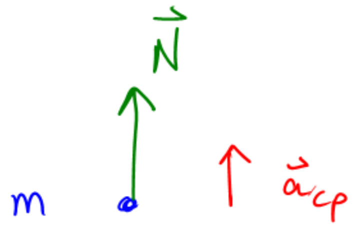
LAST NAME: STUDENT NR:

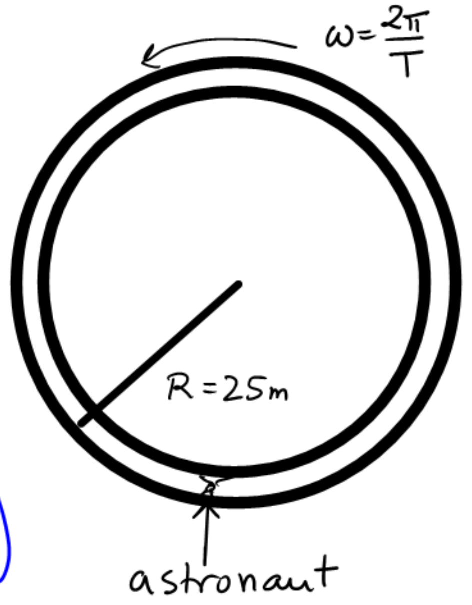
PHYS 1010 6.0: CLASS TEST 2

Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1) [5] Consider a circular space station as shown. Suppose the station has a radius of 25 m, and is designed to provide artificial gravity with the value of $g = 9.8$ m/s^2. What would be the required rotation period? Start with a free-body diagram for a person standing on the rim, and ignore the gravitational fields from earth, i.e., assume the station is in free space.



1 $\uparrow$ = towards centre



Normal force $\vec{N}$ is provided by the rim

$$a_{cp} = \frac{v^2}{R} = R\,\omega^2 \quad \left(\text{since } \omega = \frac{v}{R}\right)$$

$$= R\left(\frac{2\pi}{T}\right)^2 \qquad 1$$

We demand $a_{cp} = g = 9.8\ \text{m/s}^2$ 1

$$\therefore \left(\frac{2\pi}{T}\right)^2 = \frac{g}{R} \qquad \therefore \frac{T}{2\pi} = \sqrt{\frac{R}{g}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{R}{g}}$$

$$T = 6.28\sqrt{\frac{25}{9.8}}\ \text{s} = 10.\ \text{s} \qquad 2$$

extra: $v_{rim} = R\omega = R\frac{2\pi}{T} = \frac{25 \cdot 6.28}{10}\ \frac{\text{m}}{\text{s}} = 15.7\ \frac{\text{m}}{\text{s}}$

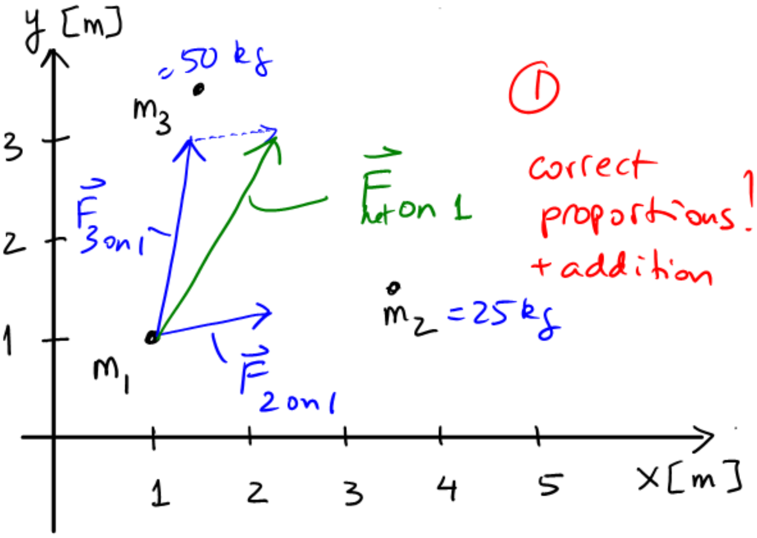
Pretty fast! $\longrightarrow$ $= 56.7\ \frac{\text{km}}{\text{h}}$

2) [5] Consider the constellation of three masses shown in the figure, where $m_1 = 12$ kg, $m_2 = 25$ kg, and $m_3 = 50$ kg. Find the force of gravity on mass m_1 due to m_2 and m_3, and state it in a notation using $\hat{i}$ and $\hat{j}$. Before you start the calculation provide arrow drawings for the forces $\vec{F}_{2 \text{ on } 1}$, and $\vec{F}_{3 \text{ on } 1}$, and a graphical addition to show $\vec{F}_{\text{net on } 1}$ in the provided figure.

