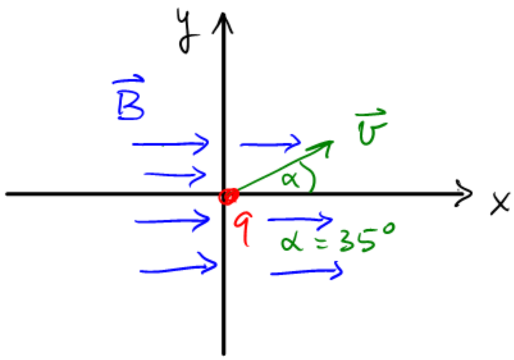
LAST NAME: STUDENT NR:

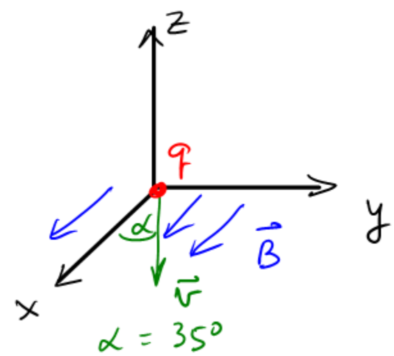
PHYS 1010 6.0: CLASS TEST 5

Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1) [5] A particle of charge $q = 15\ \mu$C is moving at an angle of 35° with respect to the $x-$axis with a speed of 150 m/s. There is a constant magnetic field of magnitude 0.75 T parallel to x. Find the magnitude of the magnetic force on the particle. Start with a figure depicting the situation.



either ↙ 2 or ↘



$$\vec{F}_M = q\,\vec{v} \times \vec{B} \quad \text{or} \quad |F_M| = |q v B \sin\alpha|$$

optional

orientation of $\vec{F}_M$: by RH rule $\vec{v} \times \vec{B}$ is along negative z (into the plane in LH figure downward in RH figure)

This is the direction of $\vec{F}_M$ (since $q > 0$)

$$F_M = q\, v\, B \sin(\measuredangle\, \vec{v}, \vec{B}) \quad \text{where } \measuredangle\, \vec{v}, \vec{B} = 360 - \alpha$$

not a magnitude

$$= -q v B \sin\alpha \quad (\text{“}-\text{”, since } \vec{F}_M \sim -\hat{k})$$

or $|F_M| = q v B \sin\alpha$ 1 (since $q > 0$, $\sin\alpha > 0$)

magnitude of $\vec{F}_M$

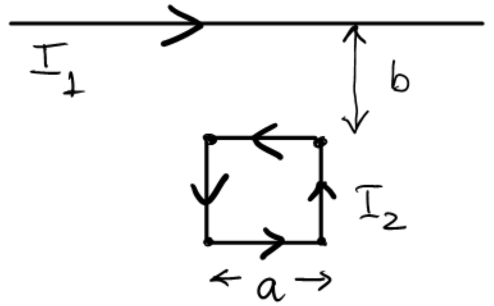
$$= 15 \times 10^{-6} \cdot 150 \cdot 0.75 \cdot \sin(35°)\ \text{N}$$

$$= 968\ \mu\text{N} = 0.97\ \text{mN} \quad 2$$

$$= 0.97 \times 10^{-3}\ \text{N} = 9.7 \times 10^{-4}\ \text{N}$$

anyone of these answers, but no “–”

2) [5] A long, straight wire carries a current $I_1 = 3.0$ A, and is a distance $b = 15$ cm from a square current loop which carries a current of $I_2 = 1.5$ A. The loop and the wire are in the same plane and parallel, the square loop has base length $a = 6$ cm, and the distance b is from wire to wire, as shown. The current orientations are indicated. What are the magnitude and direction of the force exerted by the wire on the current loop? What are the the magnitude and direction of the force exerted by the current loop on the wire?



force between parallel wires:

$$F_{2\,wires} = \frac{\mu_0}{2\pi}\frac{L}{d} I_1 I_2$$

$\vec{B}_{wire}$ at the location of the loop is into the plane (simple RH rule)

$I_2 \downarrow$ $\vec{B}$ $\otimes$ yields $\vec{F}_M \rightarrow$ while $\uparrow I_2$ yields $\vec{F}_M \leftarrow$

equal and opposite. Only the parallel segments contribute to the net force on the loop.

