LAST NAME: STUDENT NR:

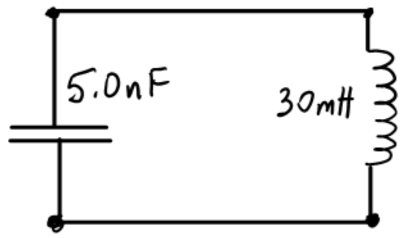
PHYS 1010 6.0: CLASS TEST 6

Time: 50 minutes; Calculators & formulae provided at the end = only aid; Total = 20 points.

1) [5] Consider the circuit shown. If the charge on the capacitor at a particular instant is 2.5×10^{-8} C, and the energy stored in the capacitor is equal to the energy stored in the inductor, what is the current at that moment?

$PE_C = \frac{1}{2C} Q^2$ $\qquad C = 5.0 \times 10^{-9}$ F

$PE_L = \frac{1}{2} L I^2$ $\qquad L = 3.0 \times 10^{-2}$ H



$LI^2 = \frac{Q^2}{C} \quad \therefore \quad I^2 = \frac{1}{LC} Q^2 \qquad Q = 2.5 \times 10^{-8}$ C

$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5.0 \cdot 3.0 \cdot 10^{-11}}} \frac{\text{rad}}{\text{s}} = \frac{1}{\sqrt{1.5}} 10^5 \frac{\text{rad}}{\text{s}} = 8.16 \cdot 10^4 \frac{\text{rad}}{\text{s}}$

$I(t) = \omega Q_0 \sin \omega t = \frac{dQ}{dt} \quad \therefore \quad Q(t) = -Q_0 \cos \omega t$

we could work from this, but it's more work!

$I = \omega Q \qquad \left(= \frac{1}{\sqrt{LC}} Q\right)$

we know Q, ω $\therefore$

$I(t_1) = 8.16 \times 10^4 \cdot \underbrace{2.5 \times 10^{-8}}_{Q(t_1)} \frac{\text{C}}{\text{s}} = 20.4 \times 10^{-4}$ A

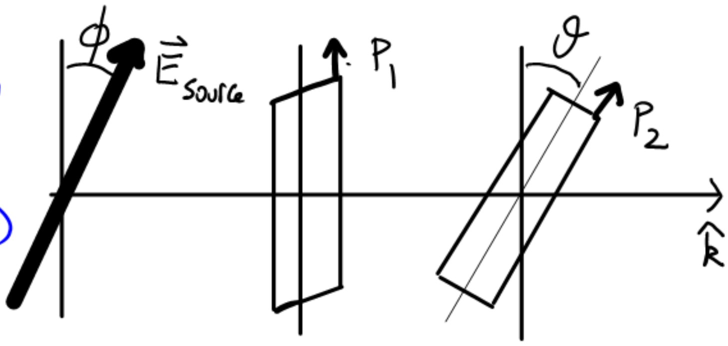
$= 2.0$ mA (5)

part marks can be given for $\omega \rightarrow$ (1)

and for deriving $I = \frac{1}{\sqrt{LC}} Q \rightarrow$ (2)

$= \omega Q$

2) [5] The law of Malus states that $I_{\text{out}} = I_{\text{in}} \cos^2\theta$, and refers to the light intensity observed after a polarizer whose axis is rotated by θ with respect to the polarization axis of the incident light (assumed to be linearly polarized). Provide a short 2-sentence explanation of the law based upon the electric field vectors $\vec{E}_{\text{in}}$ and $\vec{E}_{\text{out}}$. Now look at the figure: the original light source is linearly polarized with unknown polarization angle ϕ (with respect to the vertical). The second polarizer has a polarization axis of $\theta = 30°$ with respect to the vertical, which is the orientation for polarizer 1. If the intensity observed after the two polarizers is one half the original source intensity, what was the angle ϕ?



1) $\vec{E}_{source}$ needs to be decomposed along P_1 axis and its perpendicular (which is blocked) 1

$E_{source} \cdot \cos\phi = E_1$

2) The intensity $I \sim E^2$ 1 $\therefore \; I_1 = I_{source} \cdot \cos^2\phi$

After P_1 : $I_1 = I_{source} \cos^2\phi$

After P_2 : $I_2 = I_1 \cos^2\vartheta = I_{source} \cos^2\phi \cos^2\vartheta$ 1

$I_2 = I_{source} \cos^2\phi \cos^2(30°) = I_{source} \cos^2\phi \cdot 3/4$

$I_2 = \frac{1}{2} I_{source} = \frac{3}{4} I_{source} \cos^2\phi$

$\sin 30° = 1/2$

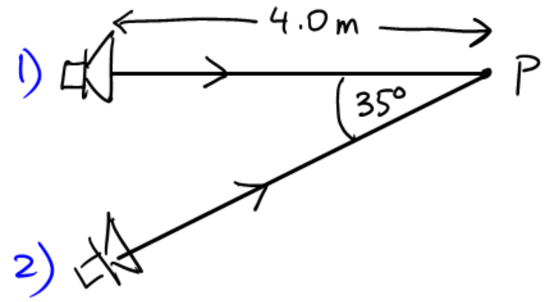
$\cos 30° = \sqrt{1 - 1/4} = \frac{\sqrt{3}}{2}$

