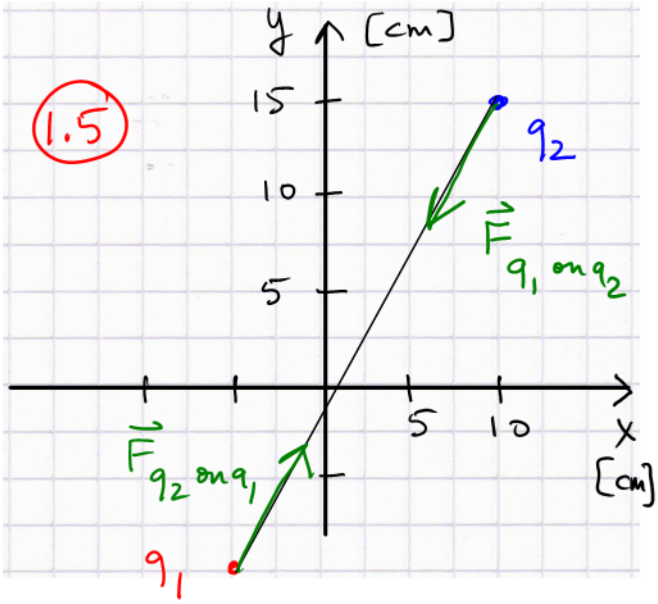
LAST NAME: STUDENT NR:

PHYS 1010 6.0: CLASS TEST 4

Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1) [5] A point charge $q_1 = 1.5\ \mu$C is located at $P_1(x,y) = (-5,-10)$ cm, and a second point charge $q_2 = -30$ nC is at $P_2(x,y) = (10,15)$ cm. Provide a drawing and indicate the forces exerted by q_1 on q_2, and by q_2 on q_1 using vector arrows. Label the forces, and calculate them. You can give them either in (x,y) representation or using magnitudes and direction angles.

solution A (magnitude + direction)



$$F_{q_1 \text{ on } q_2} = F_{q_2 \text{ on } q_1} = \frac{k|q_1 q_2|}{d^2} = F$$

$$d^2 = (x_2-x_1)^2 + (y_2-y_1)^2$$

$$= 15^2 + 25^2 = 850\ \text{cm}^2$$

$$= 0.085\ \text{m}^2$$

$$F = \frac{9.0\times10^9 \cdot 1.5\times10^{-6} \cdot 30\times10^{-9}}{0.085}$$

$$= 4.76\times10^{-3}\ \text{N} = 4.8\ \text{mN}$$

direction angle (+ve x-axis) for $\vec{F}_{q_2 \text{ on } q_1}$: $\tan\vartheta = \frac{y_2-y_1}{x_2-x_1} = \frac{25}{15}$

$\vartheta_1 = 59°$ (or 1.03 rad) $\quad \vartheta_2 = \vartheta_1 + 180° = 239°$ = angle for $\vec{F}_{q_1 \text{ on } q_2}$ (or 4.17 rad)

$\therefore\ \vec{F}_{q_2 \text{ on } q_1} = (4.8\ \text{mN}, 59°)$; $\vec{F}_{q_1 \text{ on } q_2} = (4.8\ \text{mN}, 239°)$

part marks if answer wrong → 2.5 $\quad$ 1.0

Solution B (Cartesian coord answer): (or $\vec{F}_{q_1 \text{ on } q_2} = -\vec{F}_{q_2 \text{ on } q_1}$)

