LAST NAME: STUDENT NR:

PHYS 1010 6.0: CLASS TEST 2

Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1) [5] When a fighter pilot makes a very quick turn he can pass out due to the centripetal acceleration associated with the turn. This happens typically when this acceleration reaches $8g$. Consider a pilot flying at 2200 km/h (2.2×10^3 km/h) steering into such a turn. What is the minimum radius he needs to follow to avoid a black-out?

Circular (uniform) motion: $a_{cp} = \frac{v^2}{r}$ $\therefore$ $r = \frac{v^2}{a_{cp}}$ 1

a_{cp} should not exceed $8g$; $r_c = \frac{v^2}{8g}$

$r > r_c$ guarantees $a_{cp} < 8g$ 1

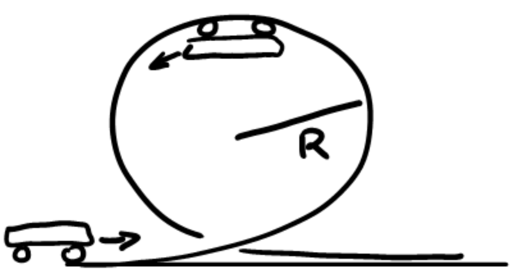
$v = 2.2 \times 10^3 \frac{km}{h} = 2.2 \times 10^3 \times \frac{10^3\ m}{3.6 \times 10^3\ s} = 0.611 \times 10^3 \frac{m}{s}$

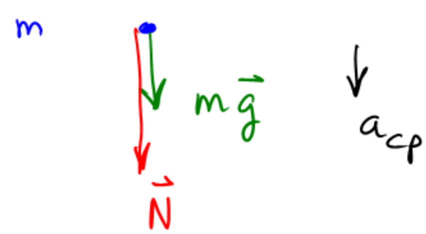
$v^2 = 0.373 \times 10^6 \frac{m^2}{s^2}$ $r_c = 4.76 \times 10^3$ m

2 part marks for getting this correct $= 4.8 \times 10^3$ m (2 significant digits)

The smallest turning radius equals 4.8 km 3

2) [5] A roller coaster track as shown in the figure is designed such that a car travels upside down when at the top. The track has a radius of 25 m. What is the minimum speed the car has to have at the top to avoid falling out of the track? Start with a free-body diagram for the general case $v > v_{min}$ and explain your steps.





1

2nd law: $m a_{cp} = mg + N$

Minimum speed: The normal force vanishes ($N = 0$) 1

...

$$\frac{\cancel{m} v_{min}^2}{R} = \cancel{m} g \quad \therefore \quad v_{min} = \sqrt{Rg} \qquad 1$$

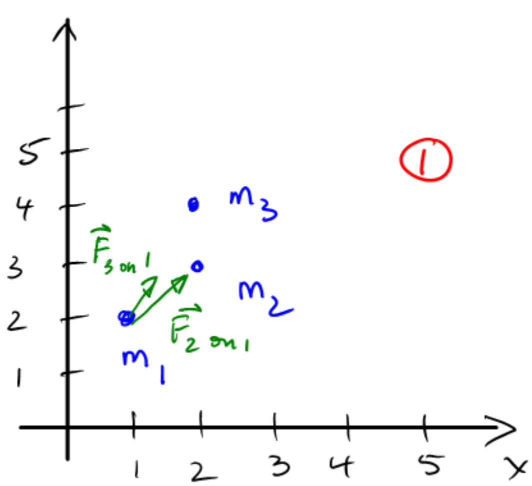
$$v_{min} = \sqrt{25\,\text{m} \times 9.8\,\frac{\text{m}}{\text{s}^2}} = 15.6\,\frac{\text{m}}{\text{s}} \rightarrow 16\,\frac{\text{m}}{\text{s}} \qquad 2$$

$$\left(\rightarrow 56\,\frac{\text{km}}{\text{h}}\right)$$

The car's speed at the top (!) has to exceed 56 km/h in order not to fall out of the track.