$\overleftarrow{I_2}$ $\otimes \vec{B}_1$ $\downarrow \vec{F}_{M_1}$ $\overrightarrow{I_2}$ $\otimes \vec{B}_2$ $\uparrow \vec{F}_{M_2}$

Since $B_2 < B_1$ a net repulsion occurs.

$$F_{on\,loop} = \frac{\mu_0}{2\pi} I_1 I_2 \left(\frac{a}{b} - \frac{a}{a+b}\right)$$; orientation is downwards ① $(\sim -\hat{j})$, away from wire

①

$$= 2\cdot 10^{-7}\ 3.0\cdot 1.5\left(\frac{6}{15} - \frac{6}{21}\right) \text{ N}$$

$$= 1.03\times 10^{-7} \text{ N} = 0.10\ \mu\text{N} = 1.0\times 10^{-7} \text{ N}$$ ①

By Newton's 3rd law, or by explicit calculation (RH rule) the force on the wire is opposite and of equal magnitude ($\vec{F}_{on\,wire} \sim \hat{j}$ or up) ①

also full marks (2)

$$F_{on\,wire} = 1.0\times 10^{-7} \text{ N}$$ ①

3) [5] A current loop with resistance $R = 350\,\Omega$ and an area $A = 0.6\ \mathrm{m}^2$ is oriented perpendicular to a magnetic field that varies in time as $B(t) = t(1-t)$ [in SI units, and $0 \le t \le 1$]. What is the current induced in the loop at times: $t = 0.3$ s, $t = 0.5$ s, $t = 1.0$ s ?

Magnetic flux $\quad \Phi_M(t) = A\,B(t)$

$$\frac{d\Phi_M}{dt} = A\frac{dB}{dt} = A\left((1-t) + t(-1)\right) = A(1-2t)$$

$$\left.\begin{aligned} \Delta V &= -\frac{d\Phi_M}{dt} \quad \text{(Faraday)} \\ I &= \frac{\Delta V}{R} \quad \text{(Ohm)} \end{aligned}\right\}\ |I| = \frac{1}{R}\left|\frac{d\Phi_M}{dt}\right|$$

$$|I| = \frac{A}{R}\,|(1-2t)| = \frac{0.6}{350}\,|1-2t| \quad 1$$

or: $I = \frac{A}{R}(2t-1)$

@ $t_1 = 0.3$ s $\quad |I_1| = \frac{0.6}{350}|1-0.6|\,A = 6.9\times 10^{-4}\,A$ $\quad 1$

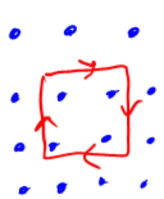
$t_2 = 0.5$ s $\quad I_2 = 0\ A$ $\quad 1$

$t_3 = 1.0$ s $\quad |I_3| = \frac{0.6}{350}\,A = 1.7\ mA = 1.7\times 10^{-3}\,A$ $\quad 1$

I_1 and I_3 have opposite orientation $\quad 1$

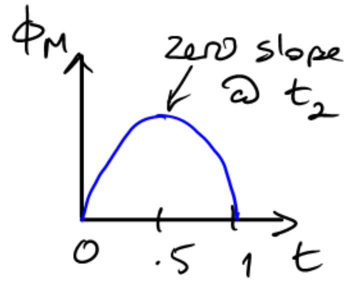
could be indicated by $I_1 = -0.69\,mA$, $I_3 = +1.7\,mA$

optional:



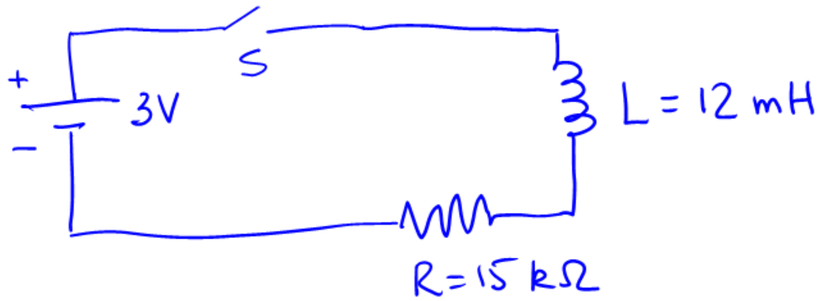
between $t=0$ and $t_2 = 0.5$ s $B(t)$ ramps up, Φ_M increases, current is CW, for $0.5 < t < 1$ it is CCW

optional:



$\therefore I_2 = 0$

4) [5] A circuit is formed using a 3V battery, a simple on-off switch, a resistor $R = 15$ kΩ, and an inductor $L = 12$ mH. They are all connected in series. Begin with a drawing of the circuit diagram. What is the current the instant after the switch is closed? What is the current a very long time after the switch is closed? Draw a figure of the current as a function of time with a correct labeling of the axes (requires the time constant!).