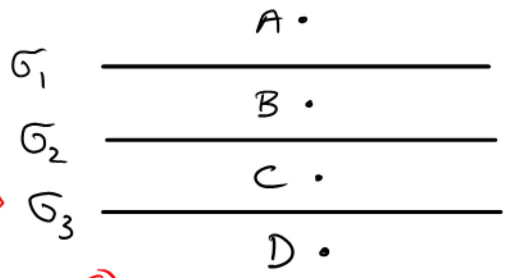
$$\vec{F}_{q_2 \text{ on } q_1} = (F\cos\vartheta_1)\,\hat{\imath} + (F\sin\vartheta)\,\hat{\jmath} = 4.8\times10^{-3}(0.515\,\hat{\imath} + 0.857\,\hat{\jmath})$$

$$= (2.5\,\hat{\imath} + 4.1\,\hat{\jmath})\times10^{-3}\ \text{N} \qquad 2.5$$

$$\vec{F}_{q_1 \text{ on } q_2} = (F\cos\vartheta_2)\,\hat{\imath} + (F\sin\vartheta_2)\,\hat{\jmath} = 4.8\times10^{-3}(-0.515\,\hat{\imath} - 0.857\,\hat{\jmath})$$

$$= -(2.5\,\hat{\imath} + 4.1\,\hat{\jmath})\times10^{-3}\ \text{N} \qquad 1.0$$

2) [5] Consider three thin large-area charged planes with surface densities $\sigma_1 = +5\ \mu\text{C/cm}^2$, $\sigma_2 = -10\ \mu\text{C/cm}^2$, and $\sigma_3 = +5\ \mu\text{C/cm}^2$ respectively. They are separated by 2.0cm from each other, as shown in a sideways cross-section. Find the electric field at points A, B, C, D.



We add the constant fields from 1, 2, 3. with proper orientations

At (A) : $\vec{E}_1 = \uparrow \quad \vec{E}_2 = \downarrow \quad E_3 = \uparrow$ (1)

$$\vec{E}^{A}_{net} = \hat{j}\left(\frac{1}{2\varepsilon_0}\right)\left(+5 + (-10) + 5\right) = 0 \quad (1)$$

Likewise at (D) the net field vanishes : $\vec{E}_1 = \downarrow, \vec{E}_2 = \uparrow, \vec{E}_3 = \uparrow$
(=0) (1)

At (B) : $\vec{E}_1 = \downarrow, \quad \vec{E}_2 = \downarrow, \quad \vec{E}_3 = \uparrow$

$$\vec{E}^{B}_{net} = \hat{j}\left(\frac{1}{2\varepsilon_0}\right)\left(-5 + (-10) + 5\right) = \frac{-10\,\mu C/cm^2}{2\varepsilon_0}\hat{j}$$

$$= -5 \times 10^{-6} \times 10^{4} \frac{C/m^2}{8.85 \times 10^{-12}\ C^2/Nm^2}\hat{j}$$

$$= -5.6 \times 10^{9} \frac{N}{C}\hat{j} \quad (1)$$

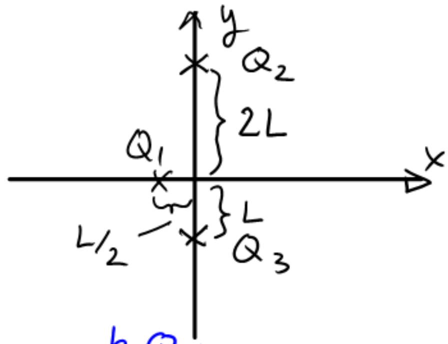
By symmetry at C : $E^{C}_{net} = +5.6 \times 10^{9} \frac{N}{C}\hat{j}$ (1)

0.5 point was given for principle of vector addition to obtain the net field)

0.5 for listing the correct formula for the field strength from a charged plate $|E| = \frac{|\sigma|}{2\varepsilon_0}$

(statements that field vanishes at A, D = 1 point)

3) [5] Three point charges with $Q_1 = 2.0\ \mu$C, $Q_2 = 3.0\ \mu$C, $Q_3 = -2.5\ \mu$C respectively are placed as shown. Calculate the *total* electric potential energy of the system by adding the three pairwise interactions.



