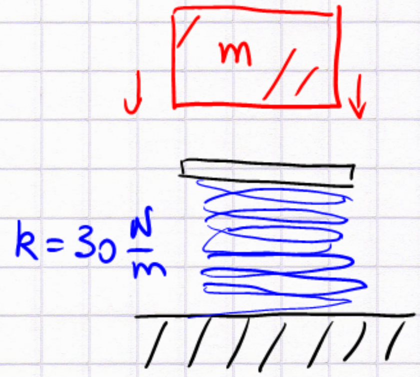
Week 7 6.59

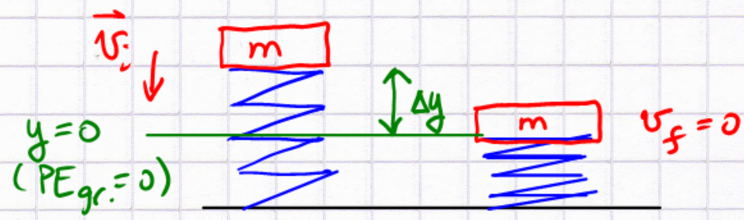
$v_i = 3.3$ m/s (just before contact)

$\Delta y = 0.12$ m ; $v_f = 0$

$m = ?$



Solution



Energy forms:

Before spring contact : $E_{tot} = \frac{1}{2} m v_i^2 + mg\,\Delta y$

(using the point of max. spring compression as $y = 0$ for the gravitational PE)

Spring fully compressed: $PE_{grav} = 0$, $PE_{spr} = \frac{1}{2} k \Delta y^2$

$KE = 0$ ($v_f = 0$)

$$E_{tot} = \frac{1}{2} k \Delta y^2$$

$$\therefore \quad m\left(\frac{1}{2} v_i^2 + g\Delta y\right) = \frac{1}{2} k \Delta y^2$$

$$m = \frac{k \Delta y^2}{v_i^2 + 2g\Delta y} = \frac{(30\ \frac{N}{m})(0.12\ m)^2}{(3.3\ \frac{m}{s})^2 + 2(9.8\ \frac{m}{s^2})(0.12\ m)}$$

$$= 0.0326\ kg$$

$$= 3.3 \times 10^{-2}\ kg = 33\ g$$

Q: will the motion stop there??

$k\Delta y = mg$?

$3.6\ N = 0.32\ N$ ↯

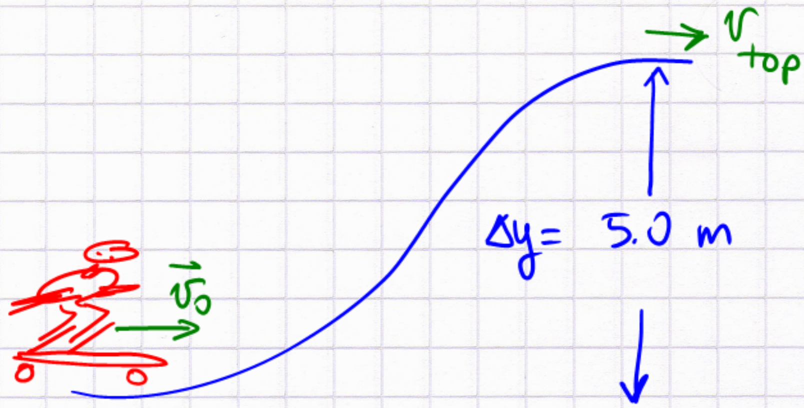
No! spring pushes back towards equilibrium!

6.70

$m = 55\ kg$

$v_0 = 20\ m/s$ (whoa!!)

$v_{top} = 12\ m/s$



How much work due to friction?

Solution.

Use total energy conservation.

Initial $PE_{grav} = 0$; Final $PE_{grav} = m g \Delta y$

Initial $KE_i = \frac{1}{2} m v_0^2$; Final $KE_f = \frac{1}{2} m v_{top}^2$

E_{tot} ; Mech. Energy + Frictional loss

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_{top}^2 + m g \Delta y + E_{lost}$$

$$E_{loss} = m \left[\frac{v_0^2 - v_{top}^2}{2} - g \Delta y \right]$$

$$= 55\ kg \left[\frac{20^2 - 12^2}{2} \left(\frac{m^2}{s^2}\right) - \left(9.8\ \frac{m}{s^2}\right)(5.0\ m) \right]$$

$$= 4345\ Nm = 4.3\ kJ$$

Work done by friction: $-E_{loss} = -4.3\ kJ$!

