LAST NAME: STUDENT NR:

PHYS 1010 6.0: CLASS TEST 3 on Nov. 30, 2012

Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1) [5] Amir loves his skateboard. He checked his weight before going for a ride, and found his mass to be 53 kg. His skateboard has a mass of 2.5 kg. He is going on a flat park trail at a speed of 25 km/h, when a small 5.5-kg dog comes running from behind at 35 km/h and jumps to join him on his board. How fast are they going together? Calculate the total mechanical energy (in SI units) before and after the jump and comment whether it is conserved in the process.

total linear momentum is conserved during this (inelastic) collision: $m_1 = m_A + m_{SB} = 55.5$ kg

$m_2 = m_{Dog} = 5.5$ kg

$m_1 \cdot 25 + m_2 \cdot 35 = (m_1 + m_2) V_{fin}$ (1) (V_{fin} in km/h)

$V_{fin} = 25.9$ km/h (1) (= 26 km/h to 2 sig. dig)

or = 7.19 m/s → 7.2 m/s

$KE_{before} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$KE_{after} = \frac{1}{2}(m_1 + m_2) V_{fin}^2$ (1)

$KE_{before} = \frac{1}{2}\left[55.5 \left(\frac{25}{3.6}\right)^2 + 5.5 \left(\frac{35}{3.6}\right)^2\right]$

(0.5) $= 1598$ J $= 1600$ J (3 digits)

$KE_{after} = \frac{1}{2}\left[61.0 \left(\frac{25.9}{3.6}\right)^2\right]$

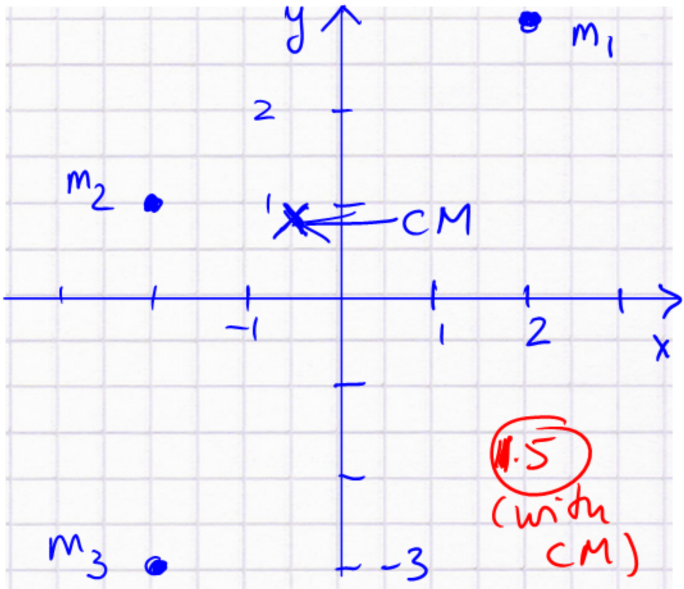
(0.5) $= 1579$ J $= 1580$ J (3 digits)

(1) $KE_{after} < KE_{before}$ → small inelasticity

(to 2 sig. digits the energy is the same, i.e., inelasticity is hard to measure)

2) [5] You are given three masses and their locations in a plane: $m_1 = 3.4$ kg located at $(x, y) = (2, 3)$ m, $m_2 = 4.8$ kg located at $(x, y) = (-2, 1)$ m, and $m_3 = 2.2$ kg located at $(x, y) = (-1, -3)$ m. Mark their positions on a graph. Calculate the location of the centre of mass, and show where it is on the graph. Suppose the set-up of masses is transferred onto a 'massless' turntable with the given coordinate origin at the centre, i.e., as pivot point. What is the rotational inertia? Would it be the same as the inertia about the centre of mass?

	x_i	y_i	r_i^2	m_i
m_1	2	3	13	3.4
m_2	-2	1	5	4.8
m_3	-1	-3	10	2.2



$M = 10.4$ kg

$M X_{CM} = \sum m_i x_i = -5$ m kg

$M Y_{CM} = \sum m_i y_i = 8.4$ m kg

$\vec{R}_{CM} = (-0.48, 0.81) = -0.48\,\hat{i} + 0.81\,\hat{j}$ m

1

$$I_0 = \sum m_i r_i^2 = 3.4 \cdot 13 + 4.8 \cdot 5 + 2.2 \cdot 10 = 90.2 \text{ kg m}^2$$

