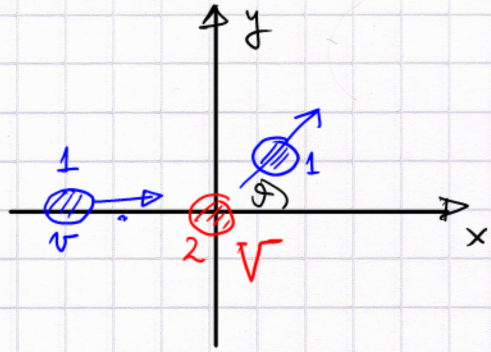
Week 8 7.27 Billiard balls $m = M$



$v_{ix} = 10\ m/s$ $v_{iy} = 0$

$V_{ix} = 0$ $V_{iy} = 0$

$v_f = 4.7\ m/s$, $\vartheta = 60°$ $V_f = ?$; direction angle $\varphi = ?$

Solution

Momentum conservation: (2 eqs)

$$m v_{ix} = m(v_{fx} + V_{fx}) \quad ①$$

$$0 = m(v_{fy} + V_{fy}) \quad ②$$

Energy conservation:

$$\frac{1}{2} m v_{ix}^2 = \frac{1}{2} m (v_f^2 + V_f^2)$$

$v_{fx} = v_f \cos\vartheta$; $v_{fy} = v_f \sin\vartheta$

$V_{fx} = V_f \cos\varphi$; $V_{fy} = V_f \sin\varphi$

m drops out of all eqs!

in SI:

① $v_{ix} = 10 = v_f \cos\vartheta + V_f \cos\varphi$

② $0 = v_f \sin\vartheta + V_f \sin\varphi$

③ $v_{ix}^2 = 100 = v_f^2 + V_f^2$

ϑ, v_f given

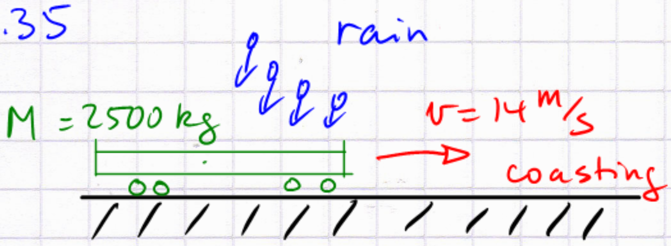
From ③: $V_f^2 = 100 - 4.7^2 \quad \therefore \quad \underline{\underline{V_f = 8.83\ \frac{m}{s}}}$

From ②: $\varphi = \sin^{-1}\left(-\frac{v_f}{V_f}\sin\vartheta\right) = -27.45°$

without energy conservation: $V_f = 8.7\ \frac{m}{s}$, $\varphi = -28°$!

$V_f = 8.8\ m/s$; $\varphi = -27°$ (below the x axis)

7.35



$v_{fin} = 11 \text{ m/s}$

how much rainwater?

Solution.

Even though this is a gradual process, we can think of it as a sequence of inelastic collisions summarized by:

$$M v_{i,x} + m_{Rain} \cdot 0 = (M + m_R)\, v_{f,x}$$

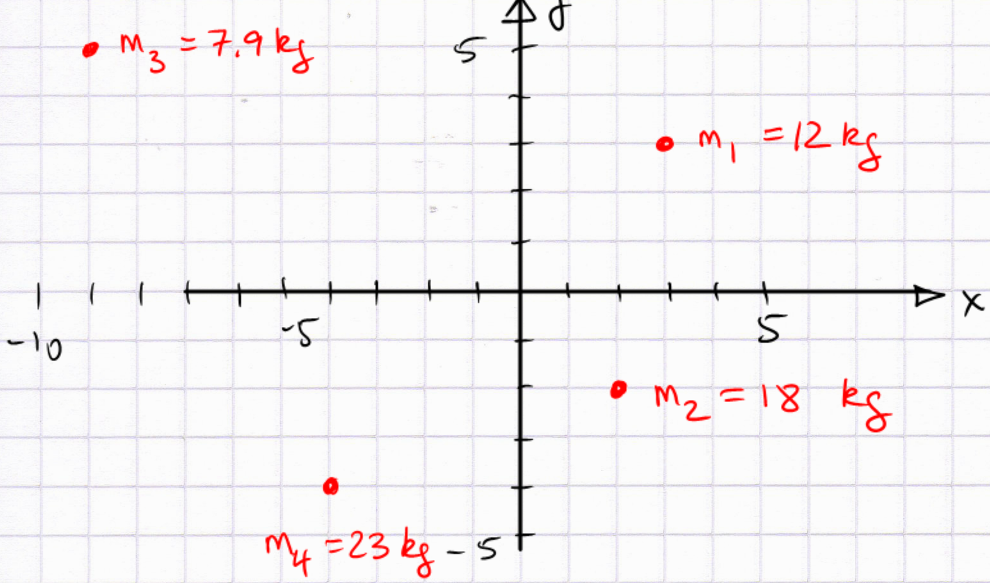
$$\frac{M v_{i,x}}{v_{f,x}} = M + m_R \quad \therefore \quad m_R = M\left(\frac{v_{i,x}}{v_{f,x}} - 1\right)$$

