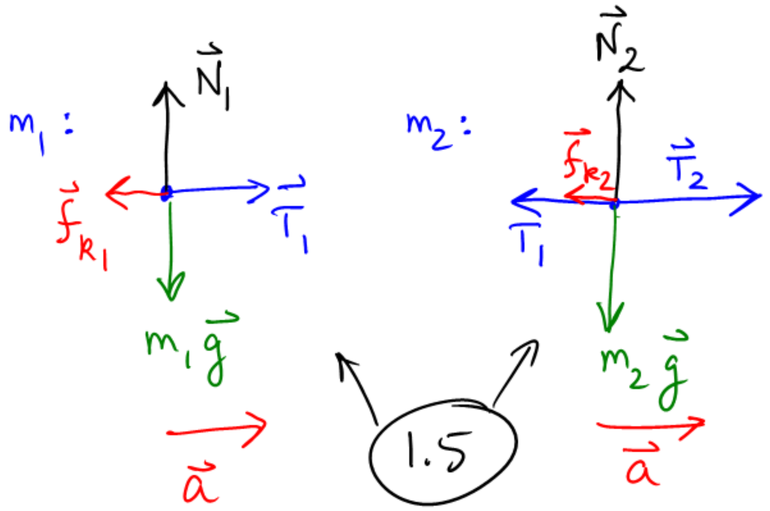
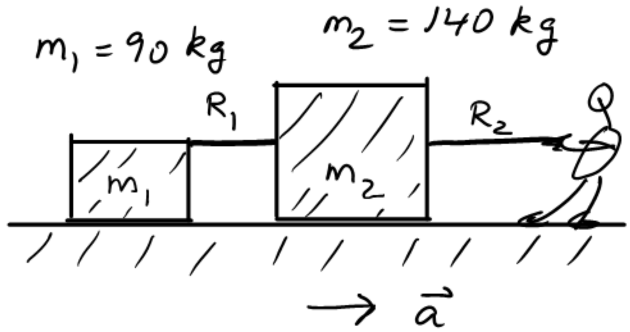
LAST NAME: STUDENT NR:

PHYS 1010 6.0: CLASS TEST 1

Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1)[5] You work for the movers, and are pulling the two crates shown over a slippery surface (kinetic friction between the crates and the floor $\mu_k = 0.1$, but you wear super-shoes which give you traction!) The boxes are accelerating with $a = 0.12$ m/s^2. What is the tension in each rope? Start with free-body diagrams for the crates.





$N_1 = m_1 g$ $\quad\quad N_2 = m_2 g$

$f_{k_1} = \mu_k m_1 g$ 0.5 $f_{k_2} = \mu_k m_2 g$

$m_1 a = T_1 - f_{k_1}$ 0.5 $\quad\quad m_2 a = T_2 - T_1 - f_{k_2}$ 0.5

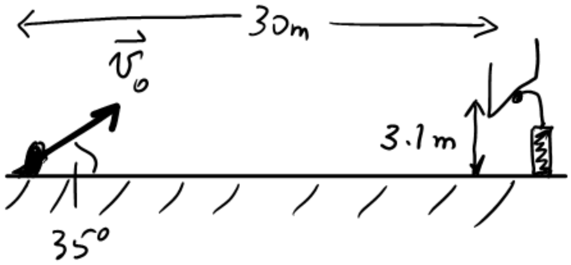
In SI:

$f_{k_1} = 0.1 \cdot 90 \cdot 9.8 = 88.2$ N $\quad\quad f_{k_2} = 0.1 \cdot 140 \cdot 9.8 = 137.0$ N

$(m_1 + m_2) a = T_2 - f_{k_1} - f_{k_2}$ $\therefore$ $T_2 = 230 \cdot 0.12 + 137 + 88.2 = 253$ N 1

$T_1 = m_1 a + f_{k_1} = 90 \cdot 0.12 + 88.2 = 99.0$ N 1

2) [5] A football player wants to kick a ball through the uprights as shown. The ball is kicked from a distance of 30 m with a velocity of magnitude 15 m/s at an angle of 35° with the horizontal. Will the ball make it over the crossbar, which is at a height of 3.1 m? Ignore air drag when you derive your answer.



