LAST NAME: STUDENT NR:

PHYS 1010 6.0: CLASS TEST 5

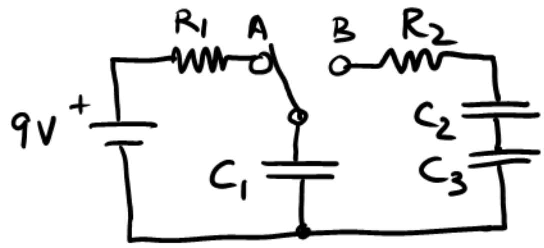
Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1) [5] Analyze this experiment: in the circuit shown the switch has been in position A for a long time, while capacitors C_2 and C_3 were uncharged. At time $t = 0$ the switch is flipped to position B, and remains there for a long time. What is the charge on each of the three capacitors at the end of the experiment? Compare your answer to what charge was held by C_1 at $t = 0$, just before the switch was flipped.

$R_1 = 50\,\Omega \qquad C_1 = 20\,\mu F$

$R_2 = 100\,\Omega \qquad C_2 = 10\,\mu F$

$C_3 = 5\,\mu F$



analyze position A:

long time → no current → $\Delta V_{R_1} = 0$; $\Delta V_C = 9V$

$Q = C\,\Delta V = 20\times10^{-6}\cdot 9.0 = 180\,\mu C$ — 1

position B: $Q_1 = 180\,\mu C$ will redistribute such that:

$Q_1 \to Q_1'$, $Q_2 = Q_3$ (same current); $Q_1' = Q_1 - Q_2$ 1

$C_{23}^{eq} = \dfrac{C_2 C_3}{C_2 + C_3} = \dfrac{50}{15} = 3.33\,\mu F$

$\Delta V_{C_1} = \Delta V_{C_{23}^{eq}}$ when current stops

current stops when $\Delta V_{C_1} = \Delta V_{C_{23}^{eq}}$

$\dfrac{Q_1'}{C_1} = \dfrac{Q_{2eq}}{C_{23}} = \dfrac{Q_1 - Q_1'}{C_{23}^{eq}}$

$\dfrac{Q_1'}{C_1} + \dfrac{Q_1'}{C_{23}^{eq}} = \dfrac{Q_1}{C_{23}^{eq}} \quad \therefore \quad Q_1'\left(\dfrac{1}{C_1} + \dfrac{1}{C_{23}^{eq}}\right) = \dfrac{Q_1}{C_{23}^{eq}}$

$\therefore \quad Q_1' \dfrac{C_{23}^{eq} + C_1}{C_1 C_{23}^{eq}} = \dfrac{Q_1}{C_{23}^{eq}} \quad \therefore \quad Q_1' = \dfrac{C_1 \cancel{C_{23}^{eq}}\, Q_1}{\cancel{C_{23}^{eq}}\,(C_{23}^{eq} + C_1)}$

$Q_1' = \dfrac{180\,\mu C \cdot 20}{20 + 3.33} = 154.3\,\mu C$ 1 $\quad \therefore \quad Q_2 = Q_1 - Q_1' = 25.7\,\mu C = 26\,\mu C$

$Q_3 = 26\,\mu C$ 1

Comparison: 180 μC splits into remaining 154 μC on C_1 while C_2 and C_3 hold 26. μC respectively 1

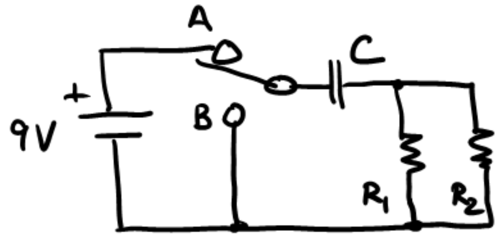
Note $Q_1 \neq Q_1' + Q_2 + Q_3$!!!