$\vec{r}_1 = \hat{i} + \hat{j}$

$\vec{r}_2 = 3.5\,\hat{i} + 1.5\,\hat{j}$

$\vec{r}_3 = 1.5\,\hat{i} + 3.5\,\hat{j}$



$$\vec{F}_{2\text{ on }1} = \frac{G m_2 m_1}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1) \quad 1$$

$$\vec{F}_{3\text{ on }1} = \frac{G m_3 m_1}{|\vec{r}_3 - \vec{r}_1|^3}(\vec{r}_3 - \vec{r}_1)$$

$\vec{r}_2 - \vec{r}_1 = 2.5\,\hat{i} + 0.5\,\hat{j}$ $\qquad r_{12} = \sqrt{2.5^2 + .5^2} = 2.55$ m

$\vec{r}_3 - \vec{r}_1 = 0.5\,\hat{i} + 2.5\,\hat{j}$ $\qquad r_{13} = 2.55$ m $= r_{12}$

$$\therefore \quad \vec{F}_{\text{net on }1} = \frac{G m_1}{r_{12}^3}\left(m_2(\vec{r}_2 - \vec{r}_1) + m_3(\vec{r}_3 - \vec{r}_1)\right) \quad 1$$

note: $m_3 = 2m_2$!

in SI! $$= \frac{6.67 \times 10^{-11}}{2.55^3} m_1 m_2 \left(\vec{r}_2 - \vec{r}_1 + 2(\vec{r}_3 - \vec{r}_1)\right)$$

$$= \frac{6.67 \times 10^{-11} \cdot 12 \cdot 25}{16.6}\left(2.5\,\hat{i} + 0.5\,\hat{j} + 1.0\,\hat{i} + 5.0\,\hat{j}\right)$$

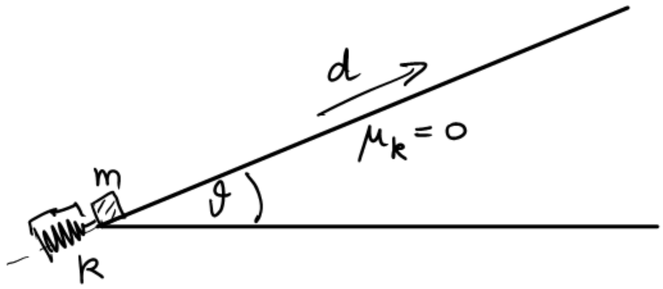
$$= 1.21 \times 10^{-11}\left(3.5\,\hat{i} + 5.5\,\hat{j}\right) = \left(4.2\,\hat{i} + 6.7\,\hat{j}\right) \times 10^{-11}\,\text{N} \quad 2$$

6.655 also OK

direction is consistent with green arrow.

3) [5] A mass $m = 1.5$ kg rests on a compressed spring ($k = 400$ N/m) which is aligned with an inclined plane ($\theta = 30°$), assumed to be frictionless. The spring is secured by a catch mechanism. When the catch is released, the mass moves up along the incline a distance of $d = 2.0$ m. What was the compression of the spring? Ignore the effects from gravity while the mass is in contact with the spring (the compression is small compared to the distance d).

$PE_g = mgy$



$\frac{y}{d} = \sin\vartheta$

1

$= \sin(30°) = 0.5$

$PE_g = mg\,d\sin\vartheta = 1.5 \cdot 9.8 \cdot 2.0 \cdot 0.5 \text{ Nm} = 14.7 \text{ J}$ 1

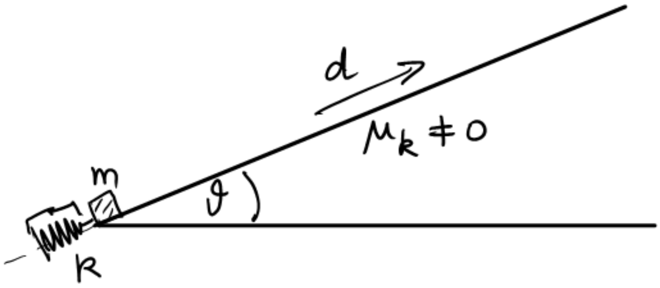
spring energy: $PE_H = \frac{1}{2} k \Delta x^2 = PE_g$ 1

$\therefore \quad \Delta x^2 = \frac{2 PE_g}{k} = \frac{2 \cdot 14.7}{400} = 0.0735 \text{ m}^2$

$\Delta x = 0.27 \text{ m} = 27 \text{ cm}$ 2

The compression is not so small, after all!

