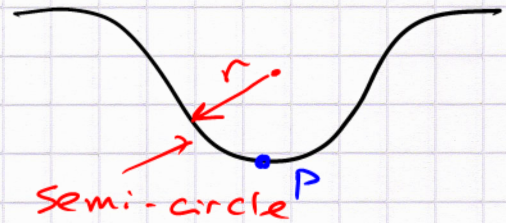
Week 5 5.23 Roller Coaster



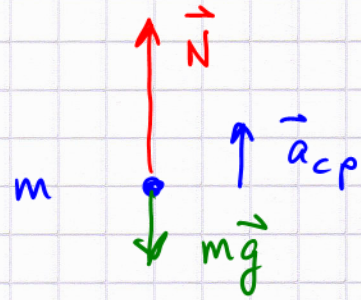
At P(•) $N = 3w = 3mg$

$v = 20 \ \frac{m}{s}$

$r = ?$

Solution.

FB diagram



Newton's 2nd law: vectors $\qquad m\vec{a}_{cp} = \vec{N} + m\vec{g}$

$\hat{j} \uparrow$ to components $\qquad m\, a_{cp} = N - mg$

magnitude $\qquad m\frac{v^2}{r} = N - mg$ (has to be > 0)

$$\cancel{m}\frac{v^2}{r} = 3mg - mg = 2\cancel{m}g$$

$$r = \frac{v^2}{2g}$$

$$r = \frac{20^2}{2 \times 9.8} \ m = 20.4 \ m$$

$r = 20 \ m$ (2 sign. digits)

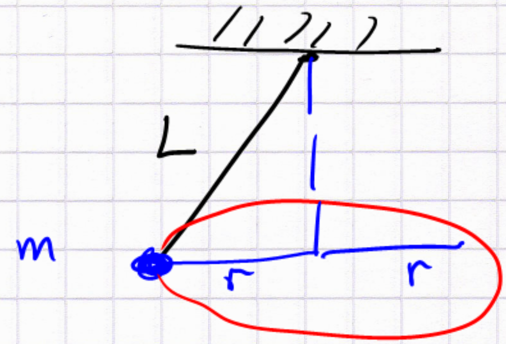
5.28 Conical Pendulum

2nd ed: 50 3.2

$m = 2.5$ kg , $L = 1.2$ m

$r = 0.7$ m

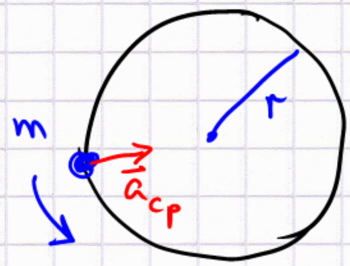
2.5



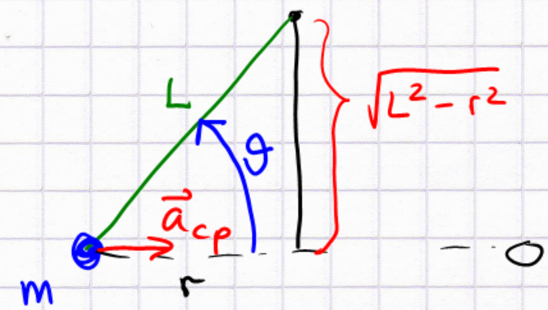
1) show the motion
2) forces, FB diagram
3) vertical + horizontal components
4) 2nd law → string tension

Solution.

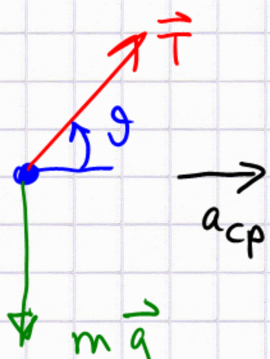
1) top view



side view



2) forces: gravity, tension force



3) $T_x = T\cos\vartheta$

$T_y = T\sin\vartheta$ $\uparrow \hat{j}$

$W_y = -mg$

4) T_x provides the centripetal acceleration

$$T\cos\vartheta = \frac{mv^2}{r} \quad ; \quad T_y + W_y = 0 \quad \therefore \quad T\sin\vartheta = mg$$