2) [5] In the circuit shown the switch has been in position A for a long time. (a) what is the voltage drop across resistor R_1? (b) what is the voltage drop across R_2? (c) How much charge is stored in the capacitor? (d) The switch is now (time $t = 0$) flipped to position B: graph the magnitude of the voltage drop across R_1 as a function of time. Calculate the time constant, and label your axes properly.

$R_1 = 1.0\ \mathrm{K\Omega}$

$R_2 = 0.5\ \mathrm{K\Omega}$

$C = 4.7\ \mu F$



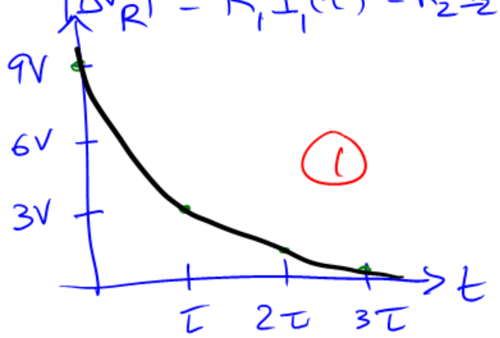
a. $\Delta V_{R_1} = 0$ 1

b. $\Delta V_{R_2} = \Delta V_{R_1}$ (Kirchhoff loop rule) $= 0$. 1

c. $\Delta V_C = \Delta V_B \quad \therefore \quad Q = C\Delta V = 4.7\mu F \cdot 9V = 42.3\ \mu C$ 1

$|\Delta V_R| = R_1 I_1(t) = R_2 I_2(t)$

d.



1.55 ms

$\tau = R_{eq} C$

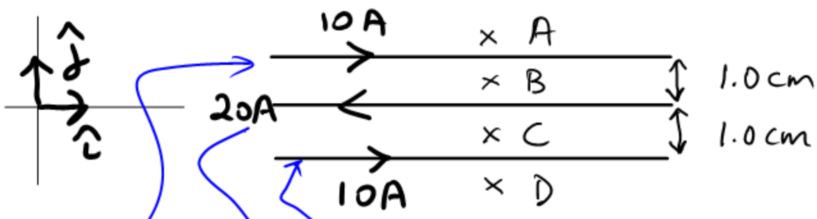
$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{0.5}{1.5}\ \mathrm{K\Omega}$

$= 330\ \Omega$

$\tau = 330 \cdot 4.7 \times 10^{-6}\ s$

$= 1.55\ ms$ 1

3) [5] Three very long current-carrying wires are shown in the figure, as well as locations A,B,C,D, which are all in the $x-y$ plane spanned by $\hat{\imath}$ and $\hat{\jmath}$. (a) Calculate $\vec{B}$ (magnitude and direction in component form) at locations A, B, C, D. (b) What can you say about the field strength far away from the wires ($|y|$ very large, or $|z|$ very large)?



$$\vec{B}_{net} = \sum_{i=1}^{3} \vec{B}_i \qquad I_1 = \qquad I_2 = \qquad I_3 =$$

$$\vec{B}_i = \left(\frac{\mu_0 I}{2\pi d} , \text{direction by simple RH rule} \right)$$

location (A): $\vec{B}_1 \sim \hat{k}$ $\quad \vec{B}_2 \sim -\hat{k}$ $\quad \vec{B}_3 \sim \vec{k}$

$$B_1 = \frac{\mu_0 \cdot 10}{2\pi \cdot 0.5\times10^{-2}} , \quad B_2 = \frac{\mu_0 \cdot 20}{2\pi \cdot 1.5\times10^{-2}} , \quad B_3 = \frac{\mu_0 \cdot 10}{2\pi \cdot 2.5\times10^{-2}}$$

$$= \frac{10\cdot10^{-7}}{2\cdot 0.5\times10^{-2}} \qquad = \frac{20\cdot10^{-7}}{2\cdot1.5\times10^{-2}} \qquad = \frac{10\cdot10^{-7}}{2\cdot2.5\times10^{-2}}$$

$$= 10\cdot10^{-5}\,T \qquad = 6.67\times10^{-5}\,T \qquad = 2\times10^{-5}\,T$$

$$B_{net} = B_1 - B_2 + B_3 = 5.3\times10^{-5}\,T \qquad \vec{B}^{A}_{net} = 5.3\times10^{-5}\,\hat{k}\,T$$

stating out of plane is OK (1)