Electric potential from a point charge Q: $V(r) = \dfrac{kQ}{r}$

Potential energy for a 2nd charge located a distance r away: $U = \dfrac{kQq}{r}$ 1

Three pairs: $U_{12} = \dfrac{kQ_1Q_2}{\sqrt{(2L)^2+(L/2)^2}}$, $U_{23} = \dfrac{kQ_2Q_3}{(3L)}$,

$U_{13} = \dfrac{kQ_1Q_3}{\sqrt{L^2+(L/2)^2}}$

$$U_{12} = \frac{k\ 6.0\times10^{-12}}{\sqrt{4L^2+L^2/4}} = \frac{9.0\times10^{9}\cdot 6.0\times10^{-12}}{\sqrt{4.25}\,L}\ J = \frac{2.62\times10^{-2}}{L}\ J$$ 1

$$U_{23} = \frac{k\ (-7.5)\times10^{-12}}{\sqrt{9L^2}} = \frac{-9.0\times10^{9}\cdot 7.5\times10^{-12}}{3L} = \frac{-2.25\times10^{-2}}{L}\ J$$ 1

$$U_{13} = \frac{k\ (-5.0)\times10^{-12}}{1.25\ L^2} = -\frac{9.0\times10^{9}\cdot 5.0\times10^{-12}}{\sqrt{1.25}\,L} = -\frac{4.02\times10^{-2}}{L}\ J$$ 1

$$U_{tot} = U_{12} + U_{23} + U_{13} = \frac{-36.5}{L}\ mJ$$ ✓

(where L is entered in SI = meters) $= \dfrac{-3.65\times10^{-2}\ J}{L}$ 1

4) [5] Using a battery a charge of $\pm Q$ is placed on the plates of a capacitor giving the initial voltage ΔV_1 across the plates. Then the capacitor is disconnected from the battery, and the plate separation is increased by a factor of 4. In this final configuration the capacitance is found to be 2.0 nF, and the charge on the plates $\pm 35\ \mu$C. What was the battery voltage in volts?

1) $\Delta V_1 = \Delta V_B$ 0.5

2) $C_1 \Delta V_B = Q$ 0.5

3) Q remains constant 0.5

4) $C_1 = \frac{\varepsilon_0 A}{d_1} \longrightarrow C_2 = \frac{\varepsilon_0 A}{d_2} = \frac{\varepsilon_0 A}{4 d_1} = \frac{1}{4} C_1$ 1.0

5) $C_2 = 2.0 \times 10^{-9}$ F ; $C_1 = 4\, C_2 = 8.0$ nF 1.0

6) $\Delta V_B = Q/C_1 = \frac{35 \times 10^{-6}\ \text{C}}{8.0 \times 10^{-9}\ \text{F}} = 4.4 \times 10^{3}$ V

$= 4.4$ kV 1.5

FORMULA SHEET

$v(t_\mathrm{f}) = v(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} a(t)\ dt \quad s(t_\mathrm{f}) = s(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} v(t)\ dt$

$v_\mathrm{f} = v_\mathrm{i} + a\Delta t \quad s_\mathrm{f} = s_\mathrm{i} + v_\mathrm{i}\Delta t + \frac{1}{2}a\Delta t^2 \quad v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2a\Delta s \qquad g = 9.8\ \mathrm{m/s^2}$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\ dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\ dt = at + C \quad F(t) =$ anti-derivative = indefinite integral

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

uniform circular m. $\vec{r}(t) = R(\cos\omega t\ \hat{\mathbf{i}} + \sin\omega t\ \hat{\mathbf{j}})$; $\vec{v}(t) = \frac{d\vec{r}}{dt} = ...$; $\vec{a}(t) = \frac{d\vec{v}}{dt} = ...$

$\exp' = \exp$; $\sin' = \cos$; $\cos' = -\sin$. $\quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}$; $\quad (fg)' = f'g + fg'$

$m\vec{a} = \vec{F}_\mathrm{net}$; $\quad F_G = \frac{Gm_1m_2}{r^2}$; $g = \frac{GM_E}{R_E^2}$; $R_E = 6370$ km; $G = 6.67 \times 10^{-11} \frac{\mathrm{Nm^2}}{\mathrm{kg^2}}$; $M_E = 6.0 \times 10^{24}$kg

$f_\mathrm{s} \leq \mu_\mathrm{s} n$; $\quad f_\mathrm{k} = \mu_\mathrm{k} n$; $\quad f_\mathrm{r} = \mu_\mathrm{r} n$; $\quad \mu_\mathrm{r} << \mu_\mathrm{k} < \mu_\mathrm{s}$. $\qquad F_H = -k\Delta x = -k(x - x_0)$.