trig: $\cos\vartheta = \frac{r}{L}$

$\sin\vartheta = \frac{\sqrt{L^2 - r^2}}{L}$

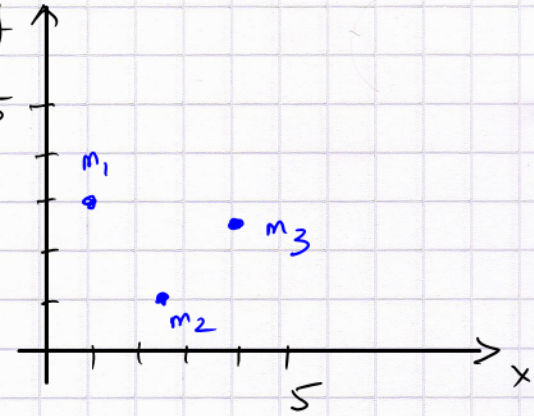
$\therefore \quad T\frac{\sqrt{L^2-r^2}}{L} = mg$

$T = \frac{Lmg}{\sqrt{L^2-r^2}} = 30$ N

2nd ed. 785 N

This is more than the weight! (24.5 N) (490 N)

5.37



$m_1 = 15\ \text{kg} \qquad \vec{P}_1 = 1.0\,\hat{i} + 3.0\,\hat{j}$

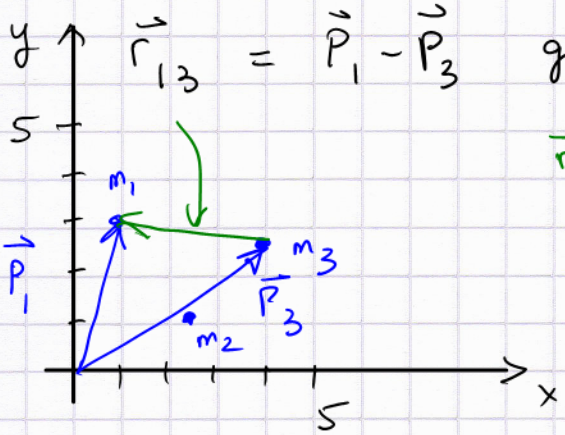
$m_2 = 25\ \text{kg} \qquad \vec{P}_2 = 2.5\,\hat{i} + 1.0\,\hat{j}$

$m_3 = 9.0\ \text{kg} \qquad \vec{P}_3 = 4.0\,\hat{i} + 2.5\,\hat{j}$

$\vec{F}_{\text{grav, on } 3} = ?$

Solution. $\vec{F}_{\text{on } 3} = \vec{F}_{1 \text{ on } 3} + \vec{F}_{2 \text{ on } 3}$ do them one at a time.

$\vec{F}_{1 \text{ on } 3} = ?$ start with magnitude?



$\vec{r}_{13} = \vec{P}_1 - \vec{P}_3$ gives the direction of this force.

$$\vec{r}_{13} = (1.0 - 4.0)\,\hat{i} + (3.0 - 2.5)\,\hat{j}$$

$$= -3.0\,\hat{i} + 0.5\,\hat{j}$$ ← understand visually!

The magnitude of the force $\vec{F}_{1 \text{ on } 3}$:

$$F_{13} = \frac{G\, m_1 m_3}{r_{13}^2} = \frac{6.67\times10^{-11} \cdot 15 \cdot 9}{(-3)^2 + (0.5)^2}\ \text{N}$$

$$= 9.73\times10^{-10}\ \text{N}$$

Many students are tempted to calculate F_{23} along the same lines and add, but that is WRONG!

