LAST NAME: STUDENT NR:

PHYS 1010 6.0: CLASS TEST 3 on Dec. 3, 2010

Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1) Two eager hockey players are entering a head-on collision with velocities $v_1 = 10$ m/s and $v_2 = -8$ m/s respectively. They grab each other, and come out of the collision together with $V = -2$ m/s. Player 1 has a mass of 90 kg.

a) [3] What is the mass of player 2 ? Explain your steps.

b) [2] What happened to the mechanical energy? Ignore frictional losses between skates and ice, ie., neither player 'applied the brakes'.

- The collision is inelastic, since they emerge with a common velocity ("sticky collision") 0.5
- Linear momentum conservation applies since no external forces act along the direction of motion (skates glide freely) 0.5

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) V \quad 1$$

$$m_2 (v_2 - V) = m_1 (V - v_1)$$

$$m_2 = m_1 \frac{V - v_1}{v_2 - V} \quad 0.5$$

in SI ∴ $m_2 = 90 \dfrac{-2-10}{-8+2} = 180$ $\quad m_2 = 180$ kg 0.5

b) $(KE_1 + KE_2)_{before} = \frac{1}{2}(m_1 v_1^2 + m_2 v_2^2) \underset{\text{in SI}}{=} \frac{1}{2}(90 \cdot 100 + 180 \cdot 64) = 1.03 \times 10^4$ J 0.5

$(KE_1 + KE_2)_{after} = \frac{1}{2}(m_1 + m_2) V^2 = 540$ J 0.5

972 kJ of mech. energy was lost (big bruises!) 1.
(almost all)

2) A roller coaster car enters a loop with a speed of 100 km/h at the bottom. The car with six passengers has a mass of 800 kg. The loop radius is 15 m. Ignore frictional/drag losses on the car's motion.

a) [3] Demonstrate that the car will make it over the top without falling out of the tracks.

b) [2] Calculate the normal force (tracks on car) at the top.

2. $v_B = 100 \frac{km}{h} = 27.8 \frac{m}{s}$ $\qquad v_B^2 = 772 \frac{m^2}{s^2}$

Energy conservation: $\frac{1}{2}\cancel{m}v_B^2 = \frac{1}{2}\cancel{m}v_{top}^2 + \cancel{m}g\underbrace{\Delta y}_{2R}$ 1

$\therefore v_{top}^2 = v_B^2 - 4gR = 184 \frac{m^2}{s^2}$

centripetal acceleration at top: $a_c = \frac{v^2}{R} = 12.2 \frac{m}{s^2}$ 1

Since a_c exceeds g, the car moves faster than with the critical speed $(v_c = \sqrt{gR})$ 1

optional $\longrightarrow v_c^2 = 147 \frac{m^2}{s^2}$

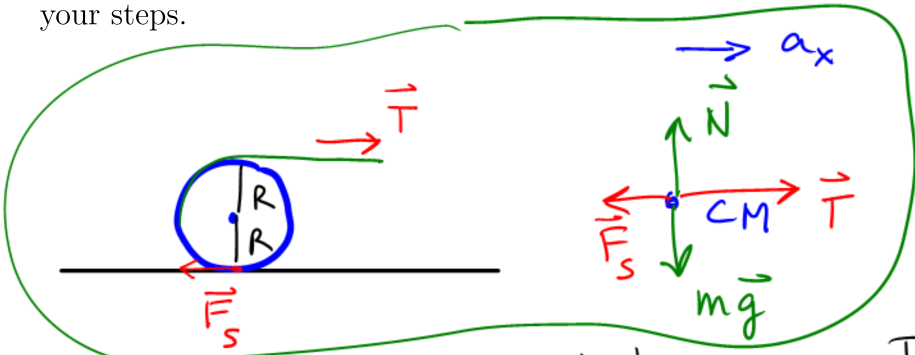
b) g provides $9.8\ m/s^2$, the balance is due to N_T:

in SI: $N_T = m(a_c - g) = m \cdot 2.44 = 800 \cdot 2.44 = 1.95\times10^3 N$ 1

The normal force provided by the track is $2.0\times10^3 N$ 1

[5]

3) A cylindrical spool has some turns of thin thread wound around it (no change in radius) and the free end comes out on top when the spool is placed on the table. The spool has a mass of 2.0 kg. Applying a tension force of 15 N causes the spool to move (it is dragged and executes no-slip rolling motion). Calculate the linear acceleration of the centre of mass (CM) of the spool. The rotational inertia of a rolling cylinder about the CM is $\frac{1}{2}mR^2$, and about a point on the rim it is $\frac{3}{2}mR^2$. Start with a sketch indicating all forces that apply and explain your steps.