(B): $\vec{B}_1 \sim -\hat{k}$; $\vec{B}_2 \sim -\hat{k}$; $\vec{B}_3 \sim \hat{k}$

$$B_1 = 10\cdot10^{-5}\,T \; ; \; B_2 = 20\cdot10^{-5}\,T \; ; \; B_3 = 3.33\cdot10^{-5}\,T$$

$$B_{net} = 26.7\cdot10^{-5}\,T \qquad \vec{B}^{B}_{net} = -26.7\times10^{-5}\,\hat{k}\,T \quad (1)$$

stating into plane is OK

(C) by symmetry $\vec{B}^{C}_{net} = -\vec{B}^{B}_{net} = +26.7\times10^{-5}\,\hat{k}\,T$ (1)

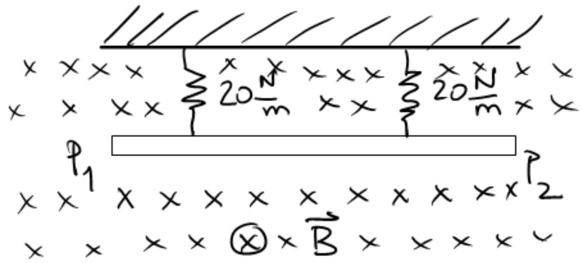
(D) " " $\vec{B}^{D}_{net} = -\vec{B}^{A}_{net} = -5.3\times10^{-5}\,\hat{k}\,T$ (1)

(C, D): not getting the right number but stating the symmetry property = full marks

b. At large distances the field from each wire $\sim \frac{1}{d}$, goes to zero (0.5)

but in addition there is a further cancellation "no net current" (expect $B \sim \frac{1}{d^2}$) fast fall-off. (0.5)

4) [5] The figure shows a copper rod of length $X = 75$ cm suspended by two springs to balance its weight (the springs are attached using plastic hooks). Both springs are characterized by a constant of $k = 20$ N/m respectively, and the rod was determined to have a mass of 500 g. A uniform magnetic field of $B = 1.5$ T acts over the entire length of the rod with an orientation into the page (as shown). Now you connect flexible low-mass electric contacts at points P_1 and P_2, and drive a current through the rod by connecting P_1 to the negative terminal, and P_2 to the positive terminal of a powerful battery. Your current meter states that 50 Amps are flowing through the battery. (a) is the rod moving up or down when you turn on the current? Explain! (b) Calculate by how much the rod is displaced. 5.0 !



a. Current convention $P_1(-) \longleftarrow P_2(+)$

By the RH rule $\vec{F}_M = q\,\vec{v} \times \vec{B}^{*}$ for $q>0$ moving left

The magnetic force pushes the wire down. — 1 for result

1 for explanation $^{*}(\vec{F}_M = \ell\,\vec{I} \times \vec{B})$ — both acceptable.

b. gravity plays no role, since the springs cancelled it at the beginning

Force balance: $k_1 \Delta x + k_2 \Delta x = I \ell B$; $\ell = X$

$\uparrow 2k\Delta x$ $\quad k_1 = k_2$

$\downarrow I X B$

$2k\Delta x = I X B$ — 1

$$\Delta x = \frac{I X B}{2k} = \frac{5.0 \cdot 0.75 \cdot 1.5}{40}$$

$$\Delta x = 0.141 \text{ m} = 14 \text{ cm}$$ 2 for result

Reasonable? The springs pre-stretched to cancel gravity by $2k\Delta x_0 = mg \quad \therefore \Delta x_0 = \frac{mg}{2k} = \frac{0.5 \cdot 9.8}{40}$

