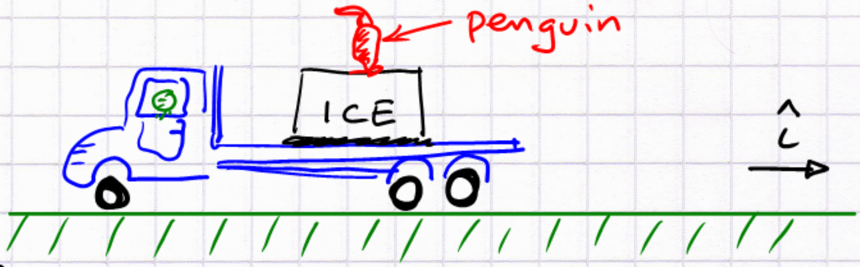
Week 2 2.40

A truck moving at 20 km/h comes to a stop. The ice block is frozen to the flatbed. What happens to the penguin? Show $v(t)$ graphs for both the truck and the penguin. Does the driver move differently from the penguin? If so, why?

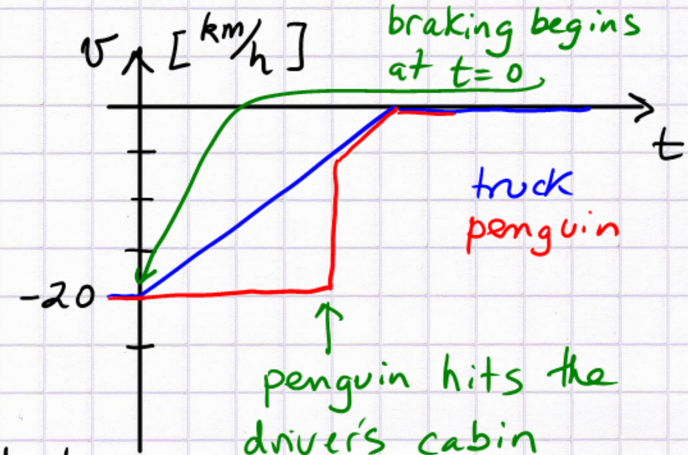


Solution.

The initial conditions for the truck and penguin motion are the same. $v_x < 0$, since it is shown moving to the left.

Application of the brakes results in a reduction of speed, v_x becomes less negative, then reaches zero, i.e., $a_x > 0$.

It is not unreasonable to assume constant-acceleration braking action.



Inertia keeps the penguin in motion until he hits an obstacle (or falls to the ground ahead of the stopped truck)

The driver is locked to the truck: 1) holds steering, 2) friction w. seat, 3) seatbelt holds him/her back

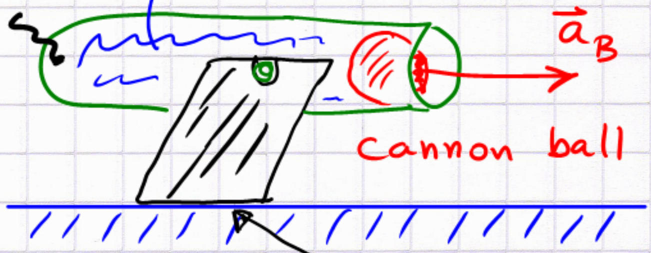
2.49

$m_B = 3.2 \text{ kg}$

$a_B = 2500 \ \frac{m}{s^2}$

$a_c = -0.76 \ m/s^2$

$M_c = ?$



Solution.

Two principles are at work here:

1) Newton's 3rd law: action-reaction force pair

The explosion pushes the ball to the right and the cannon to the left

2) Newton's 2nd law $\vec{F} = m\vec{a}$ applies separately to both objects:

$$\vec{F}_{\text{on ball}} = m_B \vec{a}_B$$

along $\hat{i}$ →

$\vec{F}_{\text{on ball}} + \vec{F}_{\text{on cannon}} = 0$!

(positive = to the right)

forces in 1d = scalar with sign!

$$\left.\begin{aligned} F_{\text{on ball}} &= m_B a_B \\ F_{\text{on cannon}} &= m_c a_c \end{aligned}\right\}$$

(to the left)

$(a_c < 0)$

$$F_{\text{on ball}} + F_{\text{on cannon}} = 0$$

$$m_c = \frac{-m_B a_B}{a_c} = 11{,}000 \text{ kg}$$

now a_B, a_c are magnitudes!

NB: we could have set the force magnitudes equal,

then $|F_{\text{on cannon}}| = m_c |a_c| > 0$!

and $m_c = \frac{m_B a_B}{|a_c|}$!

3.12 Hockey puck, $v_0 = 50$ m/s, $v_{fin} = 35$ m/s
(slowed down by friction on ice). a) it traveled $d = 35$ m
during the slowdown; $a = ?$ b) given $m = 0.11$ kg
what is the horizontal force?

