LAST NAME: STUDENT NR:

PHYS 1010 6.0: CLASS TEST 3 on Dec. 2, 2011

Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1) [5] A slowly moving car (mass 1200 kg, speed 25 km/h is hit directly from behind by an SUV (mass 2700 kg, speed 75 km/h). The two vehicles stick together and emerge with some velocity right after the collision (before braking sets in; assume one-dimensional motion). Find this velocity. Calculate how much kinetic energy is lost in the collision. Assuming a collision time of 0.1 seconds find the acceleration for both vehicles.

- sticky collision is totally inelastic
- total momentum is conserved

car: $m = 1200$ kg, $v = 25$
SUV: $M = 2700$ kg, $V = 75$ km/h

.5 $P_{tot}^{before} = mv + MV$ (along x, same direction)

$P_{tot} = 1200 \cdot 25 + 2700 \cdot 75 = 2.325 \times 10^5 \; kg\frac{km}{h}$

$= 6.46 \times 10^4 \; kg\frac{m}{s}$ (SI)

.5 $P_{tot}^{after} = (m+M)V_{fin} \simeq 3900 \cdot V_{fin} = P_{tot}^{before}$ (momentum conservation)

$V_{fin} = 16.6 \frac{m}{s}$ (17) $= 59.6 \frac{km}{h}$ (60) 1 — both OK; ← 2 sig. digits

1 $KE^{before} = \frac{1}{2}mv^2 + \frac{1}{2}MV^2 = 600 \cdot \left(\frac{25}{3.6}\right)^2 + \frac{2700}{2}\left(\frac{75}{3.6}\right)^2$ J

$= 2.89 \times 10^4 + 5.86 \times 10^5 = 6.15 \times 10^5$ J $= 615$ kJ $(= 6.1 \times 10^5 \text{ J})$

$KE^{after} = \frac{1}{2} \cdot 3900 \cdot (16.6)^2$ J $= 5.37 \times 10^5$ J $= 537$ kJ

$KE^{loss} = KE^{after} - KE^{before} = 7.8 \times 10^4$ J $(= 78$ kJ$)$ 1

$a = \frac{\Delta v}{\Delta t}$: $a_m = \frac{16.6 - 6.9}{0.1} = 96 \; m/s^2$ 0.5 $\approx 10g$

$a_M = \frac{16.6 - 20.8}{0.1} = 42.3 \; m/s^2 \gtrsim 4g$ 0.5

Note $Ma_M \simeq -ma_m$ (Newton's 3rd law relates $F_{m \, on \, M}$ to $F_{M \, on \, m}$)

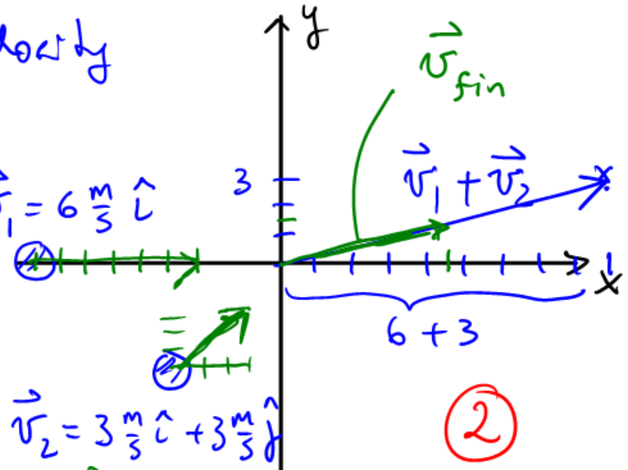
2) [5] Two pucks of equal mass approach each other to collide at $x = 0, y = 0$. Puck 1 has a speed of 6 m/s, and moves along the $x-$axis (at negative x before the collision). Puck 2 has a velocity of $(3\hat{i} + 3\hat{j})$ m/s (i.e., approaches from negative x and y). Provide a drawing which represents the system before the collision, and indicate by an arrow the total momentum vector together with the momentum vectors for both pucks. The puck sides were coated with some super-glue and stick together after the collision. What is the velocity vector for the combined system expressed by (x, y) components? What is this vector expressed through magnitude and direction angle?