$= 0.12$ m $= 12$ cm

extra bonus +1

~ the same

FORMULA SHEET

$v(t_{\rm f}) = v(t_{\rm i}) + \int_{t_{\rm i}}^{t_{\rm f}} a(t)\ dt \quad s(t_{\rm f}) = s(t_{\rm i}) + \int_{t_{\rm i}}^{t_{\rm f}} v(t)\ dt$

$v_{\rm f} = v_{\rm i} + a\Delta t \quad s_{\rm f} = s_{\rm i} + v_{\rm i}\Delta t + \frac{1}{2}a\Delta t^2 \quad v_{\rm f}^2 = v_{\rm i}^2 + 2a\Delta s \qquad g = 9.8\ {\rm m/s^2}$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\ dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\ dt = at + C \quad F(t) =$ anti-derivative $=$ indefinite integral

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

uniform circular m. $\vec r(t) = R(\cos\omega t\ \hat{\mathbf{i}} + \sin\omega t\ \hat{\mathbf{j}})$; $\vec v(t) = \frac{d\vec r}{dt} = ...$; $\vec a(t) = \frac{d\vec v}{dt} = ...$

$\exp' = \exp$; $\sin' = \cos$; $\cos' = -\sin$. $\quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}$; $\quad (fg)' = f'g + fg'$

$m\vec a = \vec F_{\rm net}$; $\quad F_G = \frac{Gm_1m_2}{r^2}$; $g = \frac{GM_E}{R_E^2}$; $R_E = 6370$ km; $G = 6.67\times10^{-11}\frac{\rm Nm^2}{\rm kg^2}$; $M_E = 6.0\times10^{24}$kg

$f_{\rm s} \le \mu_{\rm s}n$; $\quad f_{\rm k} = \mu_{\rm k}n$; $\quad f_{\rm r} = \mu_{\rm r}n$; $\quad \mu_{\rm r} << \mu_{\rm k} < \mu_{\rm s}$. $\qquad F_H = -k\Delta x = -k(x - x_0)$.

$\vec F_{\rm d} \sim -\vec v$; linear: $F_{\rm d} = dv$; quadratic: $F_{\rm d} = 0.5\rho Av^2$; $\quad A =$ cross sectional area

$W = F\Delta x = F(\Delta r)\cos\theta$. $\ W =$ area under $F_x(x)$. $\quad PE_{\rm H} = \frac{k}{2}(\Delta x)^2$; $PE_g = mg\Delta y$.

$\Delta\vec p = \vec J = \int \vec F(t)dt$; $\Delta p_x = J_x =$ area under $F_x(t) = F_x^{\rm avg}\Delta t$; $\quad \vec p = m\vec v$; $K = \frac{m}{2}v^2$

$\Delta\vec p_1 + \Delta\vec p_2 = 0$; $K_1^{\rm in} + K_2^{\rm in} = K_1^{\rm fin} + K_2^{\rm fin}$ for elastic collisions. $\quad \vec a_{\rm CM} = \frac{m_1\vec a_1 + m_2\vec a_2}{m_1 + m_2}$

$\vec\tau = \vec r\times\vec F$; $\quad \tau_z = rF\sin(\alpha)$ for $\vec r$, $\vec F$ in xy plane. $\quad I = \sum_i m_ir_i^2$; $\quad I\alpha_z = \tau_z$; ($\hat k =$ rot. axis)

$K_{\rm rot} = \frac{I}{2}\omega^2$; $L_z = I\omega_z$; $\frac{d}{dt}L_z = \tau_z$; $\vec L = \vec r\times\vec p$; $\frac{d}{dt}\vec L = \vec\tau$

$x(t) = A\cos(\omega t + \phi)$; $\quad \omega = \frac{2\pi}{T} = 2\pi f$; $\quad v_x(t) = ...$; $\quad v_{\rm max} = ...$

$m_{\rm e} = 9.11\times10^{-31}$kg $\quad m_{\rm p} = 1.67\times10^{-27}$kg $\quad e = 1.60\times10^{-19}$C $\quad k = \frac{1}{4\pi\epsilon_0} = 9.0\times10^9\frac{\rm Nm^2}{\rm C^2}$

