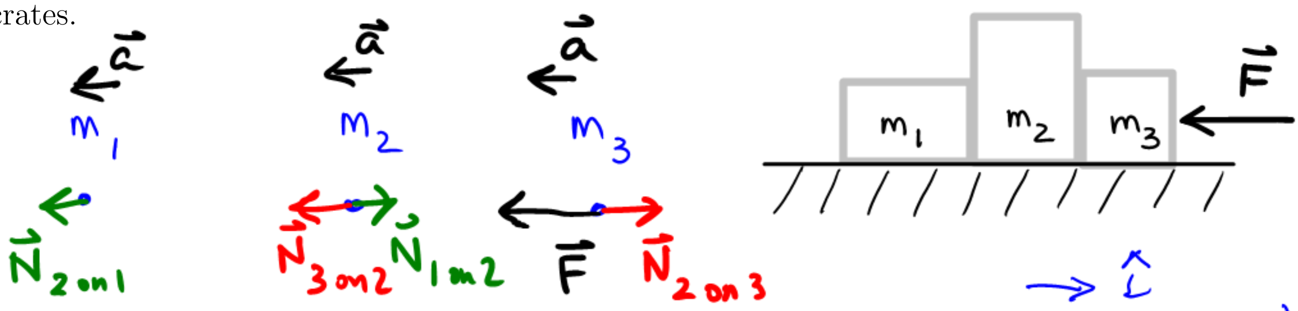
LAST NAME: STUDENT NR:

PHYS 1010 6.0: CLASS TEST 1

Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1)[5] Three crates rest on a frictionless, horizontal ice surface as shown with $m_1 = 80$ kg, and $m_3 = 110$ kg. A horizontal force of $F = -1150$ N is applied to block 3 (which is on the right), and the acceleration of all three blocks is found to be $a = -3.4$ m/s^2. What is m_2 ? What is the normal force from block 2 on 1 (magnitude and direction)? Start with free-body diagrams for the crates.



1.5

using $\hat{\imath}$ we obtain (F, $N_{i\,on\,j}$ are with sign, not magnitudes)

① $m_3 a = -N_{2\,on\,3} + F$

② $m_2 a = +N_{3\,on\,2} - N_{1\,on\,2}$

③ $m_1 a = +N_{2\,on\,1}$

⑤ $N_{3\,on\,2} = -N_{2\,on\,3}$

⑥ $N_{2\,on\,1} = -N_{1\,on\,2}$

④ $(m_1 + m_2 + m_3)\, a = F$

unknowns : m_2 , $N_{2\,on\,1}$, $N_{1\,on\,2}$, $N_{2\,on\,3}$, $N_{3\,on\,2}$

equations : 6

From ④ : in SI

$$m_2 = \frac{F - (m_1 + m_3)a}{a} = \frac{-1150 + (190)3.4}{-3.4}$$

$$m_2 = 148 \text{ kg} \longrightarrow 150 \text{ kg}$$

1.5 (significant)

From ③: $N_{2\,on\,1} = m_1 a = 80\,(-3.4)$

$$= -272 \text{ N} \longrightarrow -270 \text{ N}$$

1.5 (significant)

(the force is to the left)

0.5 for intermediate steps

P.S.: A smart choice: switch to $\leftarrow \hat{\imath}$, then $a > 0$, $F > 0$, the equations are the same! A positive $N_{2\,on\,1}$ is then to the left.

2) [5] A car is speeding on a highway at $v = 130$ km/h under foggy conditions. Suddenly, the driver notices that at a distance of $d = 250$ m in front the traffic has come to a complete stop. The maximum braking acceleration his car can sustain corresponds to $a = 0.8g$ (where g is the free-fall acceleration at the surface of the earth). Calculate whether the driver will manage without hitting the stopped cars.

constant acceleration kinematics applies →

1 $\quad v_f^2 = v_i^2 + 2a\Delta x \qquad v_f = 0, \quad a < 0$

$$0 = v_i^2 + 2a\Delta x$$

0.5 $\quad \Delta x = -v_i^2/2a$

$$v_i = 130\ \frac{km}{h} = 130\ \frac{10^3}{3.6\cdot 10^3}\ \frac{m}{s} = \frac{130}{3.6}\ \frac{m}{s} = 36.1\ \frac{m}{s}$$ 0.5

In SI:

$$\Delta x = -\frac{1304}{2(-0.8\cdot 9.8)}\ m$$

$$= 83.2\ m$$

Under perfect braking conditions* the car stops after $\Delta x = 83$ m, i.e., well before $d = 250$ m.

2 1

*) A value of $a = -0.8g$ is among the best-possible for a normal car → dry road; ABS → no skid

In practice we would need to add reaction time on the order of 0.1 - 0.2 s before braking sets in.

3) [5] A crate of mass $m = 500$ kg is loaded on a flatbed truck. The truck moves with a velocity of $v = -25\ \hat{\imath}$ m/s, i.e., it moves to the left. The driver applies the brakes, and manages to reduce the speed at a rate of 3 m/s^2. The crate does not slip. The coefficient of static friction is given as $\mu_s = 0.55$. Provide a free-body diagram for the crate. Calculate the force on the crate (magnitude and direction). Does the answer make sense?