equal mass: momentum and velocity vectors scale equally

$$\vec{P}_{tot} = m(\vec{v}_1 + \vec{v}_2)$$

(length of $\vec{v}_2$: $\sqrt{9+9}$

$= \sqrt{18} \simeq 4.2$



After the collision: $\vec{P}_{tot} = 2m\vec{v}_{fin} \therefore \vec{v}_{fin} = \frac{1}{2}(\vec{v}_1 + \vec{v}_2)$

$$\vec{v}_{comb} = (6+3)\hat{i} + 3\hat{j} \ \frac{m}{s}$$

$$= 9\hat{i} + 3\hat{j} \ \frac{m}{s}$$

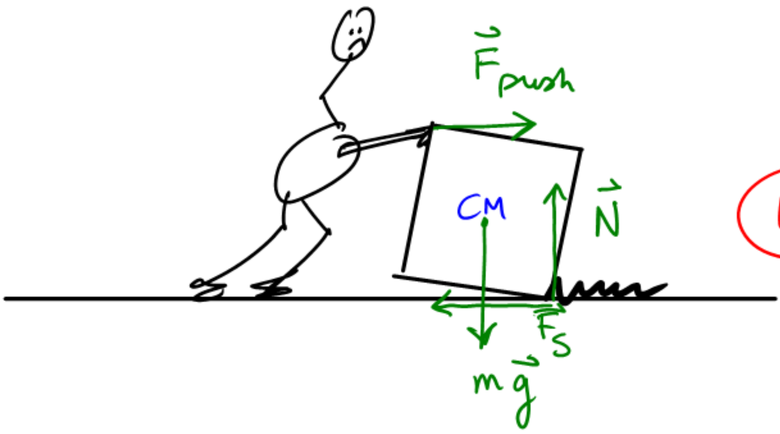
$$\vec{v}_{fin} = \frac{1}{2}\vec{v}_{comb} = 4.5\hat{i} + 1.5\hat{j} \ \frac{m}{s} \quad 1$$

SI: $v_{fin} = \frac{1}{2}\sqrt{81+9} = \frac{1}{2}\sqrt{90} = 4.75 \ \frac{m}{s}$ 1

$$\vartheta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{3}{9} = 18.4^\circ = 0.32 \text{ rad}$$

1

3) [5] You are pushing a crate (cube-shaped box, mass 250 kg evenly distributed, edge length 0.8 m) with constant speed over a smooth floor ($\mu_k = 0.2$). Your hands apply the required force at the top edge of the back of the crate. Suddenly the crate hits an obstacle (floor carpet), the front edge of the crate jams at the bottom. Calculate whether the crate will tip over or not (using the front bottom edge as a pivot point). Start with a drawing in which you show the forces that apply (and where they apply) assuming that the crate is about to tip, i.e., the contact with the floor is just at the pivot. Make sure that the magnitudes of the forces are reasonable relative to each other!



1.0

translational equilibrium:

$\hat{i}$: $F_{push} + F_S = 0$ 0.5

$\hat{j}$: $-mg + N = 0$ 0.5

rotation about pivot:

@ point of tipping: $\vec{N}$ applies at P (no torque) 0.5

$\vec{F}_S$: no torque about P

τ_{grav} is out of plane (by RH rule) → CCW rotation

τ_{push} is into plane → CW rotation

.5

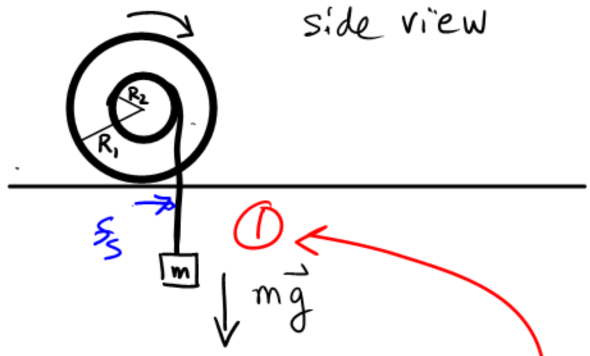
$F_{push} = \mu_k N = \mu_k mg = 0.2 \cdot 250 \cdot 9.8\ N = 490\ N$ 0.5

$mg = 2450\ N$

$|\tau_{grav}| = mg \cdot \frac{L\sqrt{2}}{2} \cdot |\sin\varphi|$ 0.5 ; $|\tau_{push}| = F_{push} \, L\sqrt{2}\, |\sin\varphi|$ 0.5

clearly $|\tau_{push}| << |\tau_{grav}|$ ∴ crate won't tip. .5

4) [5] A large cylinder (radius R_1, small length L, mass M) rolls on a flat surface, since attached to it is another cylinder (radius $R_2 = R_1/2$, same length L, mass $M/4$), and a mass m suspended freely from a rope falls under gravity and is unwinding rope at radius R_2. Formulate Newton's second law for: (i) mass m, (ii) the rotation of the combined two cylinders about their centre of mass, (iii) the linear motion of the centre of mass for the combined cylinders. Keeping in mind this last equation draw the friction force f_s (show where it applies and its correct direction). The inertia for a disk about the CM is given as $I_d = MR^2/2$.