3) [5] Three rocks are placed at the following coordinates in the $x - y$ plane: $m_1 = 10$ kg at $P_1 = (1.0, 2.0)$ m, $m_2 = 20$ kg at $P_2 = (2.0, 3.0)$ m, and $m_3 = 30$ kg at $P_3 = (2.0, 4.0)$ m. Calculate the gravitational attraction that m_2 and m_3 exert on m_1. Start with a drawing indicating the positions of the masses and the force vectors acting on m_1. Express the net force on m_1 as Cartesian components (using $\hat{\imath}$ and $\hat{\jmath}$).

$$\left.\begin{aligned} \vec{F}_{2\text{ on }1} &= -\frac{G m_1 m_2}{r_{12}^2}\,\hat{r}_{12} \\ \text{where} \quad \vec{r}_{12} &= \vec{r}_1 - \vec{r}_2 \end{aligned}\right\} \quad 1$$



$$= (1.0 - 2.0)\,\hat{\imath} + (2.0 - 3.0)\,\hat{\jmath}$$

$$= -\hat{\imath} - \hat{\jmath} \quad ; \quad r_{12}^2 = 1 + 1 = 2.$$

$$\text{or:} \quad \vec{F}_{2\text{ on }1} = \frac{G m_1 m_2}{2}\,\frac{\hat{\imath} + \hat{\jmath}}{\sqrt{2}}$$

$$= \frac{6.67 \times 10^{-11} \times 10 \times 20}{2 \times 1.414}\,(\hat{\imath} + \hat{\jmath}) \quad \text{N}$$

$$= \left(4.72 \times 10^{-9}\,\hat{\imath} + 4.72 \times 10^{-9}\,\hat{\jmath}\right)\,\text{N} \qquad 1$$

...

$$\vec{r}_{13} = \vec{r}_1 - \vec{r}_3 = -1.0\,\hat{i} \quad -2.0\,\hat{j} \;;\; r_{13}^2 = 1+4=5$$

$$\vec{F}_{3\,on\,1} = -\frac{G m_1 m_3}{r_{13}^2}\,\hat{r}_{13}$$

$$= \frac{6.67\times10^{-11}\times 10\times 30}{5\sqrt{5}}\,(\hat{i}+2\hat{j})\ \text{N}$$

$$= (1.79\times10^{-9}\,\hat{i} + 3.58\times10^{-9}\,\hat{j})\ \text{N} \quad 1$$

$$\vec{F}_{net\ on\ 1} = \left((4.72+1.79)\times10^{-9}\,\hat{i} + (4.72+3.58)\times10^{-9}\,\hat{j}\right)\text{N}$$

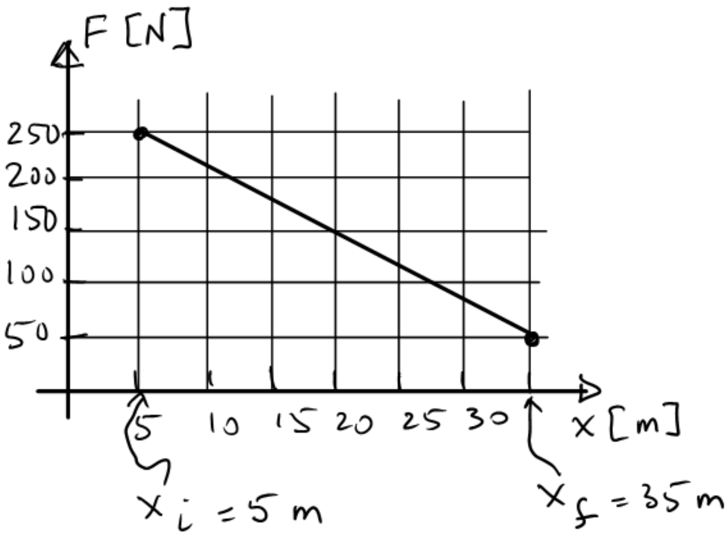
$$= (6.51\,\hat{i} + 8.30\,\hat{j})\ \text{nN} \qquad (\text{or } 10^{-9}\ \text{N})$$

$$= (6.5\,\hat{i} + 8.3\,\hat{j})\times10^{-9}\ \text{N} \quad 1$$

4) A car is pushed along a long road that is straight, flat, and parallel to the x direction. The horizontal force on the car varies with x as shown in the figure.