4) [5] Reconsider problem (3). Now the incline is no longer assumed frictionless, but the coefficient of kinetic friction μ_k is unknown. For the same experiment as before, the mass is found to travel only a distance of $d = 1.5$ m. Calculate μ_k. Note: to find the answer it is not necessary to solve (3) completely.



The total energy provided by the spring is given by the gravitational PE at maximum travel d in problem (3).

$$E_{tot} = m g y_0 = m g d_0 \sin\vartheta = 14.7 \text{ J} \quad (\text{from } 3)$$

With kinetic friction the mass travels only to $d = 1.5$ m $\therefore$ $E_{fin} = m g d \sin\vartheta = 11.0$ J 1

$\therefore$ The friction force did negative work on the mass of $(11.0 - 14.7)$ J $= -3.7$ J 1

The force magnitude is given as $\mu_k N = \mu_k m g_\perp$

in SI: $|F_k| = \mu_k m g \cos\vartheta = \mu_k \cdot 1.5 \cdot 9.8 \cdot 0.866 = 12.7 \mu_k$ 1

The work done:

$$W = F_k d = -12.7 \mu_k \cdot 1.5 = -\mu_k \cdot 19.1 \text{ J}$$ 1

$\therefore$ $-3.7 = -19.1 \mu_k$ $\therefore$ $\mu_k = 0.19$ 1

reasonable, since $0 < \mu_k < 1$

FORMULA SHEET

$v(t_\mathrm{f}) = v(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} a(t)\ dt \quad s(t_\mathrm{f}) = s(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} v(t)\ dt$

$v_\mathrm{f} = v_\mathrm{i} + a\Delta t \quad s_\mathrm{f} = s_\mathrm{i} + v_\mathrm{i}\Delta t + \frac{1}{2}a\Delta t^2 \quad v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2a\Delta s \qquad g = 9.8\ \mathrm{m/s^2}$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\ dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\ dt = at + C \quad F(t) =$ anti-derivative = indefinite integral

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

$\exp' = \exp$; $\sin' = \cos$; $\cos' = -\sin$. $\quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}$; $\quad (fg)' = f'g + fg'$

$m\vec{a} = \vec{F}_\mathrm{net}$; $\quad F_G = \frac{Gm_1m_2}{r^2}$; $g = \frac{GM_E}{R_E^2}$; $R_E = 6370$ km; $G = 6.67 \times 10^{-11}\frac{\mathrm{Nm^2}}{\mathrm{kg^2}}$; $M_E = 6.0 \times 10^{24}$kg

$f_\mathrm{s} \leq \mu_\mathrm{s} n$; $\quad f_\mathrm{k} = \mu_\mathrm{k} n$; $\quad \mu_\mathrm{k} < \mu_\mathrm{s}$; $\qquad F_H = -k\Delta x = -k(x - x_0)$.

$\vec{F}_\mathrm{d} \sim -\vec{v}$; linear: $F_\mathrm{d} = dv$; quadr.: $F_\mathrm{d} = 0.5\rho A v^2$; $\quad A$ = cross s'n area; ρ = density of medium

Sphere: $V = \frac{4}{3}\pi R^3$; Total Surface: $A_S = 4\pi R^2$; Cross Section=?

uniform circular m.: $\vec{r}(t) = R(\cos\omega t\ \hat{\mathbf{i}} + \sin\omega t\ \hat{\mathbf{j}})$; $\vec{v}(t) = \frac{d\vec{r}}{dt} = ...$; $\vec{a}(t) = \frac{d\vec{v}}{dt} = ...$; $\omega = \frac{2\pi}{T}$.

$a_\mathrm{cp} = \frac{v^2}{r} \quad v = \omega r.$

$W = F\Delta x = F(\Delta r)\cos\theta$. W = area under $F_x(x)$. $\quad PE_\mathrm{H} = \frac{k}{2}(\Delta x)^2$; $PE_g = mgy$.

$K = \frac{m}{2}v^2$