$\vec{F}_\mathrm{d} \sim -\vec{v}$; linear: $F_\mathrm{d} = dv$; quadratic: $F_\mathrm{d} = 0.5\rho A v^2$; $\quad A =$ cross sectional area

$W = F\Delta x = F(\Delta r)\cos\theta$. $W =$ area under $F_x(x)$. $\quad PE_\mathrm{H} = \frac{k}{2}(\Delta x)^2$; $PE_g = mg\Delta y$.

$\Delta\vec{p} = \vec{J} = \int \vec{F}(t)dt$; $\Delta p_x = J_x =$ area under $F_x(t) = F_x^\mathrm{avg}\Delta t$; $\quad \vec{p} = m\vec{v}$; $K = \frac{m}{2}v^2$

$\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$; $K_1^\mathrm{in} + K_2^\mathrm{in} = K_1^\mathrm{fin} + K_2^\mathrm{fin}$ for elastic collisions. $\quad \vec{a}_\mathrm{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$

$\vec{\tau} = \vec{r} \times \vec{F}$; $\quad \tau_z = rF\sin(\alpha)$ for $\vec{r}$, $\vec{F}$ in xy plane. $\quad I = \sum_i m_i r_i^2$; $\quad I\alpha_z = \tau_z$; ($\hat{k} =$ rot. axis)

$K_\mathrm{rot} = \frac{I}{2}\omega^2$; $L_z = I\omega_z$; $\frac{d}{dt}L_z = \tau_z$; $\vec{L} = \vec{r} \times \vec{p}$; $\frac{d}{dt}\vec{L} = \vec{\tau}$

$x(t) = A\cos(\omega t + \phi)$; $\quad \omega = \frac{2\pi}{T} = 2\pi f$; $\quad v_x(t) = ...$; $\quad v_\mathrm{max} = ...$

$m_\mathrm{e} = 9.11 \times 10^{-31}$kg $\quad m_\mathrm{p} = 1.67 \times 10^{-27}$kg $\quad e = 1.60 \times 10^{-19}$C $\quad k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \frac{\mathrm{Nm^2}}{\mathrm{C^2}}$

$\vec{F}_\mathrm{C} = \frac{kq_1q_2}{r^2}\hat{\mathbf{r}} \quad \vec{F}_\mathrm{E} = q\vec{E} \quad E_\mathrm{line} = \frac{2k|\lambda|}{r} = \frac{2K|Q|}{Lr} \quad E_\mathrm{plane} = \frac{|\sigma|}{2\epsilon_0} = \frac{|Q|}{2A\epsilon_0} \quad \vec{E}_\mathrm{cap} = \left(\frac{Q}{\epsilon_0 A}, \mathrm{pos} \to \mathrm{neg}\right)$

$\frac{mv^2}{2} + U_\mathrm{el}(s) = \frac{mv_0^2}{2} + U_\mathrm{el}(s_0)$, $(U \equiv PE_\mathrm{el}) \quad U_\mathrm{el} = qEx$ for $\vec{E} = -E\,\hat{i} \quad V_\mathrm{el} = U_\mathrm{el}/q \quad E_x = -\frac{dV_\mathrm{el}}{dx}$

point charge: $V_\mathrm{el} = \frac{kQ}{r} \quad Q = C\Delta V_C \quad$ farad = F = $\frac{\mathrm{C}}{\mathrm{V}} \quad C = \frac{\epsilon_0 A}{d} \quad \epsilon_0 = 8.85 \times 10^{-12} \frac{\mathrm{C^2}}{\mathrm{Nm^2}}$

parallel C_1, C_2: $C_\mathrm{eq} = C_1 + C_2 \quad$ series C_1, C_2: $C_\mathrm{eq}^{-1} = C_1^{-1} + C_2^{-1}$