1

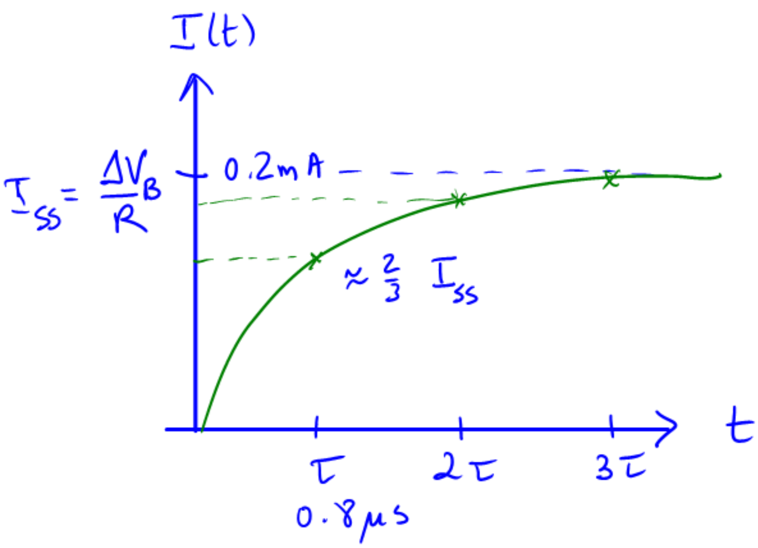
$I(t=0) = 0$ 1

inductor provides counter-EMF

$I(t\to\infty) = \frac{\Delta V_B}{R} = \frac{3}{15}\text{ mA} = 0.2\text{ mA}$ 1

optional: (assumes ideal inductor, or $R_L \ll R$)

time constant $\tau = \frac{L}{R} = \frac{12}{15}\ \mu s = 0.8\ \mu s$



$I(t) = \underbrace{\frac{\Delta V_B}{R}}_{I_{ss}}\left(1 - e^{-t/\tau}\right)$

I_{ss} = steady state current

2 (includes $\tau = 0.8\ \mu s$ and $I_{ss} = 0.2$ mA labels)

FORMULA SHEET

$v(t_f) = v(t_i) + \int_{t_i}^{t_f} a(t)\ dt \quad s(t_f) = s(t_i) + \int_{t_i}^{t_f} v(t)\ dt$

$v_f = v_i + a\Delta t \quad s_f = s_i + v_i\Delta t + \frac{1}{2}a\Delta t^2 \quad v_f^2 = v_i^2 + 2a\Delta s \qquad g = 9.8\ \mathrm{m/s^2}$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\ dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\ dt = at + C \quad F(t) =$ anti-derivative $=$ indefinite integral

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

uniform circular m. $\vec{r}(t) = R(\cos\omega t\ \hat{\mathbf{i}} + \sin\omega t\ \hat{\mathbf{j}})$; $\vec{v}(t) = \frac{d\vec{r}}{dt} = ...$; $\vec{a}(t) = \frac{d\vec{v}}{dt} = ...$

$\exp' = \exp$; $\sin' = \cos$; $\cos' = -\sin$. $\quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}$; $\quad (fg)' = f'g + fg'$

$m\vec{a} = \vec{F}_{\text{net}}$; $\quad F_G = \frac{Gm_1m_2}{r^2}$; $g = \frac{GM_E}{R_E^2}$; $R_E = 6370$ km; $G = 6.67 \times 10^{-11}\frac{\mathrm{Nm^2}}{\mathrm{kg^2}}$; $M_E = 6.0 \times 10^{24}$kg

$f_s \le \mu_s n$; $\quad f_k = \mu_k n$; $\quad f_r = \mu_r n$; $\quad \mu_r << \mu_k < \mu_s$. $\qquad F_H = -k\Delta x = -k(x - x_0)$.

$\vec{F}_d \sim -\vec{v}$; linear: $F_d = dv$; quadratic: $F_d = 0.5\rho A v^2$; $\quad A =$ cross sectional area

$W = F\Delta x = F(\Delta r)\cos\theta$. $W =$ area under $F_x(x)$. $\quad PE_H = \frac{k}{2}(\Delta x)^2$; $PE_g = mg\Delta y$.

$\Delta\vec{p} = \vec{J} = \int \vec{F}(t)dt$; $\Delta p_x = J_x =$ area under $F_x(t) = F_x^{\text{avg}}\Delta t$; $\quad \vec{p} = m\vec{v}$; $K = \frac{m}{2}v^2$

$\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$; $K_1^{\text{in}} + K_2^{\text{in}} = K_1^{\text{fin}} + K_2^{\text{fin}}$ for elastic collisions. $\quad \vec{a}_{\text{CM}} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$

$\vec{\tau} = \vec{r} \times \vec{F}$; $\quad \tau_z = rF\sin(\alpha)$ for $\vec{r}$, $\vec{F}$ in xy plane. $\quad I = \sum_i m_i r_i^2$; $\quad I\alpha_z = \tau_z$; ($\hat{k} =$ rot. axis)

$K_{\text{rot}} = \frac{I}{2}\omega^2$; $L_z = I\omega_z$; $\frac{d}{dt}L_z = \tau_z$; $\vec{L} = \vec{r} \times \vec{p}$; $\frac{d}{dt}\vec{L} = \vec{\tau}$