Solution. a) Constant acceleration kinematics:

$$v_f^2 = v_0^2 + 2a\,\Delta x \qquad \Delta x = d$$

$$\frac{v_f^2 - v_0^2}{2d} = a \qquad \therefore\ a = -18\ \mathrm{m/s^2}$$

This is $\approx 2g$ in magnitude, not very reasonable! (very rough ice)

NB: $a < 0$ means negative acceleration (slowdown)
it happens since $v_f^2 < v_0^2$

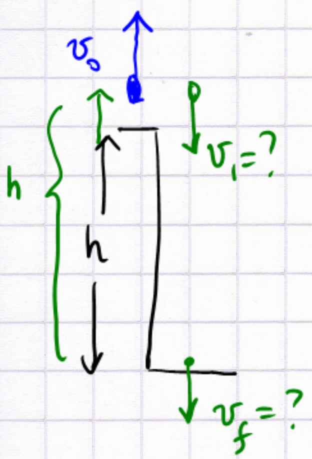
b) Now use the 2nd law $F = ma$ (1dim. vector, with sign!)

$$F = 0.11\ \mathrm{kg}\left(-18\ \frac{\mathrm{m}}{\mathrm{s^2}}\right) = -2.0\ \mathrm{N}$$

$$\left(1\ \mathrm{N} = 1\ \frac{\mathrm{kg\,m}}{\mathrm{s^2}}\right)$$

Is this force reasonable?
Compare to the weight: $mg = 1.1$ N
very large friction coefficient (>1)

3.22



A bullet is fired vertically at the edge of a cliff of height h (neglect the height of the person holding the rifle; neglect air drag, neglect horizontal displacement)

a) speed of bullet as it passes the cliff edge on the way down $v_1 = ?$

b) what is the final bullet speed before impact?

Solution. Constant-acceleration Kinematics $(t_0 = 0)$

① $v = v_0 + at$, ② $s = s_0 + v_0 t + \frac{1}{2} a t^2$,

③ $v^2 = v_0^2 + 2a(s - s_0)$ ← follows from the first two, by eliminating t

In our case: $\uparrow = \hat{j}$, s = vertical displacement $\uparrow y$

$\therefore \quad a = -g$

a) From ③ it follows that the returning bullet has the same speed when it passes the original height.

$v_1 = -v_0$ (velocity) $\quad |v_1| = |v_0|$

(specify whether v_1 is speed or velocity!)

b) The same problem as shooting straight down the cliff!

$$v_f^2 = v_0^2 + 2a(s_f - s_0)$$

watch out for signs!

$$v_f^2 = v_0^2 - 2g(0 - h)$$

↙ chose origin at the bottom, $s_0 = h$, $s_f = 0$.

$$v_f = \pm\sqrt{v_0^2 + 2gh}$$

I could have chosen: $s_0 = 0$, $s_f = -h$

Need "−" for velocity, + for speed

3.26 Motivated mule, applies constant force to cart whether empty or loaded

(how realistic is that?) perhaps quite reasonable!

$m_c = 180\ kg$

$v_f = 5.0\ m/s$ after $t_f = 10\ s$

Now add wood: $m_W = 540\ kg$; when does it reach v_f ?

Solution Data from part 1 allows us to determine the constant acceleration a

$v_f = a\, t_f \quad (t_0 = 0) \quad \therefore \quad a = \frac{v_f}{t_f} = 0.5\ \frac{m}{s^2}$

With this we determine the force the motivated mule can apply from the 2nd law: $F_0 = m_c a = 90\ N$

Now we get the acceleration when the cart is loaded

$(m_c + m_W)\, a_L = F_0 \quad \therefore \quad a_L = \frac{F_0}{m_c + m_W} = \frac{90\ N}{720\ kg}$

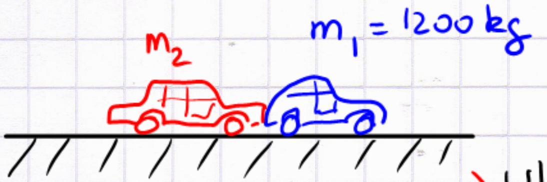
$a_L = 0.125\ \frac{m}{s^2}$ (keep an extra digit) "1/8" is exact

When does it reach v_f ? $\quad v_f = v_0 + a_L \tilde{t}_f$

$v_0 = 0$

$\tilde{t}_f = \frac{v_f}{a_L} = \frac{5\ m/s}{0.125\ m/s^2} = 40\ s$

3.33



Red car (m_2) pushes m_1.
It accelerates (constantly) for 1 min to reach $v = 2.5\ m/s$.

a) What is the normal force between the bumpers?

b) The total distance ΔS to be covered is 2 km. How long does it take?

Solution. a) Newton's 2^{nd} law allows us to determine

$$F_{on\ m_1} = m_1\, a, \quad \text{where} \quad a = \frac{2.5\ m/s}{60\ s} = 4.17 \times 10^{-2}\ \frac{m}{s^2}$$

one extra.

$$F_{on\ m_1} = 1200 \cdot 4.17 \times 10^{-2}\ N$$
$$= 50.0\ N$$

[By Newton's 3^{rd} we know $F_{on\ m_2} = -50.0\ N$]

b) We need to separate the two phases, with and without acceleration!

$$\Delta S_1 = \frac{1}{2} a t_1^2 \quad \text{where } t_1 = 60\,s$$

$$\Delta S_1 = 75\ m \quad \therefore \quad \Delta S_2 = 2000 - 75\ m = 1925\,m$$

In phase 2 $a = 0$, i.e., $\Delta S_2 = v_0 t_2 \ \therefore\ t_2 = \frac{\Delta S_2}{v_0}$

$$t_2 = \frac{1925\,m}{2.5\ m/s} = 770\,s$$

(answer)

[The total time: $\Delta t = t_1 + t_2 = 830\ s$
$\approx 1 + 13$ min.]

Note, in phase 2 there is no force between the bumpers (in the absence of drag), both are coasting.