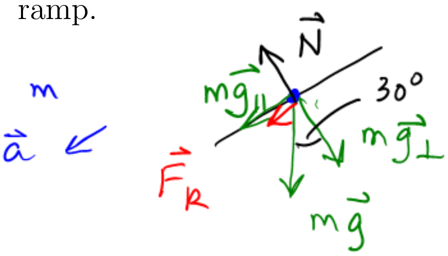
LAST NAME: STUDENT NR:

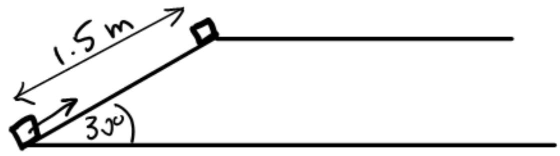
PHYS 1010 6.0: CLASS TEST 2 on Oct. 26, 2012

Time: 50 minutes; Calculators & formulae provided at the end = only aid; ask no questions!

1) [5] A 2.0 kg parcel is launched up a ramp which ends in a horizontal surface with known initial speed ~~3.5~~ 6.5 m/s. The ramp makes a 30-degree angle with the horizontal and is ~~2.5~~ 1.5 m long. The coefficient of kinetic friction between parcel and ramp is given as 0.2. Calculate the velocity vector of the parcel as it arrives at the top of the ramp (it will launch into the air at this point!). Begin your solution with a complete free-body diagram for the parcel on the ramp.



1.0 (all details)



$g_{\parallel} = g\sin\alpha = g\sin 30^\circ = g/2 = 0.5g$ 0.5

$g_{\perp} = g\cos\alpha = g\cos 30^\circ = \frac{\sqrt{3}}{2}g = 0.866g$ 0.5

choose $-\vec g_{\parallel}$ direction as $\hat\imath$ or $\hat s$

$F_{net,s} = \underbrace{-mg_{\parallel} - \mu_k m g_{\perp}}_{0.5} = -mg(\underbrace{0.5 + 0.2\cdot 0.866}_{0.673}) = -13.2\,N$ 0.5

or $a_s = -6.60\ m/s^2$ (used in (B) below)

Now use either a work calculation, or const.-a kinematics

(A) $W = F_s\Delta s = -19.8\ J$ 0.5

$K_i = \frac{1}{2}mv_i^2 = 1.0\cdot 6.5^2 = 42.25\ J$

0.5 $\rightarrow K_f = \frac{1}{2}mv_f^2 = K_i + W = 22.45\ J$

$\therefore v_f = 4.74\ \frac{m}{s}$ $\vec v_f = \underset{v_f\cos 30^\circ}{4.1}\hat\imath + \underset{v_s}{2.4}\hat\jmath\ (\frac{m}{s})$

$\vec v_f = v_f\cos\alpha\,\hat\imath + v_f\sin\alpha\,\hat\jmath = 4.74\cdot 0.866\,\hat\imath + 4.74\cdot 0.5\,\hat\jmath$

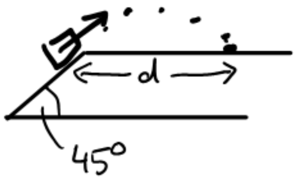
$= 4.1\,\hat\imath + 2.4\,\hat\jmath\ (\frac{m}{s})$ 1

alternative: (B) 0.5 $v_f^2 = v_i^2 + 2a\Delta s \Rightarrow$ constant acceleration kinematics

0.5 $v_f^2 = 42.25 - 2\cdot 6.60\cdot 1.5 = 22.45\ \frac{m^2}{s^2}$

↳ then the same as above. 1 for final answer

2) [5] A 3.0 kg parcel arrives at the top of a ramp which makes a 45-degree angle with the horizontal with a speed of 2.4 m/s. It lands some distance d away on the horizontal surface. Calculate this distance while neglecting air drag. Do not use a (memorized) formula for the range, but derive your result for d from the kinematic equations given in the formula sheet. Provide a free-body diagram for the parcel while it is airborne.



$$\vec{v}_0 = v_0 \cos 45^\circ \,\hat{i} + v_0 \sin 45^\circ \,\hat{j}$$

$$= v_0 \left(\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}} \right)$$

$$\left.\begin{aligned} v_{0x} &= 1.70 \text{ m/s} \\ v_{0y} &= 1.70 \text{ m/s} \end{aligned}\right\} 1$$

$$\left.\begin{aligned} v_x(t) &= v_{0x}, & x(t) &= x_0 + v_{0x} t \\ v_y(t) &= v_{0y} - gt, & y(t) &= y_0 + v_{0y} t - \tfrac{1}{2} g t^2 \end{aligned}\right\} 1$$

use $x_0 = 0$ and $t = 0$ for the start of the "flight".