(a) [4] What is the work done on the car by this force?

(b) [1] What is the work done by gravity on the car? (explain!)



a) $$W = \underbrace{\int_{x_i}^{x_f} F\,dx}_{\text{area law}} \qquad (F = F_x(x))$$

area can be obtained from a triangle 1 + rectangle : $(W = W_T + W_R)$

$$W_T = \frac{1}{2}(x_f - x_i)(F_{max} - F_{min})$$

$$= \frac{1}{2}(30)\times 200\ \text{Nm} = 3000\ \text{Nm} \quad 1$$

$$W_R = (x_f - x_i)\cdot F_{min} = 30\cdot 50\ \text{Nm} = 1500\ \text{Nm}$$

$W_T + W_R = W = 4500 \text{ Nm}$ $(= 4500 \text{ J})$ 1
...
$= 4.5 \times 10^3 \text{ J}$ 1 (indicates
Nm 2 significant fig., consistent with plot)

b) Gravity acts at a right angle to the displacement 0.5

No contribution to the work 0.5

$\left(W = F \Delta \vec{r} \cos\alpha \right.$ where $\left. \alpha = \sphericalangle \vec{F}, \Delta\vec{r} \right)$

FORMULA SHEET

$v(t_\mathrm{f}) = v(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} a(t)\ dt \quad s(t_\mathrm{f}) = s(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} v(t)\ dt$

$v_\mathrm{f} = v_\mathrm{i} + a\Delta t \quad s_\mathrm{f} = s_\mathrm{i} + v_\mathrm{i}\Delta t + \frac{1}{2}a\Delta t^2 \quad v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2a\Delta s \qquad g = 9.8\ \mathrm{m/s^2}$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\ dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\ dt = at + C \qquad F(t) =$ anti-derivative $=$ indefinite integral

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

$\exp' = \exp;\ \sin' = \cos;\ \cos' = -\sin. \quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}; \quad (fg)' = f'g + fg'$

$m\vec{a} = \vec{F}_\mathrm{net}; \quad F_G = \frac{Gm_1m_2}{r^2};\ g = \frac{GM_E}{R_E^2};\ R_E = 6370\ \mathrm{km};\ G = 6.67\times10^{-11}\frac{\mathrm{Nm^2}}{\mathrm{kg^2}};\ M_E = 6.0\times10^{24}\mathrm{kg}$

$f_\mathrm{s} \le \mu_\mathrm{s} n; \quad f_\mathrm{k} = \mu_\mathrm{k} n; \quad \mu_\mathrm{k} < \mu_\mathrm{s}.$

$\vec{F}_\mathrm{d} \sim -\vec{v}$; linear: $F_\mathrm{d} = dv$; quadr.: $F_\mathrm{d} = 0.5\rho A v^2$; $\quad A =$ cross s'n area; $\rho =$ density of medium

Sphere: $V = \frac{4}{3}\pi R^3$; Total Surface: $A_S = 4\pi R^2$; Cross Section=?

uniform circular m.: $\vec{r}(t) = R(\cos\omega t\ \hat{\mathbf{i}} + \sin\omega t\ \hat{\mathbf{j}});\ \vec{v}(t) = \frac{d\vec{r}}{dt} = ...;\ \vec{a}(t) = \frac{d\vec{v}}{dt} = ...;\ \omega = \frac{2\pi}{T}.$

$a_\mathrm{cp} = \frac{v^2}{r} \qquad v = \omega r.$

$W = F\Delta x = F(\Delta r)\cos\theta \quad$ For $F(x)$ the work is given as area under the F_x vs x curve.