(i) mass m:

$$m a_y = mg - T \qquad 1$$

(ii) $$I_{CM} = \frac{1}{2} M R_1^2 + \frac{1}{2}\frac{M}{4} R_2^2$$

$$= \frac{17}{8} M R_2^2 \qquad 1$$

$$I_{CM}\,\alpha = T R_2 - f_s R_1 \qquad 1$$

f_s is to the right!
f_s-torque opposes the torque from tension T

(iii) $$\left(M + \frac{M}{4} \underbrace{(+m)}_{\text{negligible for } m \ll M}\right) a_{CM} = f_s \qquad 1$$

Note: this can be solved, since:

$$a_y = R_2 \alpha \quad \text{and} \quad a_{CM} = R_1 \alpha = 2 R_2 \alpha = 2 a_y$$

- The linear motion of m has net force mg
- The inertia is complicated, but for $m \ll M$ we find from the 3 equations

$$\underbrace{\boxed{\frac{57}{8} M}}_{\text{effective inertia for vertical motion caused by } mg} a_y = mg$$

Using the rolling contact point as pivot, and ignoring $m \ll M$ terms: $I_P = \frac{57}{8} M R_2^2$; solution follows from 2 eqs

FORMULA SHEET

$v(t_\mathrm{f}) = v(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} a(t)\ dt \quad s(t_\mathrm{f}) = s(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} v(t)\ dt$

$v_\mathrm{f} = v_\mathrm{i} + a\Delta t \quad s_\mathrm{f} = s_\mathrm{i} + v_\mathrm{i}\Delta t + \frac{1}{2}a\Delta t^2 \quad v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2a\Delta s \qquad g = 9.8\ \mathrm{m/s^2}$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\ dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\ dt = at + C \quad F(t) =$ anti-derivative = indefinite integral

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

$\exp' = \exp$; $\sin' = \cos$; $\cos' = -\sin$. $\quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}$; $\quad (fg)' = f'g + fg'$

$m\vec{a} = \vec{F}_\mathrm{net}$; $\quad F_G = \frac{Gm_1m_2}{r^2}$; $g = \frac{GM_E}{R_E^2}$; $R_E = 6370$ km; $G = 6.67 \times 10^{-11}\frac{\mathrm{Nm^2}}{\mathrm{kg^2}}$; $M_E = 6.0 \times 10^{24}$kg

$f_\mathrm{s} \le \mu_\mathrm{s} n$; $\quad f_\mathrm{k} = \mu_\mathrm{k} n$; $\quad \mu_\mathrm{k} < \mu_\mathrm{s}$; $\qquad F_H = -k\Delta x = -k(x - x_0)$.

$\vec{F}_\mathrm{d} \sim -\vec{v}$; linear: $F_\mathrm{d} = dv$; quadr.: $F_\mathrm{d} = 0.5\rho A v^2$; $\quad A$ = cross s'n area; ρ = density of medium

Sphere: $V = \frac{4}{3}\pi R^3$; Total Surface: $A_S = 4\pi R^2$; Cross Section=?

uniform circular m.: $\vec{r}(t) = R(\cos\omega t\ \hat{\mathbf{i}} + \sin\omega t\ \hat{\mathbf{j}})$; $\vec{v}(t) = \frac{d\vec{r}}{dt} = ...$; $\vec{a}(t) = \frac{d\vec{v}}{dt} = ...$; $\omega = \frac{2\pi}{T}$.

$a_\mathrm{cp} = \frac{v^2}{r} \quad v = \omega r$.

$W = F\Delta x = F(\Delta r)\cos\theta$. W = area under $F_x(x)$. $\quad PE_\mathrm{H} = \frac{k}{2}(\Delta x)^2$; $PE_g = mg\Delta y$.

$\Delta\vec{p} = \vec{J} = \int \vec{F}(t)dt$; $\Delta p_x = J_x$ = area under $F_x(t) = F_x^\mathrm{avg}\Delta t$; $\quad \vec{p} = m\vec{v}$; $K = \frac{m}{2}v^2$

$\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$; $K_1^\mathrm{in} + K_2^\mathrm{in} = K_1^\mathrm{fin} + K_2^\mathrm{fin}$ for elastic collisions. $\quad \vec{a}_\mathrm{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$

$\vec{\tau} = \vec{r} \times \vec{F}$; $\quad \tau_z = rF\sin(\alpha)$ for $\vec{r}$, $\vec{F}$ in xy plane. $\quad I = \sum_i m_i r_i^2$; $\quad I\alpha_z = \tau_z$; ($\hat{k}$ = rot. axis)

$K_\mathrm{rot} = \frac{I}{2}\omega^2$; $L_z = I\omega_z$; $\frac{d}{dt}L_z = \tau_z$; $\vec{L} = \vec{r} \times \vec{p}$; $\frac{d}{dt}\vec{L} = \vec{\tau}$