= 1

$F_{net,y} = 0 \quad (N = mg)$

$ma_x = T - F_s$ 0.5

Rotational motion (either option is ok)

pivot = CM : $I_{CM}\alpha = -RT - RF_s$ (a)

pivot = contact : $I_0\alpha = -2RT$ 1 (b)

Condition of no-slip rolling: $\Delta x = -R\,\Delta\vartheta$ positive translation

$v_x = -R\omega$ $=$ CW rot'n

$a_x = -R\alpha$ $\therefore$ − sign

0.5

Using option (a): $I_{CM} = \frac{1}{2} m R^2$

$$\frac{1}{2} m \cancel{R^2}\left(-\frac{a_x}{\cancel{R}}\right) = -\cancel{R}(T + F_s) \quad \therefore \quad a_x = \frac{2(T+F_s)}{m}$$

Using (b): $I_0 = \frac{3}{2} m R^2$:

$$\frac{3}{2} m \cancel{R^2}\left(-\frac{a_x}{\cancel{R}}\right) = -2\cancel{R}T \quad \therefore \quad a_x = \frac{4T}{3m}$$ 1

Now combine: (a) $m a_x = T - F_s$ and $m a_x = 2(T + F_s)$

$\therefore \quad 2T + 2F_s = T - F_s \quad \therefore \quad T = -3F_s \quad \therefore \quad F_s = -\frac{1}{3}T$

OR: (b) $\quad m a_x = \frac{4}{3}T = T - F_s \quad \therefore \quad \frac{1}{3}T = -F_s \quad \therefore \quad F_s = \frac{-T}{3}$

Motion of CM: $\quad a_x = \frac{1}{m}(T - F_s) = \frac{1}{m}\left(T - \left(-\frac{T}{3}\right)\right) = \frac{4T}{3m}$

in SI: $\quad a_x = \frac{4}{3}\,\frac{15}{2.0} = 10\ \frac{m}{s^2}$ 1 || NB: $\vec{F}_s$ is actually in the direction of $\vec{a}$!

optional:

cf: slipping (no rolling): $m a_x = T$ ($F_s = 0$) $\quad a_x = \frac{15}{2.0} = 7.5\ m/s^2$

4) [5] A bowling ball arrives with a speed of 12 m/s at the bottom of an inclined plane. What is the vertical height Δy the ball will reach under no-slip rolling motion? The rotational inertia of a sphere is $\frac{2}{5}mR^2$. Note that even though you don't know the angle of inclination, nor the mass of the ball, or its radius, a numerical answer can be obtained. Compare this height to the height obtained under perfect sliding (no rolling, no friction).

perfect sliding: $\quad \frac{1}{2}\cancel{m}v_i^2 = \cancel{m}g\,\Delta y \quad \therefore \quad \Delta y = \frac{72}{9.8} = 7.3\,m$ 1

Rolling: $\quad \Delta x = R\,\Delta\vartheta \quad$ or $\quad v_x = R\omega \quad$ (no-slip)

$$\underbrace{\text{initial KE}: \quad \frac{1}{2} m v_x^2 + \frac{1}{2} I_{CM}\,\omega^2}_{1} = \frac{1}{2} m\left[v_x^2 + \frac{2}{5}R^2\frac{v_x^2}{R^2}\right]$$

$$= \frac{1}{2} m\left[\frac{7}{5}v_x^2\right] = m \cdot 101 \text{ SI}$$ 1

final PE: $m g \Delta y$ 1 $\quad \therefore \quad \Delta y = 10.3\,m$

. The final height under rolling: $\Delta y = 10$ m ①

(it is more than under sliding conditions, since the rotational energy is in addition to the translation of the CM)

Additional remark for Q3: When pulling the spool under rolling vs sliding we get $a_x = 10 \frac{m}{s^2}$ vs $7.5 \frac{m}{s^2}$ from the same T! In addition we are storing rotational energy. Are we getting something for free??

