LAST NAME: STUDENT NR:

PHYS 1010 6.0: CLASS TEST 4

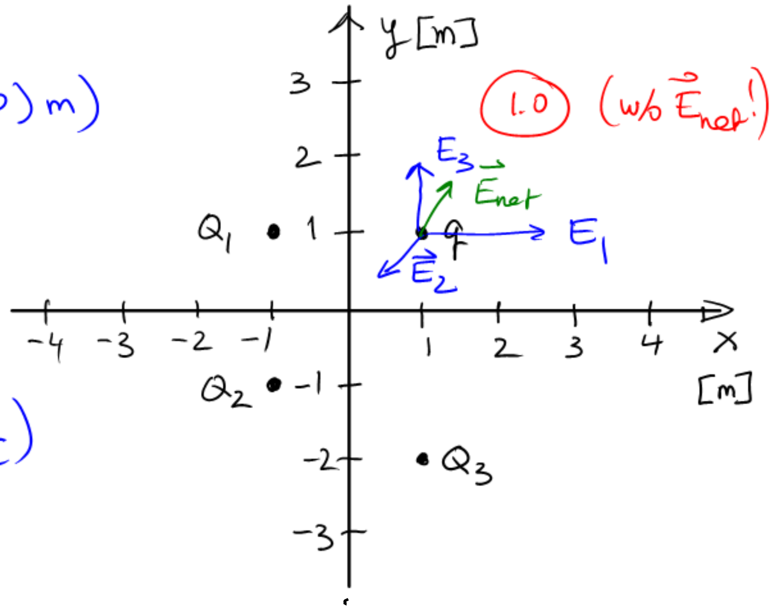
Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1) [5] Three charges ($Q_1 = +2,\ Q_2 = -3,\ Q_3 = +4$ all in μC) are placed in a plane as shown in the diagram. Calculate the force on charge $q = +1$ nC in component form.

~two significant digits

At the location of q ((1.0, 1.0) m)

1.0 (w/o $\vec{E}_{net}$!)



Q_1 contributes field

$$\vec{E}_1 = \frac{kQ_1}{2^2}\hat{\imath},$$

$$Q_2:\quad \vec{E}_2 = -\frac{k|Q_2|}{(2^2+2^2)}\left(\frac{\hat{\imath}}{\sqrt{2}} + \frac{\hat{\jmath}}{\sqrt{2}}\right)$$

$$Q_3:\quad \vec{E}_3 = \frac{kQ_3}{3^2}\hat{\jmath}$$

$$\therefore\ \vec{E}_{net} = \left[k\left(\frac{2}{4} - \frac{3}{8\cdot 1.414}\right)\hat{\imath} + k\left(\frac{-3}{8\cdot 1.414} + \frac{4}{9}\right)\hat{\jmath}\right]\underset{\uparrow \text{ from } \mu C}{\times 10^{-6}}$$

$$= 9.0\times10^{9}\cdot 10^{-6}\,(0.235\,\hat{\imath} + 0.179\,\hat{\jmath})$$

$$= (2.1\,\hat{\imath} + 1.6\,\hat{\jmath})\cdot 10^{3}\ \frac{N}{C}$$

$$\vec{F}_{on\ q} = q\,\vec{E}_{net}\underbrace{(1,1)}_{\text{location}}$$

$$= 1.0\times10^{-9}\cdot 10^{3}\,(2.1\,\hat{\imath} + 1.6\,\hat{\jmath})\ N$$

$$= (2.1\,\hat{\imath} + 1.6\,\hat{\jmath})\ \mu N$$

4.0

with part marks depending on method of calculation

[could be done by pair-wise $(Q_i - q)$ Coulomb forces]

2) [5] Consider the same set-up as in question 1. What is the potential energy of the charge q in the field of charges Q_1, Q_2, Q_3?