$x_{fin} = d = v_{0x} t_f$ time of flight: $t_f = \dfrac{d}{v_{0x}}$ 1

1

$y(t_f) = y_0 \Rightarrow v_{0y} t_f - \frac{1}{2} g t_f^2 = 0$

$\therefore t_f \left(v_{0y} - \frac{1}{2} g t_f \right) = 0$

We are interested in $(\;) = 0$, since $t_f = 0$ is the start of the flight

$\therefore 2 v_{0y} - g t_f = 0 \quad \therefore t_f = \dfrac{2 v_{0y}}{g}$ 2 1

Thus, $\dfrac{d}{v_{0x}} = \dfrac{2 v_{0y}}{g} \quad \therefore d = \dfrac{2 v_{0y} v_{0x}}{g}$

1 = 2

$$d = \frac{2 v_0^2 \cos 45^\circ \sin 45^\circ}{g} = \frac{v_0^2}{g} \longrightarrow d = 0.59 \text{ m} \quad 1$$

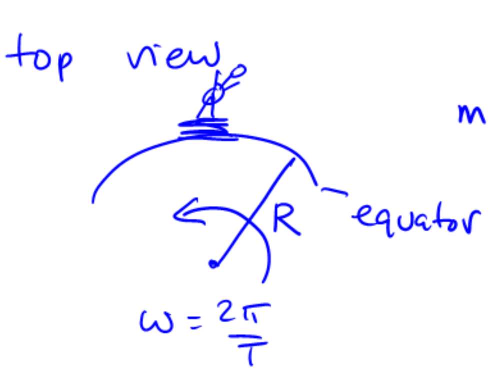
3) An astronaut with mass $m = 80$ kg (including her gear) steps on a spring scale on the equator of planet X, and finds a reading of 90 kg. Mission control estimated that the planet has a mass of $M = 3.5 \times 10^{24}$kg, and a radius of 4,500 km, but was unable to provide the period T at which the planet is spinning about its north-south axis.

a) [2] Calculate the $g-$value for this planet, and determine what the reading on the spring scale should be if the planet wasn't spinning. Hint: a spring scale measures the normal force, and is calibrated on earth based on $N = mg_e$, where $g_e = 9.8$ m/s^2.

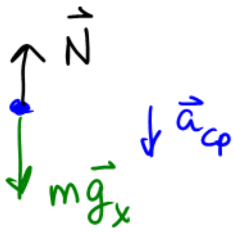
$$g_x = \frac{GM}{R^2} = \frac{6.67 \cdot 10^{-11} \cdot 3.5 \cdot 10^{24}}{4.5^2 \cdot (10^6)^2} = 11.5 \ \frac{m}{s^2} \qquad 1$$

$$N = m g_x = \underset{0.5}{922\ N}, \text{ but } \ m_{display} = \frac{N}{g_e} = \underset{0.5}{94.1\ kg}$$

b) [3] Determine what the astronaut finds T to be on the basis of her measurement. Begin with a free-body diagram.



m



1

$$m a_{cp} = m g_x - N$$

$$1 \qquad m R \omega^2 = m g_x - N \qquad N = 90 \cdot 9.8 \ N$$

reading

$$\left(\frac{2\pi}{T}\right)^2 = \frac{g_x}{R} - \frac{N}{mR}$$

$$\therefore \ \frac{1}{T^2} = \frac{1}{4\pi^2 R}\left(g_x - \frac{N}{m}\right) = \frac{1}{4\pi^2 R}\Big(\underbrace{11.5 - \frac{90 \cdot 9.8}{80}}_{0.475}\Big)$$