(1) $x(t) = v_{0x}\, t = (v_0 \cos\alpha)\, t$

(1) $y(t) = v_{0y} t - \frac{1}{2} g t^2$

time of flight t_f : (in SI units)

$$30 = x(t_f) = (v_0 \cos\alpha)\, t_f = 15 \cos(35^\circ)\, t_f$$

$$\therefore t_f = \frac{30}{15 \cos(35^\circ)} = \frac{2}{\cos(35^\circ)} = 2.44\,\text{s} \quad (1)$$

$$y(t_f) = (v_0 \sin\alpha)\, t_f - \frac{1}{2} g t_f^2$$

Q: above 3.1 m?

$$= 15 \cdot 0.574 \cdot 2.44 - 0.5 \cdot 9.8 \cdot 5.96$$

$$= -8.20\,\text{m} \quad (1)$$

$\therefore$ the ball falls short (1)

Bonus: when does the ball hit the ground?

$$y(t_g) = 0 = t_g\left(v_0 \sin\alpha - \tfrac{g}{2} t_g\right) \quad \therefore \quad t_g = \frac{2 v_0 \sin\alpha}{g}$$

$$t_g = \frac{30 \cdot 0.574}{9.8} = 1.76\,\text{s}$$

$$\Delta x = v_{0x}\, t_g$$

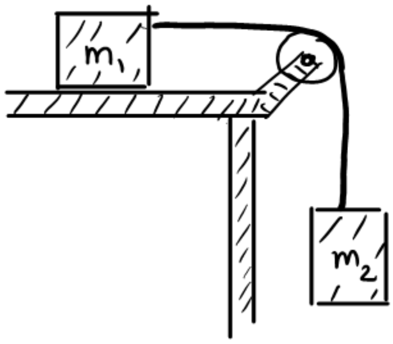
$$= 15 \cos(35^\circ) \cdot 1.76$$

$$= 21.6\,\text{m}$$

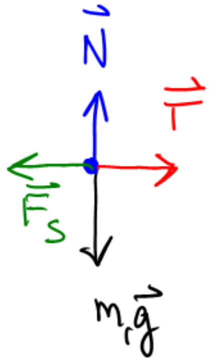
Bonus (+1)

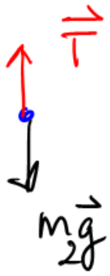
3) [5] Two blocks of mass $m_1 = 35$ kg and $m_2 = 13$ kg are connected by a massless, non-stretchable string that passes over a massless and frictionless pulley as shown. (a) What is the coefficient of static friction between mass m_1 and the table if the system is in equilibrium?* Start with free-body diagrams for the two masses. (b) What is the tension force in the string?

* Assume that the system is barely stable, almost beginning to move



1 m_1:





m_2 0.5

$a_1 = 0$ $\qquad$ $a_2 = 0$

$N = m_1 g$ $\qquad$ $T = m_2 g$ 0.5

$F_s = \mu_s N = \mu_s m_1 g$ 1 $\qquad$ $F_s = T = m_2 g$

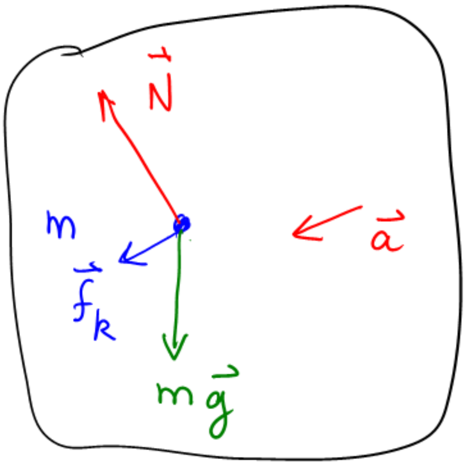
↑ limit

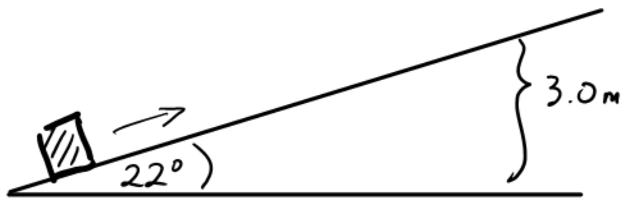
$\therefore \quad m_2 \cancel{g} = \mu_s m_1 \cancel{g}$ $\qquad$ $\mu_s = \frac{m_2}{m_1} = \frac{13}{35} = 0.371$ 1