The potential from each charge Q_i: $V_i = \frac{kQ_i}{d_i}$

(d_i = distance from q-location)

Potential energy of q: $U = q \sum_{i=1}^{3} V_i$

distances: $d_1 = 2.0\,\text{m}$

$d_2 = \sqrt{2.0^2 + 2.0^2} = \sqrt{8.0}\,\text{m} = 2.83\,\text{m}$

$d_3 = 3.0\,\text{m}$

$$\underset{V(x,y)}{\overset{}{V(1.0, 1.0)}} = k\left(\frac{2}{2.0} + \frac{-3}{2.83} + \frac{4}{3.0}\right) \times 10^{-6}$$

$$= 9.0 \times 10^9 \times 10^{-6} (1.0 - 1.06 + 1.33)$$

$$= 11.4 \times 10^3 \text{ V(olts)} = 1.1 \times 10^4\,\text{V}$$

$$U_q = q\,V(1.0, 1.0) = 1.0 \times 10^{-9} \cdot 1.1 \times 10^4 \underbrace{\text{VC}}_{\text{J = Nm}}$$

$$= 1.1 \times 10^{-5}\,\text{J}$$

5

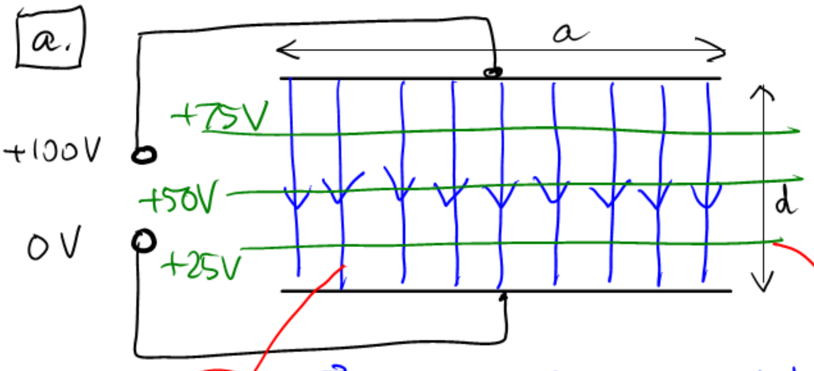
with part-marks for computational steps

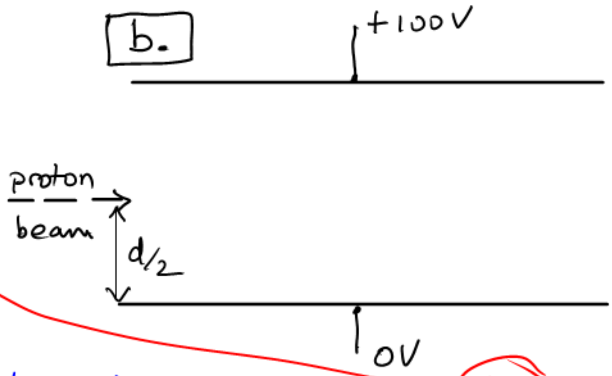
since PE_q can be obtained also by summing pair-wise ($q \sim Q_i$) PE expressions.

(less economical, as in Q1 would be avoidance of $\vec{E}_{net}$)

3) [5] A parallel-plate set-up is connected to a DC voltage source of 100 V, as shown in Fig a. The square plates (base length $a = 20$ cm) are separated by $d = 2.0$ cm. a. Draw field lines inside the capacitor, and also show (and label) some equipotential lines (using Fig. a).

b. a proton beam with speed $v = 1.5 \times 10^5$ m/s is sent through the plates as show in Fig b. Will the protons make it through the plates? Calculate.





1.0 $\vec{E}$-field from + plate to minus (equidistant lines)

= lines = equipotentials ⊥ to $\vec{E}$ 1.0

proton has charge $+e$, $e = 1.60 \times 10^{-19}$ C

$$\vec{E} = \frac{\Delta V}{d}(-\hat{j}) \quad \text{and} \quad \vec{F}_{on\,p} = +e\vec{E} = -\frac{e\Delta V}{d}\hat{j}$$

ignore gravity, since $\vec{E}$ dominates. (*)

2nd law: $m_p a_y = -\frac{e\Delta V}{d}$ $\therefore$ $a_y = -\frac{e}{m_p}\frac{\Delta V}{d}$ 0.5

$$a_y = -\frac{1.60\times10^{-19}\cdot 100}{1.67\times10^{-27}\cdot 2.0\times10^{-2}} = 0.479 \cdot 10^{-19+2}\cdot 10^{27+2}$$

$$= 4.79\cdot 10^{11}\ \frac{m}{s^2}$$ 0.5 $\leftarrow g = 9.8\ \frac{m}{s^2}$ irrelevant! (*)