NO: we are unwinding the string. The end of the string (where $\vec{T}$ acts) travels with $\Delta X = 2\,\Delta X_{CM}$ due to the unwinding. Thus, the work put into the system $W = T\,\Delta X$ is twice (for constant T) as compared to dragging the spool without rolling. Some of this extra energy goes towards an increase in translational motion.

Why does static friction do the counter-intuitive thing? → slowing rotation, +boosting translation?

Under rolling the point of contact is at rest. Electrostatic bonding can pull in either direction (up to the limit $\mu_s N$).

FORMULA SHEET

$v(t_f) = v(t_i) + \int_{t_i}^{t_f} a(t)\, dt \qquad s(t_f) = s(t_i) + \int_{t_i}^{t_f} v(t)\, dt$

$v_f = v_i + a\Delta t \qquad s_f = s_i + v_i\Delta t + \frac{1}{2}a\Delta t^2 \qquad v_f^2 = v_i^2 + 2a\Delta s \qquad g = 9.8\ \mathrm{m/s^2}$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\, dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\, dt = at + C \qquad F(t) =$ anti-derivative = indefinite integral

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

$\exp' = \exp$; $\sin' = \cos$; $\cos' = -\sin$. $\quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}$; $\quad (fg)' = f'g + fg'$

$m\vec{a} = \vec{F}_{\text{net}}$; $\quad F_G = \frac{Gm_1m_2}{r^2}$; $g = \frac{GM_E}{R_E^2}$; $R_E = 6370$ km; $G = 6.67 \times 10^{-11} \frac{\mathrm{Nm^2}}{\mathrm{kg^2}}$; $M_E = 6.0 \times 10^{24}$kg

$f_s \le \mu_s n$; $\quad f_k = \mu_k n$; $\quad \mu_k < \mu_s$; $\qquad F_H = -k\Delta x = -k(x - x_0)$.

$\vec{F}_d \sim -\vec{v}$; linear: $F_d = dv$; quadr.: $F_d = 0.5\rho A v^2$; $\quad A$ = cross s'n area; ρ = density of medium

Sphere: $V = \frac{4}{3}\pi R^3$; Total Surface: $A_S = 4\pi R^2$; Cross Section=?

uniform circular m.: $\vec{r}(t) = R(\cos\omega t\ \hat{\mathbf{i}} + \sin\omega t\ \hat{\mathbf{j}})$; $\vec{v}(t) = \frac{d\vec{r}}{dt} = ...$; $\vec{a}(t) = \frac{d\vec{v}}{dt} = ...$; $\omega = \frac{2\pi}{T}$.

$a_{cp} = \frac{v^2}{r} \qquad v = \omega r.$

$W = F\Delta x = F(\Delta r)\cos\theta$. $\ W$ = area under $F_x(x)$. $\qquad PE_H = \frac{k}{2}(\Delta x)^2$; $PE_g = mg\Delta y$.

$\Delta\vec{p} = \vec{J} = \int \vec{F}(t)dt$; $\Delta p_x = J_x$ = area under $F_x(t) = F_x^{\text{avg}}\Delta t$; $\quad \vec{p} = m\vec{v}$; $K = \frac{m}{2}v^2$

$\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$; $K_1^{\text{in}} + K_2^{\text{in}} = K_1^{\text{fin}} + K_2^{\text{fin}}$ for elastic collisions. $\quad \vec{a}_{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$

$\vec{\tau} = \vec{r} \times \vec{F}$; $\quad \tau_z = rF\sin(\alpha)$ for $\vec{r}$, $\vec{F}$ in xy plane. $\quad I = \sum_i m_i r_i^2$; $\quad I\alpha_z = \tau_z$; ($\hat{k}$ = rot. axis)

$K_{\text{rot}} = \frac{I}{2}\omega^2$; $L_z = I\omega_z$; $\frac{d}{dt}L_z = \tau_z$; $\vec{L} = \vec{r} \times \vec{p}$; $\frac{d}{dt}\vec{L} = \vec{\tau}$