6.74

$v_i = 0$

coasts (no power input)

$m_{\text{rider + bike}} = 100\,kg$

a) Work due to losses (friction + drag) ?

b) estimate average drag force (assuming friction ≈ 0) based on power + average (const) speed

↳ requires excellent hubs + superinflated tires!

Solution.

Start with energy balance :

$$E_{tot} = \underbrace{\cancel{KE}_{in}}_{=0} + PE_{in} = \underbrace{\cancel{KE}_{fin}}_{=0} + \underbrace{PE_{fin}}_{0.6\,PE_{in}} + E_{loss}$$

$$\therefore \quad E_{loss} = 0.4\, mg\,\Delta y$$

Note: we can move E_{loss} to the LHS, it becomes the negative work done by air drag (+ friction)

a) $W_{loss} = -0.4 \cdot 100\,kg \cdot 9.8\,\frac{m}{s} \cdot 20\,m$
$= -7.8(4)\ kJ$

quick estimate:
$W_{loss} = F_{loss} \cdot \Delta s$
$\Delta s \approx 30 + 20\ m$
$F_{loss} \approx 157\,N = 160\,N$

b) • estimate average speed w/o energy loss @ bottom $v_{max} = \sqrt{2gh}$ (cf free fall!; $\approx 20\,m/s$ mech. energy cons.)

without mech. energy cons. : $h \to 0.6\,h$ in $v_{max} \to 15\,m/s$

$v_{bottom} \approx \frac{1}{2}(15+20)\frac{m}{s} \approx 18\,\frac{m}{s}$; const. acc. ⇒ $v_{avg} = \frac{1}{2}(0+18)\frac{m}{s} = 9\,\frac{m}{s}$

Power: $\frac{W_{loss}}{\Delta t} = P_{aver.} = F_{loss}\, v_{avg}$ $\quad \therefore \quad F_{loss} = \frac{W_{loss}}{v_{avg}\,\Delta t} = \frac{7.8 \times 10^3}{9 \times 5.5} \approx 160\,N$

$\Delta t = S/v_{avg} \approx (30\,m + 20\,m)/9\,m/s \approx 5.5\,s$

6.77 Electric Motor, $\underline{P} = 0.75$ hp NB: 1 hp = 745.7 W

Lifts crate, $M = 200$ kg $\longrightarrow$ how fast?

hint: it works against gravity.

Solution.

$$P = \frac{\Delta E}{\Delta t} \qquad \Delta E = mg\,\Delta y$$

$$P = mg\,\frac{\Delta y}{\Delta t} = mg\,v_y$$

recognize $\rightarrow$ for constant force: $\underline{P} = \underline{F} \cdot \underline{v}$

$$v_y = \frac{P}{mg} = \frac{0.75 \cdot 0.746 \times 10^3\,\mathrm{W}}{200\ \mathrm{kg} \cdot 9.8\ \mathrm{m/s^2}}$$

$$W = \frac{Nm}{s} = \frac{kg\,m}{s^2}\,\frac{m}{s}$$

$$= 0.286\ \frac{m}{s} = 0.29\ \frac{m}{s}$$

in real life: frictional losses when winding cable!

(29 cm/sec is fast for moving 200 kg up)

7.10

Rubber ball bounces from floor. $m = 0.15\,kg$

$v_i = -3.0\ m/s \rightarrow v_f = +2.5\ m/s$ $\uparrow \hat{j}$

$F^{avg}_{\text{floor on ball}} = ?$ when $\Delta t = 0.12\ s$ (contact time)

Solution.

Momentum-Impulse theorem: $\Delta p_y = F_y^{avg}\,\Delta t \quad (= J_y)$

$$\Delta p_y = p_f - p_i = m(v_f - v_i) = 0.15 \cdot (2.5 - (-3.0))\ \frac{kg\,m}{s}$$
$$= 0.825\ \frac{kg\,m}{s}$$

$$F_y^{avg} = \frac{\Delta p_y}{\Delta t} = \frac{0.825}{0.12}\ \frac{kg\,m}{s^2} = 6.88\ N$$
$$= 6.9\ N$$

by comparison: the weight of the ball $= mg = 1.5\ N$