in SI:

$$m_R = 2{,}500\left(\frac{14}{11} - \frac{11}{11}\right) = 2500 \times \frac{3}{11} = 681.8 \text{ kg}$$

The accumulated rain has a mass of 680 kg

7.42



Find the CM

Solution.

$$\vec{R}_{CM} = X_{CM}\,\hat{\imath} + Y_{CM}\,\hat{\jmath}$$

$$= \frac{1}{M}\left(\left(\sum m_i x_i\right)\hat{\imath} + \left(\sum m_i y_i\right)\hat{\jmath}\right)$$

$$M = \sum m_i = 60.9 \text{ kg}$$

$$\sum m_i x_i = -91.1 \text{ kg m}$$

$$\sum m_i y_i = -52.5 \text{ kg m}$$

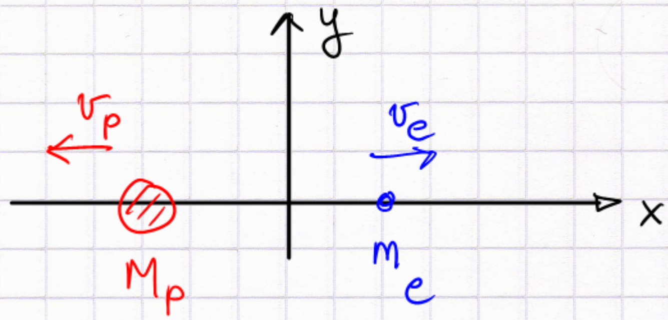
i	x_i	y_i	m_i
1	3.0	3.0	12
2	2.0	-2.0	18
3	-9.0	5.0	7.9
4	-4.0	-4.0	23

$$X_{CM} = \frac{-91.1}{60.9}\text{ m} = -1.496 \text{ m} \qquad Y_{CM} = -0.862 \text{ m}$$

$$\vec{R}_{CM} = (-1.5, -0.86)\text{ m} = \left(-1.5\,\hat{\imath} - 0.86\,\hat{\jmath}\right)\text{ m}$$

7.56

$$\frac{M_p}{m_e} = 1800$$



v_p = known ; find v_e

Solution.

Idea: original particle was at rest. $\vec{P}_{tot} = \vec{0}$

$$M_p v_p + m_e v_e = P_{tot,x} = 0, \text{ and } P_{tot,y} = 0$$

$$\therefore \quad m_e v_e = - M_p v_p$$

$$v_e = - \frac{M_p}{m_e} v_p = - 1800 \, v_p$$

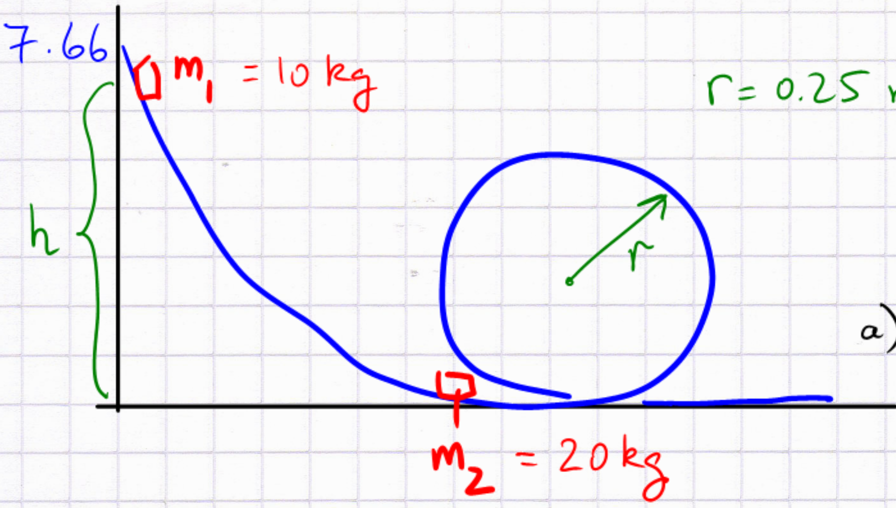
The elementary particle can be an electron e^-