1.5

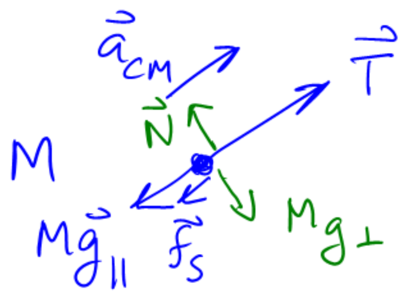
I_{CM} would be different (less) since 1

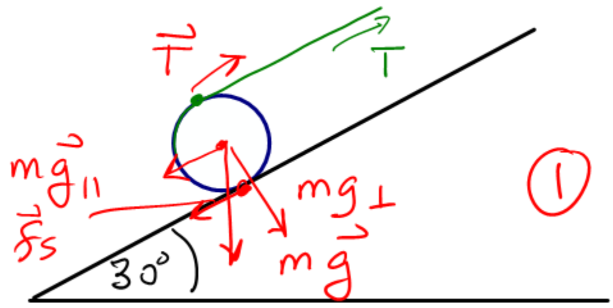
$$I_0 = I_{CM} + M \underbrace{r_{CM}^2}_{a^2}$$

$a^2 = \left(\text{distance of rot. axis from CM}\right)^2$

3) [5] A cylindrical spool of mass $m =$ ~~4.0~~ 1.0 kg has a diameter of 20 cm. It has a strong thread wound around it many times (the turns side-by-side, so the radius doesn't change), such that the end emerges at the top, as shown. A tension force of 5.0 N is applied to drag the spool up a 30-degree incline. Use the plot to indicate all forces that apply. Then draw a free-body diagram for the motion of the CM. Formulate the equations of motion for the centre of mass, and for the rotation of the spool. Note that the inertia about the CM is given as $0.5MR^2$, and about a point on the rim it is $1.5MR^2$. What is the acceleration up the incline?

1





1

$|f_s| \leq \mu_s M g_{\perp}$

$$M a_{CM} = T - Mg_{\parallel} - f_s \quad \text{up incline}$$

$$= T - Mg\sin 30^\circ - f_s$$

1

$$= T - Mg\tfrac{1}{2} - f_s$$

$$I_p \alpha = 2TR - Mg_{\parallel}R \quad \text{CW direction}$$

1

$$1.5MR^2\alpha = (2T - Mg/2)R$$

OR:

$$I_{CM}\alpha = f_s R + TR$$

$$0.5MR^2\alpha = f_s R + TR$$

use $\alpha = \frac{a_{CM}}{R}$

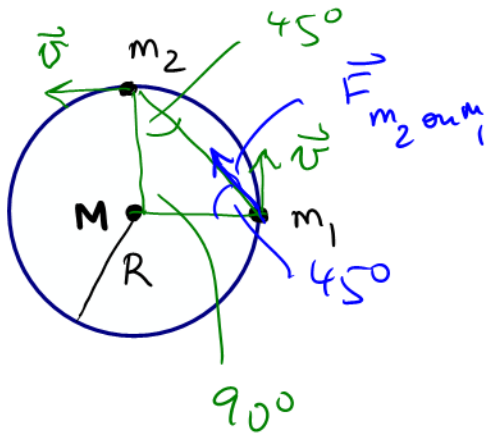
+ divide by R

$$\therefore\ 1.5 M a_{CM} = 2T - \frac{Mg}{2}$$

$$1.5\, a_{CM} = 10.0 - \frac{1.0\ 9.8}{2}$$

$$a_{CM} = 3.4\ \text{m/s}^2 \quad 1$$

4) [5] The drawing shows a planet m_1 under the gravitational influence of a star (M at the origin) and another (hypothetical) planet $m_2 = m_1$ on the same circular orbit, but advanced by 90 degrees. Calculate the angular momentum vector for planet m_1 with respect to the star. Calculate the gravitational torque vector about the origin planet m_2 exerts on planet m_1. Is the angular momentum of m_1 with respect to the star conserved? Explain.