$x = a = 0.2$ m $\qquad x = v_0 t_f$ $\therefore$ $t_f = \frac{x}{v_0} = \frac{0.2}{1.5\times10^5}$ s

time of flight through plates: $t_f = 0.133\times10^{-5}$ s $= 1.33\ \mu s$ 0.5

$\Delta y_f = \frac{1}{2} a_y t_f^2 = 0.5 \cdot 4.79\times10^{11}\cdot 1.33^2 \cdot 10^{-12}$ m 0.5

$= 4.24\times10^{-1}$ m $= 0.42$ m $\gg \frac{d}{2} = 0.01$ m 0.5

$\therefore$ The protons hit the botom plate! 0.5

4) [5] A cylindrical piece of copper wire has dimensions: length = 10 cm, diameter = 1 mm. Calculate the resistance. Aluminum has a resistivity larger than copper by a ratio of 2.7/1.7. Suppose you want to replace the copper resistor by one of the same length but made of aluminum. To what value should you change the diameter? (This determines the wire gauge.)

$$R = \rho \frac{L}{A}$$

$$A = \pi \frac{d^2}{4} = \frac{3.14}{4} \cdot 10^{-6} \ m^2$$

$$L = 0.1 \, m, \quad \rho = 1.7 \times 10^{-8} \, \Omega m$$

$$R = 1.7 \cdot 10^{\cancel{-8}\,-3} \frac{0.1 \cdot 4}{3.14 \ \cancel{10^{-6}}} = 0.216 \times 10^{-2} \, \Omega = \underline{\underline{2.2 \, m\Omega}}$$

$2.2 \times 10^{-3} \Omega$

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(with part-marks)

$$\frac{R}{L} = \rho' \cdot \frac{\cancel{L}}{A'} = \rho \frac{\cancel{L}}{A}$$

$$\therefore \quad \frac{A'}{A} = \frac{\rho'}{\rho} = \frac{2.7}{1.7} = 1.59$$

we need to increase the cross-sectional area to make up for the poorer conductivity (higher resistivity).

But area $A \sim d^2$, so if $\frac{A'}{A} \sim \left(\frac{d'}{d}\right)^2$, then

$$\frac{d'}{d} \sim \sqrt{\frac{A'}{A}} = 1.26$$

$\therefore$ The diameter of the Al wire should be 1.3 mm

3

with part-marks

FORMULA SHEET

$v(t_\mathrm{f}) = v(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} a(t)\, dt \quad s(t_\mathrm{f}) = s(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t_\mathrm{f}} v(t)\, dt$

$v_\mathrm{f} = v_\mathrm{i} + a\Delta t \quad s_\mathrm{f} = s_\mathrm{i} + v_\mathrm{i}\Delta t + \frac{1}{2}a\Delta t^2 \quad v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2a\Delta s \qquad g = 9.8\ \mathrm{m/s}^2$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\, dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\, dt = at + C \quad F(t) =$ anti-derivative = indefinite integral

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

uniform circular m. $\vec{r}(t) = R(\cos\omega t\ \hat{\mathbf{i}} + \sin\omega t\ \hat{\mathbf{j}})$; $\vec{v}(t) = \frac{d\vec{r}}{dt} = ...$; $\vec{a}(t) = \frac{d\vec{v}}{dt} = ...$

$\exp' = \exp$; $\sin' = \cos$; $\cos' = -\sin$. $\quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}$; $\quad (fg)' = f'g + fg'$

$m\vec{a} = \vec{F}_\mathrm{net}$; $\quad F_G = \frac{Gm_1m_2}{r^2}$; $g = \frac{GM_E}{R_E^2}$; $R_E = 6370$ km; $G = 6.67 \times 10^{-11} \frac{\mathrm{Nm}^2}{\mathrm{kg}^2}$; $M_E = 6.0 \times 10^{24}$kg

$f_\mathrm{s} \leq \mu_\mathrm{s} n$; $\quad f_\mathrm{k} = \mu_\mathrm{k} n$; $\quad f_\mathrm{r} = \mu_\mathrm{r} n$; $\quad \mu_\mathrm{r} << \mu_\mathrm{k} < \mu_\mathrm{s}$. $\qquad F_H = -k\Delta x = -k(x - x_0)$.