In SI:

$T = m_2 g = 13 \cdot 9.8 = 127$ N 1

4) [5] A block slides up a [redacted], inclined plane with $\theta = 22°$. The coefficient of kinetic friction between block and plane equals $\mu_k = 0.25$. What initial speed is required for the block to reach a maximum height of $h = 3.0$ m? Start with a free-body diagram.

(1) 



$\vec{g} = \vec{g}_{\parallel} + \vec{g}_{\perp}$ $\quad |\vec{g}_{\perp}| = g\cos\alpha$ $\quad |\vec{g}_{\parallel}| = g\sin\alpha$ (1)

$N = mg_{\perp} = mg\cos\alpha$ $\qquad |\vec{f}_k| = \mu_k N = \mu_k mg\cos\alpha$

use $\hat{\imath}$ along incline: (1) $ma_x = -mg_{\parallel} - f_k$

since $\vec{g}_{\parallel}$ is to the left; opposes $v_x > 0$

$\therefore \; a_x = -g\sin\alpha - \mu_k g\cos\alpha$

constant acceleration!

$\sin\alpha = \frac{3.0}{\Delta x}$

use $v_f^2 = v_i^2 + 2a\,\Delta x$ $\qquad \Delta x = ?$ $\qquad$ Δx, 3.0

$v_f = 0$

(1) $0 = v_i^2 - 2g(\sin\alpha + \mu_k\cos\alpha)\frac{3.0}{\sin\alpha}$

$\therefore \; v_i^2 = 6.0\,g\left(1 + \frac{\mu_k}{\tan\alpha}\right) = 6.0 \cdot 9.8\left(1 + \frac{0.25}{\tan 22°}\right)$

$= 95.2 \text{ m}^2/\text{s}^2$

(1) $v_i = 9.76$ m/s $\quad (\approx 35$ km/h$)$

FORMULA SHEET

$v(t_\mathrm{f}) = v(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} a(t)\ dt \quad s(t_\mathrm{f}) = s(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} v(t)\ dt$

$v_\mathrm{f} = v_\mathrm{i} + a\Delta t \quad s_\mathrm{f} = s_\mathrm{i} + v_\mathrm{i}\Delta t + \frac{1}{2}a\Delta t^2 \quad v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2a\Delta s \qquad g = 9.8\ \mathrm{m/s^2}$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\ dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\ dt = at + C \quad F(t)$ = anti-derivative = indefinite integral

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

$\exp' = \exp$; $\sin' = \cos$; $\cos' = -\sin$. $\quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}$; $\quad (fg)' = f'g + fg'$

$m\vec{a} = \vec{F}_\mathrm{net}$; $\quad F_G = \frac{Gm_1m_2}{r^2}$; $g = \frac{GM_E}{R_E^2}$; $R_E = 6370$ km; $G = 6.67 \times 10^{-11}\frac{\mathrm{Nm^2}}{\mathrm{kg^2}}$; $M_E = 6.0 \times 10^{24}$kg

$f_\mathrm{s} \leq \mu_\mathrm{s} n$; $\quad f_\mathrm{k} = \mu_\mathrm{k} n$; $\quad \mu_\mathrm{k} < \mu_\mathrm{s}$.

$\vec{F}_\mathrm{d} \sim -\vec{v}$; linear: $F_\mathrm{d} = dv$; quadr.: $F_\mathrm{d} = 0.5\rho A v^2$; $\quad A$ = cross s'n area; ρ = density of medium

Sphere: $V = \frac{4}{3}\pi R^3$; Total Surface: $A_S = 4\pi R^2$; Cross Section=?