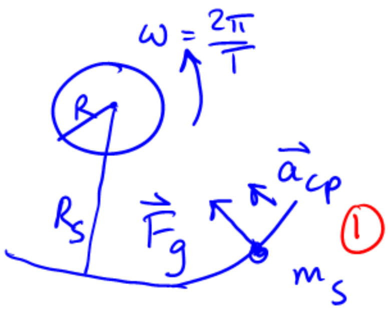
$$\frac{1}{T^2} = \frac{0.475}{4\pi^2 \, 4.5 \times 10^6} = 2.68 \times 10^{-9}$$

$$T^2 = 0.373 \times 10^9 = 3.73 \times 10^8$$

$$T = 1.9 \times 10^4 \ s \quad \simeq 5.3 \text{ hours}. \qquad 1$$

4) [5] Derive the result for the (circular) geosynchronous satellite orbit radius. Begin with a free-body diagram for the satellite over the earth's equator. Comment on how accurate this result may be. The formula sheet lists most of the required data. Hint: a geosynchronous satellite does not move with respect to a fixed location on the equator, which means it can be used for direct-line communication all the time.

$$T = 24^h = 24 \times 3{,}600 \text{ s}$$



$$a_{cp} = R_S \omega^2 \quad \text{where } \omega = \frac{2\pi}{T}$$

(synchronous)

$$m_S a_{cp} = \frac{G M_E m_S}{R_S^2} \quad 1 \quad \left(2^{nd} \text{ law}\right)$$

$$\therefore \quad R_S \left(\frac{2\pi}{T}\right)^2 = \frac{G M_E}{R_S^2}$$

$$R_S^3 = \frac{G M_E T^2}{4\pi^2}$$

$$R_S = \sqrt[3]{\frac{G M_E T^2}{4\pi^2}} \quad 1$$

$$R_S = \sqrt[3]{\frac{6.67 \times 10^{-11} \cdot 6.0 \times 10^{24} \cdot (24 \cdot 3{,}600)^2}{4\pi^2}}$$

$$= 4.2 \times 10^7 \text{ m} = 42{,}000 \text{ km} \approx 6.6\, R_E$$

2

FORMULA SHEET

$v(t_\mathrm{f}) = v(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} a(t)\ dt \quad s(t_\mathrm{f}) = s(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} v(t)\ dt$

$v_\mathrm{f} = v_\mathrm{i} + a\Delta t \quad s_\mathrm{f} = s_\mathrm{i} + v_\mathrm{i}\Delta t + \frac{1}{2}a\Delta t^2 \quad v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2a\Delta s \qquad g = 9.8\ \mathrm{m/s^2}$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\ dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\ dt = at + C \quad F(t) =$ anti-derivative = indefinite integral

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

$\exp' = \exp$; $\sin' = \cos$; $\cos' = -\sin$. $\quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}$; $\quad (fg)' = f'g + fg'$

$m\vec{a} = \vec{F}_\mathrm{net}$; $\quad F_G = \frac{Gm_1m_2}{r^2}$; $g = \frac{GM_E}{R_E^2}$; $R_E = 6370$ km; $G = 6.67 \times 10^{-11} \frac{\mathrm{Nm^2}}{\mathrm{kg^2}}$; $M_E = 6.0 \times 10^{24}$kg

$f_\mathrm{s} \le \mu_\mathrm{s} n$; $\quad f_\mathrm{k} = \mu_\mathrm{k} n$; $\quad \mu_\mathrm{k} < \mu_\mathrm{s}$; $\qquad F_H = -k\Delta x = -k(x - x_0)$.

$\vec{F}_\mathrm{d} \sim -\vec{v}$; linear: $F_\mathrm{d} = dv$; quadr.: $F_\mathrm{d} = 0.5\rho A v^2$; $\quad A$ = cross s'n area; ρ = density of medium

Sphere: $V = \frac{4}{3}\pi R^3$; Total Surface: $A_S = 4\pi R^2$; Cross Section=?

uniform circular m.: $\vec{r}(t) = R(\cos\omega t\ \hat{\mathbf{i}} + \sin\omega t\ \hat{\mathbf{j}})$; $\vec{v}(t) = \frac{d\vec{r}}{dt} = ...$; $\vec{a}(t) = \frac{d\vec{v}}{dt} = ...$; $\omega = \frac{2\pi}{T}$.

$a_\mathrm{cp} = \frac{v^2}{r} \quad v = \omega r.$

$W = F\Delta x = F(\Delta r)\cos\theta$. W = area under $F_x(x)$. $\quad PE_\mathrm{H} = \frac{k}{2}(\Delta x)^2$; $PE_g = mgy$.

$K = \frac{m}{2}v^2$