This process could be the decay of a free neutron, which is unstable (half-life ~ 11 min.)

$$n \rightarrow p + e^- + \underbrace{\bar{\nu}_{e^-}}_{\text{antineutrino}}$$

ignored in the present problem

7.66



$r = 0.25$ m

- Elastic collision
- cart 2 to complete loop just not falling out of track

a) $h = ?$

b) swap cars $h = ?$

Solution.

Part 1 : elastic collision following conversion of $PE_{grav} \rightarrow KE$ for car 1

a) $m_1 g h = \frac{1}{2} m v_1^2 \longrightarrow v_1 = \sqrt{2gh}$

b) collision (elastic) :

$$m_1 v_1 = m_1 v_1^f + m_2 v_2^f \quad (1)$$

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 (v_1^f)^2 + \frac{1}{2} m_2 (v_2^f)^2 \quad (2)$$

case 1 : $m_2 = 2 m_1$

(1) : $\cancel{m_1} v_1 = \cancel{m_1} v_1^f + 2 \cancel{m_1} v_2^f$

(2) $\cancel{m_1} v_1^2 = \cancel{m_1} (v_1^f)^2 + 2 \cancel{m_1} (v_2^f)^2$

use (1) to eliminate v_1^f in (2) (we want v_2^f)

$v_{1f} = v_1 - 2 v_{2f} \xrightarrow{\text{into (2)}} v_1^2 = (v_1 - 2v_{2f})^2 + 2 v_{2f}^2$

$$0 = -4 v_1 \cancel{v_{2f}} + 6 v_{2f}^{\cancel{2}} \quad \Big| \; v_{2f} \neq 0$$

$$\boxed{v_{2f} = \frac{2}{3} v_1 = \frac{2}{3} \sqrt{2gh}}$$

Look at case 2 (swapped cars)

① $2\cancel{m_1} v_1 = 2\cancel{m_1} v_1^f + \cancel{m_1} v_2^f$ ② : $2\cancel{m_1} v_1^2 = 2\cancel{m_1}(v_1^f)^2 + \cancel{m_1}(v_2^f)^2$

$v_1^f = v_1 - \frac{1}{2} v_2^f \longrightarrow 2v_1^2 = 2\left(v_1 - \frac{1}{2} v_2^f\right)^2 + (v_2^f)^2$

$0 = -2\cancel{v_2^f} v_1 + \frac{1}{2}(v_2^f)^{\cancel{2}} + (v_2^f)^{\cancel{2}}$

$2v_1 = \frac{3}{2} v_2^f \therefore v_2^f = \frac{4}{3} v_1$

twice as fast.

$\boxed{v_2^f = \frac{4}{3}\sqrt{2gh}}$

Now solve the hoop the loop

$KE_{top} = KE_{bottom} - mg(2r)$

Critical speed @ top

$v_c = \sqrt{gr}$ (Eq. 5.11 in book) $\longrightarrow$ derived on test 2!

$\frac{1}{2}\cancel{m} v_c^2 = \frac{1}{2}\cancel{m}(v_2^f)^2 - 2\cancel{m}gr \therefore \frac{1}{2}gr + 2gr = \frac{1}{2}(2gh)F$

$F \nearrow (2/3)^2$ case 1
$F \searrow (4/3)^2$ case 2

$\therefore \cancel{g}h = \frac{1}{F}\frac{5}{2}\cancel{g}r$

case 1: $h = \frac{5}{2}r \times \frac{9}{4} = \frac{45}{8}r = \frac{45}{32} = \underline{1.40 \text{ m}}$

case 2: $h = \frac{5}{2}r \times \frac{9}{16} = \frac{45}{32 \times 4} = \underline{0.35 \text{ m}}$