$\vec{a} = +3\ \hat{\imath}\ \frac{m}{s^2}$ or $a = +3\ \frac{m}{s^2}$ 0.5

1 

$ma = F_s$ 0.5

$\vec{a} = 3\ m/s^2$ in SI: $F_s = 500\ kg \cdot 3\ \frac{m}{s^2} = 1500\ N$ 1

to the right

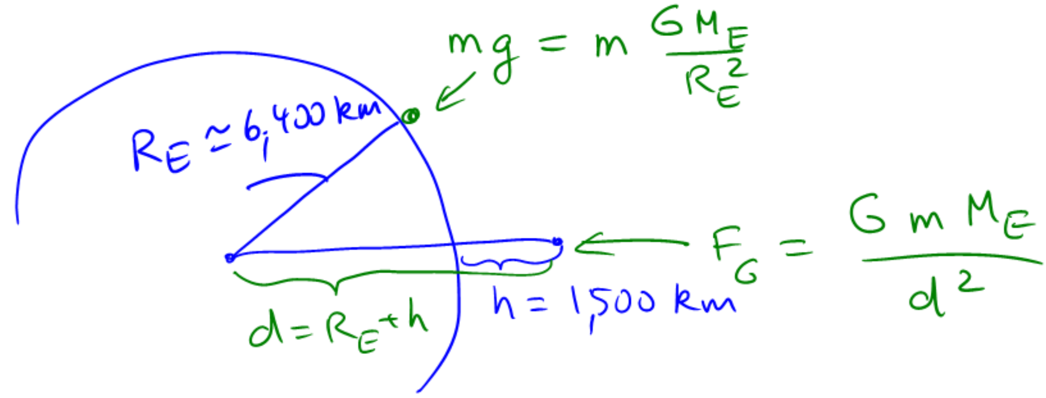
This is reasonable if $F_s \le \mu_s N = \mu_s mg = F_s^{max}$

1 $F_s^{max} = 0.55 \cdot 500 \cdot 9.8\ N = 2{,}700\ N$

1 $F_s < F_s^{max}$ means that it makes sense

4) [5] A small rocket has a mass of $m = 1,500$ kg (without fuel). Calculate the gravitational force of the earth on the rocket when it has reached a final height of $1,500$ km above the earth's surface (ignore the mass of the fuel). What is the ratio of the (pure rocket) weight at the final altitude compared to the value at lift-off. Start with a drawing and explain your reasoning.

1



$$\text{weight} = mg = 1.5 \cdot 10^3 \cdot 9.8 \ \frac{kg\,m}{s^2} = 14,700 \text{ N} \quad 1$$

(at earth's surface)

at altitude $h = 1.5 \times 10^6$ m :

$$\text{in SI:} \quad F_G = G \frac{m M_E}{(R_E + h)^2} = 6.67 \cdot 10^{-11} \frac{1.5 \times 10^3 \cdot 6.0 \times 10^{24}}{(6.37 \cdot 10^6 + 1.5 \cdot 10^6)^2}$$

1

$$F_G = 9692 \text{ N} \rightarrow 9.69 \times 10^3 \text{ N} \quad 1$$

$$\frac{F_G\ (@\ 1500 \text{ km})}{W} = \frac{9.69}{14.7} = 0.659 \rightarrow 0.66 \quad 1$$

The ratio is about 2/3

FORMULA SHEET

$v(t_\mathrm{f}) = v(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} a(t)\ dt \quad s(t_\mathrm{f}) = s(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} v(t)\ dt$

$v_\mathrm{f} = v_\mathrm{i} + a\Delta t \quad s_\mathrm{f} = s_\mathrm{i} + v_\mathrm{i}\Delta t + \frac{1}{2}a\Delta t^2 \quad v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2a\Delta s \qquad g = 9.8\ \mathrm{m/s^2}$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\ dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\ dt = at + C \quad F(t) =$ anti-derivative = indefinite integral

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

$\exp' = \exp$; $\sin' = \cos$; $\cos' = -\sin$. $\quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}$; $\quad (fg)' = f'g + fg'$

$m\vec{a} = \vec{F}_\mathrm{net}$; $\quad F_G = \frac{Gm_1m_2}{r^2}$; $g = \frac{GM_E}{R_E^2}$; $R_E = 6370$ km; $G = 6.67 \times 10^{-11}\frac{\mathrm{Nm^2}}{\mathrm{kg^2}}$; $M_E = 6.0 \times 10^{24}$kg

$f_\mathrm{s} \leq \mu_\mathrm{s} n$; $\quad f_\mathrm{k} = \mu_\mathrm{k} n$; $\quad \mu_\mathrm{k} < \mu_\mathrm{s}$.

$\vec{F}_\mathrm{d} \sim -\vec{v}$; linear: $F_\mathrm{d} = dv$; quadr.: $F_\mathrm{d} = 0.5\rho A v^2$; $\quad A =$ cross s'n area; $\rho =$ density of medium

Sphere: $V = \frac{4}{3}\pi R^3$; Total Surface: $A_S = 4\pi R^2$; Cross Section=?