$x(t) = A\cos(\omega t + \phi)$; $\quad \omega = \frac{2\pi}{T} = 2\pi f$; $\quad v_x(t) = ...$; $\quad v_{\max} = ...$

$m_e = 9.11 \times 10^{-31}$kg $\quad m_p = 1.67 \times 10^{-27}$kg $\quad e = 1.60 \times 10^{-19}$C $\quad K = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9\frac{\mathrm{Nm^2}}{\mathrm{C^2}}$

$\vec{F}_C = \frac{Kq_1q_2}{r^2}\hat{\mathbf{r}} \quad \vec{F}_E = q\vec{E} \quad E_{\text{line}} = \frac{2K|\lambda|}{r} = \frac{2K|Q|}{Lr} \quad E_{\text{plane}} = \frac{|\eta|}{2\epsilon_0} = \frac{|Q|}{2A\epsilon_0} \quad \vec{E}_{\text{cap}} = \left(\frac{Q}{\epsilon_0 A}, \text{pos} \to \text{neg}\right)$

$\frac{mv^2}{2} + U_{\text{el}}(s) = \frac{mv_0^2}{2} + U_{\text{el}}(s_0)$, $(U \equiv PE_{\text{el}})$ $\quad U_{\text{el}} = qEx$ for $\vec{E} = -E\ \hat{i}$ $\quad V_{\text{el}} = U_{\text{el}}/q \quad E_x = -\frac{dV_{\text{el}}}{dx}$

$Q = C\Delta V_C \quad$ farad $= \mathrm{F} = \frac{\mathrm{C}}{\mathrm{V}} \quad C = \frac{\epsilon_0 A}{d} \quad \epsilon_0 = 8.85 \times 10^{-12}\frac{\mathrm{C^2}}{\mathrm{Nm^2}}$

parallel C_1, C_2: $C_{\text{eq}} = C_1 + C_2 \quad$ series C_1, C_2: $C_{\text{eq}}^{-1} = C_1^{-1} + C_2^{-1}$

$\Delta V_{\text{loop}} = \sum_i \Delta V_i = 0 \quad \sum I_{\text{in}} = \sum I_{\text{out}}$

$P = \Delta V I \quad$ watt $= \mathrm{W} = \mathrm{VA} \quad P_R = \Delta V_R I = I^2 R$

$\tau = RC \quad Q(t) = Q_0 e^{-t/\tau} \quad I(t) = -\frac{dQ}{dt} = \frac{\Delta V_0}{R}e^{-t/\tau}$

$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \vec{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s} \times \vec{r}}{r^3} \quad B_{\text{wire}} = \frac{\mu_0}{2\pi}\frac{I}{d}$ (use RH rule) $\quad \frac{\mu_0}{4\pi} = 10^{-7}\frac{\mathrm{Tm}}{\mathrm{A}} \quad$ tesla $= \mathrm{T} = \frac{\mathrm{N}}{\mathrm{Am}}$

short coil, $R >> L$ (N turns): $B_{\text{coil,centre}} = \frac{\mu_0 NI}{2R} \qquad$ solenoid, $L >> R$: $B_{\text{sol,inside}} = \frac{\mu_0 NI}{L}$

mag dipole: $\vec{\mu} = (AI, \text{from south to north}) \quad \vec{B}_{\text{dip}} = \frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{z^3}$ on axis, far away

$\vec{F}_{\text{on}q} = q\vec{v} \times \vec{B} \quad$ force on current $\perp$ to $\vec{B}$: $F_{\text{wire}} = ILB$

force betw. parallel wires: $F_{2\text{wires}} = \frac{\mu_0 L I_1 I_2}{2\pi d} \qquad$ torque on mag dipole: $\vec{\mu}$ in $\vec{B}$: $\vec{\tau} = \vec{\mu} \times \vec{B}$

bar (length L) moves w. $\vec{v} \perp \vec{B}$ gen. EMF: $\varepsilon = vLB$;

$\Phi_m = \vec{A} \cdot \vec{B} \quad \Phi_m = AB\cos\theta \quad \varepsilon = \left|\frac{d\Phi_m}{dt}\right| = \left|\vec{B} \cdot \frac{d\vec{A}}{dt} + \vec{A} \cdot \frac{d\vec{B}}{dt}\right|$

$L = \frac{\Phi_m}{I} \quad$ henry $= \mathrm{H} = \frac{\mathrm{Tm^2}}{\mathrm{A}} \quad \varepsilon_{\text{coil}} = L\left|\frac{dI}{dt}\right| \quad \Delta V_L = -L\frac{dI}{dt} \quad \mathrm{PE}_L = \frac{L}{2}I^2$

series L and R: $\tau = \frac{L}{R} \quad I(t) = I_0(1 - e^{-t/\tau})$