1 $\vec{L}_0 = \vec{R} \times \vec{p} = R m_1 v \hat{k}$

(magnitude $m_1 R v$; out of plane)

1 $F_{m_2 \text{ on } m_1} = \dfrac{G m_1 m_2}{2R^2} = \dfrac{G m_1^2}{2R^2}$

$\vec{\tau}_{\substack{m_2 \text{ on } m_1 \\ \text{about } 0}} = \vec{R}_1 \times \vec{F}_{m_2 \text{ on } m_1}$

is out of the plane by RH rule 1

$\tau_{\substack{m_2 \text{ on } m_1 \\ \text{about } 0}} = \dfrac{R\, G m_1 m_2}{2R^2} \sin 45° = \dfrac{G m_1^2}{2\sqrt{2}\, R}$ 1

Angular momentum of m_1 about M (the origin 0) is not conserved, since m_1 is not torque-free

$\dfrac{dL}{dt} = \tau \neq 0$ 1

(The orientation of $\vec{L}_{m_1}$ remains unchanged,
bonus +1 though, since torque and $\vec{L}$ are aligned)

FORMULA SHEET

$v(t_\mathrm{f}) = v(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} a(t)\ dt \quad s(t_\mathrm{f}) = s(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} v(t)\ dt$

$v_\mathrm{f} = v_\mathrm{i} + a\Delta t \quad s_\mathrm{f} = s_\mathrm{i} + v_\mathrm{i}\Delta t + \frac{1}{2}a\Delta t^2 \quad v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2a\Delta s \qquad g = 9.8\ \mathrm{m/s^2}$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\ dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\ dt = at + C \quad F(t) =$ anti-derivative = indefinite integral

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

$\exp' = \exp$; $\sin' = \cos$; $\cos' = -\sin$. $\quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}$; $\quad (fg)' = f'g + fg'$

$m\vec{a} = \vec{F}_\mathrm{net}$; $\quad F_G = \frac{Gm_1m_2}{r^2}$; $g = \frac{GM_E}{R_E^2}$; $R_E = 6370$ km; $G = 6.67 \times 10^{-11}\frac{\mathrm{Nm^2}}{\mathrm{kg^2}}$; $M_E = 6.0 \times 10^{24}$kg

$f_\mathrm{s} \leq \mu_\mathrm{s} n$; $\quad f_\mathrm{k} = \mu_\mathrm{k} n$; $\quad \mu_\mathrm{k} < \mu_\mathrm{s}$; $\qquad F_H = -k\Delta x = -k(x - x_0)$.

$\vec{F}_\mathrm{d} \sim -\vec{v}$; linear: $F_\mathrm{d} = dv$; quadr.: $F_\mathrm{d} = 0.5\rho A v^2$; $\quad A$ = cross s'n area; ρ = density of medium

Sphere: $V = \frac{4}{3}\pi R^3$; Total Surface: $A_S = 4\pi R^2$; Cross Section=?

uniform circular m.: $\vec{r}(t) = R(\cos\omega t\ \hat{\mathbf{i}} + \sin\omega t\ \hat{\mathbf{j}})$; $\vec{v}(t) = \frac{d\vec{r}}{dt} = ...$; $\vec{a}(t) = \frac{d\vec{v}}{dt} = ...$; $\omega = \frac{2\pi}{T}$.

$a_\mathrm{cp} = \frac{v^2}{r} \quad v = \omega r$.

$W = F\Delta x = F(\Delta r)\cos\theta$. W = area under $F_x(x)$. $\quad PE_\mathrm{H} = \frac{k}{2}(\Delta x)^2$; $PE_g = mg\Delta y$.

$\Delta\vec{p} = \vec{J} = \int \vec{F}(t)dt$; $\Delta p_x = J_x$ = area under $F_x(t) = F_x^\mathrm{avg}\Delta t$; $\quad \vec{p} = m\vec{v}$; $K = \frac{m}{2}v^2$

$\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$; $K_1^\mathrm{in} + K_2^\mathrm{in} = K_1^\mathrm{fin} + K_2^\mathrm{fin}$ for elastic collisions. $\quad \vec{a}_\mathrm{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$

$\vec{\tau} = \vec{r} \times \vec{F}$; $\quad \tau_z = rF\sin(\alpha)$ for $\vec{r}$, $\vec{F}$ in xy plane. $\quad I = \sum_i m_i r_i^2$; $\quad I\alpha_z = \tau_z$; ($\hat{k}$ = rot. axis)

$K_\mathrm{rot} = \frac{I}{2}\omega^2$; $L_z = I\omega_z$; $\frac{d}{dt}L_z = \tau_z$; $\vec{L} = \vec{r} \times \vec{p}$; $\frac{d}{dt}\vec{L} = \vec{\tau}$