Vector addition of the forces is required!

$$\vec{F}_{13} = F_{13}\,\frac{\vec{r}_{13}}{r_{13}} = \frac{F_{13}}{\sqrt{(-3)^2 + (0.5)^2}}\left(-3.0\,\hat{i} + 0.5\,\hat{j}\right)$$

Now $\vec{F}_{23}$: $\vec{r}_{23} = \vec{P}_2 - \vec{P}_3 = -1.5\,\hat{i} - 1.5\,\hat{j} \quad \therefore\ r_{23}^2 = (1.5)^2 + (1.5)^2$

$$F_{23} = \frac{G\, m_2 m_3}{r_{23}^2} = \frac{6.67\times10^{-11} \cdot 25 \cdot 9}{2.25 + 2.25}\ \text{N} = 3.33\times10^{-9}\ \text{N}$$

5.37 continued:

$$\vec{F}_{23} = F_{23}\,\frac{\vec{r}_{23}}{r_{23}} = \frac{F_{23}}{\sqrt{2.25\times 2}}\left(-1.5\,\hat{\imath} - 1.5\,\hat{\jmath}\right)$$

Now summarize:

$$\vec{F}_{13} = 3.20\times 10^{-10}\left(-3.0\,\hat{\imath} + 0.5\,\hat{\jmath}\right)\ \mathrm{N}$$

$$\vec{F}_{23} = 1.57\times 10^{-9}\left(-1.5\,\hat{\imath} - 1.5\,\hat{\jmath}\right)\ \mathrm{N}$$

$$\vec{F}_{on\,3} = \left[-3.3\cancel{2}\times 10^{-9}\,\hat{\imath} - 2.2\cancel{0}\times 10^{-9}\,\hat{\jmath}\right]\mathrm{N}$$

round non-significant digits

Now express this as a magnitude / direction.

$$|\vec{F}_{on\,3}| = 3.98\times 10^{-9}\ \mathrm{N} \longrightarrow 4.0\times 10^{-9}\ \mathrm{N}$$

$$\tan\vartheta = \frac{F_y}{F_x} = \frac{2.20}{3.32} \quad \therefore \quad \vartheta = 33.5^\circ \quad (33.55^\circ) = 34^\circ$$

BUT: $F_y < 0$, $F_x < 0$ $\therefore$ 3rd quadrant

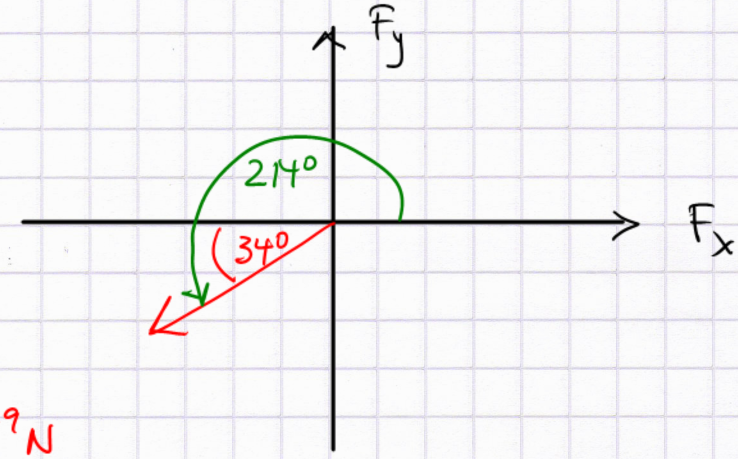
$$\vartheta \to \vartheta + 180^\circ = 213.5^\circ \simeq 214^\circ$$

This is the angle w.r.t. the +ve x-axis

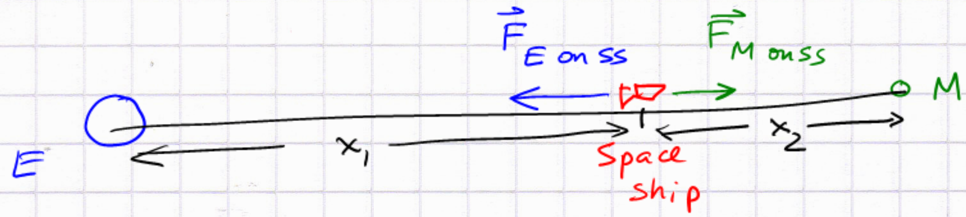
Note:

$$\boxed{|\vec{F}_{on\,3}| \neq |\vec{F}_{1\,on\,3}| + |\vec{F}_{2\,on\,3}|\,!}$$

4.0×10^{-9} N $\qquad \underbrace{9.7\times 10^{-10}\ \mathrm{N} \qquad 3.3\times 10^{-9}\ \mathrm{N}}_{\text{adds up to more!}}$



5.43



Q: For which distance x_1 (where $x_1 + x_2 = d_{ME}$) do the gravitational forces on the SS cancel?