$\vec{F}_\mathrm{d} \sim -\vec{v}$; linear: $F_\mathrm{d} = dv$; quadratic: $F_\mathrm{d} = 0.5\rho A v^2$; $\quad A =$ cross sectional area

$W = F\Delta x = F(\Delta r)\cos\theta$. $W =$ area under $F_x(x)$. $\quad PE_\mathrm{H} = \frac{k}{2}(\Delta x)^2$; $PE_g = mg\Delta y$.

$\Delta\vec{p} = \vec{J} = \int \vec{F}(t)dt$; $\Delta p_x = J_x =$ area under $F_x(t) = F_x^\mathrm{avg}\Delta t$; $\quad \vec{p} = m\vec{v}$; $K = \frac{m}{2}v^2$

$\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$; $K_1^\mathrm{in} + K_2^\mathrm{in} = K_1^\mathrm{fin} + K_2^\mathrm{fin}$ for elastic collisions. $\quad \vec{a}_\mathrm{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$

$\vec{\tau} = \vec{r} \times \vec{F}$; $\quad \tau_z = rF\sin(\alpha)$ for $\vec{r}$, $\vec{F}$ in xy plane. $\quad I = \sum_i m_i r_i^2$; $\quad I\alpha_z = \tau_z$; ($\hat{k} =$ rot. axis)

$K_\mathrm{rot} = \frac{I}{2}\omega^2$; $L_z = I\omega_z$; $\frac{d}{dt}L_z = \tau_z$; $\vec{L} = \vec{r} \times \vec{p}$; $\frac{d}{dt}\vec{L} = \vec{\tau}$

$x(t) = A\cos(\omega t + \phi)$; $\quad \omega = \frac{2\pi}{T} = 2\pi f$; $\quad v_x(t) = ...$; $\quad v_\mathrm{max} = ...$

$m_\mathrm{e} = 9.11 \times 10^{-31}$kg $\quad m_\mathrm{p} = 1.67 \times 10^{-27}$kg $\quad e = 1.60 \times 10^{-19}$C $\quad k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \frac{\mathrm{Nm}^2}{\mathrm{C}^2}$

$\vec{F}_\mathrm{C} = \frac{kq_1q_2}{r^2}\hat{\mathbf{r}} \quad \vec{F}_\mathrm{E} = q\vec{E} \quad E_\mathrm{line} = \frac{2k|\lambda|}{r} = \frac{2K|Q|}{Lr} \quad E_\mathrm{plane} = \frac{|\sigma|}{2\epsilon_0} = \frac{|Q|}{2A\epsilon_0} \quad \vec{E}_\mathrm{cap} = \left(\frac{Q}{\epsilon_0 A}, \mathrm{pos} \to \mathrm{neg}\right)$

$\frac{mv^2}{2} + U_\mathrm{el}(s) = \frac{mv_0^2}{2} + U_\mathrm{el}(s_0)$, $(U \equiv PE_\mathrm{el})$ $\quad U_\mathrm{el} = qEx$ for $\vec{E} = -E\,\hat{i} \quad V_\mathrm{el} = U_\mathrm{el}/q \quad E_x = -\frac{dV_\mathrm{el}}{dx}$

point charge: $V_\mathrm{el} = \frac{kQ}{r} \quad Q = C\Delta V_C \quad$ farad = F = $\frac{\mathrm{C}}{\mathrm{V}} \quad C = \frac{\epsilon_0 A}{d} \quad \epsilon_0 = 8.85 \times 10^{-12}\frac{\mathrm{C}^2}{\mathrm{Nm}^2}$

parallel C_1, C_2: $C_\mathrm{eq} = C_1 + C_2 \quad$ series C_1, C_2: $C_\mathrm{eq}^{-1} = C_1^{-1} + C_2^{-1}$

resistance vs resisitivity: $R = \rho\frac{L}{A}$ (L =length, A =cross section); copper: $\rho = 1.7 \times 10^{-8}\ \Omega\mathrm{m}$