$\vec F_{\rm C} = \frac{kq_1q_2}{r^2}\hat{\mathbf{r}} \quad \vec F_{\rm E} = q\vec E \quad E_{\rm line} = \frac{2k|\lambda|}{r} = \frac{2K|Q|}{Lr} \quad E_{\rm plane} = \frac{|\sigma|}{2\epsilon_0} = \frac{|Q|}{2A\epsilon_0} \quad \vec E_{\rm cap} = \left(\frac{Q}{\epsilon_0A}, \text{pos} \to \text{neg}\right)$

$\frac{mv^2}{2} + U_{\rm el}(s) = \frac{mv_0^2}{2} + U_{\rm el}(s_0)$, $(U \equiv PE_{\rm el}) \quad U_{\rm el} = qEx$ for $\vec E = -E\,\hat i \quad V_{\rm el} = U_{\rm el}/q \quad E_x = -\frac{dV_{\rm el}}{dx}$

point charge: $V_{\rm el} = \frac{kQ}{r} \quad Q = C\Delta V_C \quad$ farad $= {\rm F} = \frac{\rm C}{\rm V} \quad C = \frac{\epsilon_0 A}{d} \quad \epsilon_0 = 8.85\times10^{-12}\frac{\rm C^2}{\rm Nm^2}$

parallel C_1, C_2: $C_{\rm eq} = C_1 + C_2 \quad$ series C_1, C_2: $C_{\rm eq}^{-1} = C_1^{-1} + C_2^{-1}$

resistance vs resisitivity: $R = \rho\frac{L}{A}$ ($L =$length, $A =$cross section); copper: $\rho = 1.7\times10^{-8}\ \Omega$m

$\Delta V_{\rm loop} = \sum_i \Delta V_i = 0 \quad \sum I_{\rm in} = \sum I_{\rm out}$

$P = \Delta VI \quad$ watt $=$ W $=$ VA $\quad P_R = \Delta V_RI = I^2R$

$\tau = RC \quad Q(t) = Q_0e^{-t/\tau} \quad I(t) = -\frac{dQ}{dt} = \frac{\Delta V_0}{R}e^{-t/\tau}$

$\vec B = \frac{\mu_0}{4\pi}\frac{q\vec v\times\vec r}{r^3} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec s\times\vec r}{r^3} \quad B_{\rm wire} = \frac{\mu_0}{2\pi}\frac{I}{d}$ (use RH rule) $\quad \frac{\mu_0}{4\pi} = 10^{-7}\frac{\rm Tm}{\rm A} \quad$ tesla $=$ T $= \frac{\rm N}{\rm Am}$

short coil, $R >> L$ (N turns): $B_{\rm coil,centre} = \frac{\mu_0NI}{2R} \qquad$ solenoid, $L >> R$: $B_{\rm sol,inside} = \frac{\mu_0NI}{L}$

mag dipole: $\vec\mu = (AI, \text{from south to north}) \quad \vec B_{\rm dip} = \frac{\mu_0}{4\pi}\frac{2\vec\mu}{z^3}$ on axis, far away

$\vec F_{\rm onq} = q\vec v\times\vec B \quad$ force on current $\perp$ to $\vec B$: $F_{\rm wire} = ILB$

force betw. parallel wires: $F_{\rm 2wires} = \frac{\mu_0LI_1I_2}{2\pi d} \qquad$ torque on mag dipole: $\vec\mu$ in $\vec B$: $\vec\tau = \vec\mu\times\vec B$

bar (length L) moves w. $\vec v \perp \vec B$ gen. EMF: $\varepsilon = vLB$;

$\Phi_m = \vec A\cdot\vec B \quad \Phi_m = AB\cos\theta \quad \varepsilon = \left|\frac{d\Phi_m}{dt}\right| = \left|\vec B\cdot\frac{d\vec A}{dt} + \vec A\cdot\frac{d\vec B}{dt}\right|$