Solution. Cancellation is possible along the direct line connecting E and M.

$$\frac{\cancel{G}\, \cancel{m_{ss}}\, M_E}{x_1^2} = \frac{\cancel{G}\, \cancel{m_{ss}}\, M_M}{x_2^2}$$

$$\frac{M_E}{x_1^2} = \frac{M_M}{(d_{ME} - x_1)^2}$$

$$\left(\frac{d_{ME} - x_1}{x_1}\right)^2 = \frac{M_M}{M_E}$$

$$\frac{d_{ME}}{x_1} - 1 = \sqrt{\frac{M_M}{M_E}}$$

$$\frac{d_{ME}}{x_1} = 1 + \sqrt{\frac{M_M}{M_E}}$$

$$x_1 = \frac{d_{ME}}{1 + \sqrt{\frac{M_M}{M_E}}}$$

Numbers:

$d_{ME} = 3.85 \times 10^8$ m

$M_M = 0.0735 \times 10^{24}$ kg

$M_E = 5.97 \times 10^{24}$ kg

$\therefore\ x_1 = 3.5 \times 10^8$ m from earth (close to the moon!)

5.55 speed of satellite in geosynchronous orbit ?

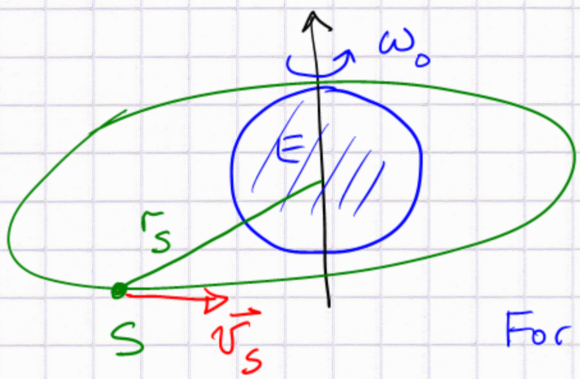
cf.: speed of earth about sun

Solution:

Example 5.10 $\longrightarrow$ geosynchronous orbit ?

E spins about an axis with $T = 24\ h = 24 \times 3.6 \times 10^3\ s$

$$\omega_0 = \frac{2\pi}{T}$$



$$v_s = \omega_0 r_s$$

For this geosynchronous orbit it must be true that:

$$\cancel{m_s} \frac{v_s^2}{r_s} = G \frac{\cancel{m_s} M_E}{r_s^2} \quad \text{with } v_s = \omega_0 r_s$$

$$\omega_0^2 r_s = G M_E r_s^{-2}$$

$$r_s^3 = \frac{G M_E}{\omega_0^2} = \frac{G M_E T^2}{4\pi^2}$$

$$r_s = \sqrt[3]{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 24^2 \times 3.6^2 \times 10^6}{4 \times 3.14^2}}\ m$$

$$= 4.22 \times 10^7\ m = 42{,}000\ km \approx 6.6\ R_E$$

$(R_E = 6370\ km)$

$$v_s = \omega_0 r_s = \frac{6.28 \times 4.2 \times 10^7\ m}{\underbrace{86400\ s}_{day}} = 3100\ \frac{m}{s}$$

Earth's orbit about Sun: $v = \dfrac{2\pi (1.50 \times 10^{11}\ m)}{365 \times 86400\ s} \simeq 3.0 \times 10^4\ \frac{m}{s}$

Earth is orbiting Sun ~ 10 times faster than a geos. satellite orbiting E