$\therefore \; \cos^2\phi = 2/3$ 1 $\quad \cos\phi = \sqrt{2/3}$

$\phi = \cos^{-1}\left(\sqrt{2/3}\right) = 0.615 \text{ rad} = 35.3°$

$\phi = 35°$ 1

3) [5] The figure shows an experiment where two loudspeakers are driven by exactly the same signal. If the interference at point P is destructive, provide two possible values for the frequency of the sound.



The path length difference should be $\lambda/2$, or $\frac{3\lambda}{2}$, 1

(or $\frac{2n+1}{2}\lambda$ for $n = 0, 1, 2, \ldots$)

The path for speaker 2 is : $\frac{4.0}{L_2} = \cos 35° = 0.819$

$L_2 = \frac{4.0}{\cos 35°}$ m $\quad \therefore \quad \Delta L = L_2 - L_1 = 4.0\left(\frac{1}{\cos 35°} - 1\right)$

$= 0.884$ m 1

Condition 1 : $\Delta L = \frac{\lambda}{2} \quad \therefore \quad \lambda_1 = 2\,\Delta L = 1.77$ m

2 $\quad \Delta L = \frac{3\lambda}{2} \quad \therefore \quad \lambda_2 = \frac{2}{3}\,\Delta L = 0.589$ m

Frequencies follow from $\lambda_n f_n = c_s = 343$ m/s 1

$f_1 = \frac{c_s}{\lambda_1} = \frac{343}{2\,\Delta L} = 194$ Hz 1

$\simeq 190$ Hz

$f_2 = \frac{c_s}{\lambda_2} = \frac{343}{2/3\,\Delta L} = 582$ Hz 1

$\simeq 580$ Hz

($f_3 = \ldots$)

4) [5] The siren in an ambulance is approaching you with a speed v, and you perceive the sound to have a frequency of 1200 Hz. The ambulance makes a U turn, and when it reaches speed v again you hear the siren at 1000 Hz. What was the speed v?

Two unknowns: f_{src}, v ; given f_+, f_-

$$f_+ = \frac{f_{src}}{1 - v/v_s} \quad 1 \qquad f_- = \frac{f_{src}}{1 + v/v_s} \quad 1 \qquad v_s = 343\ \frac{m}{s}$$

eliminate f_{src} :

$$\left(1 - \frac{v}{v_s}\right) f_+ = \left(1 + \frac{v}{v_s}\right) f_- \qquad \text{call } x = \frac{v}{v_s}$$

$$(1-x)\ 12\cancel{00} = (1+x)\ 10\cancel{00}$$

$$12 - 12x = 10 + 10x$$

$$22x = 2 \qquad x = \frac{1}{11} \qquad v = \frac{v_s}{11}$$

$$v = \frac{343}{11}\ \frac{m}{s} = 31.2\ m/s$$

3

$$\left(= 112\ \frac{km}{h}\right)$$

which is possible (fast)

FORMULA SHEET

$v(t_f) = v(t_i) + \int_{t_i}^{t_f} a(t)\ dt \qquad s(t_f) = s(t_i) + \int_{t_i}^{t_f} v(t)\ dt$

$v_f = v_i + a\Delta t \qquad s_f = s_i + v_i\Delta t + \frac{1}{2}a\Delta t^2 \qquad v_f^2 = v_i^2 + 2a\Delta s \qquad g = 9.8\ \text{m/s}^2$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\ dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\ dt = at + C \qquad F(t) = \text{anti-derivative} = \text{indefinite integral}$

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2+px+q=0$ factored by: $x_{1,2}=-\frac{p}{2}\pm\sqrt{\frac{p^2}{4}-q}$

uniform circular m. $\vec{r}(t)=R(\cos\omega t\,\hat{\mathbf{i}}+\sin\omega t\,\hat{\mathbf{j}})$; $\vec{v}(t)=\frac{d\vec{r}}{dt}=...$; $\vec{a}(t)=\frac{d\vec{v}}{dt}=....$

$\exp'=\exp$; $\sin'=\cos$; $\cos'=-\sin$. $\frac{d}{dx}[f(g(x))]=\frac{df}{dg}\frac{dg}{dx}$; $(fg)'=f'g+fg'$

$m\vec{a}=\vec{F}_{\rm net}$; $F_G=\frac{Gm_1m_2}{r^2}$; $g=\frac{GM_E}{R_E^2}$; $R_E=6370$ km; $G=6.67\times10^{-11}\frac{\rm Nm^2}{\rm kg^2}$; $M_E=6.0\times10^{24}$kg

$f_{\rm s}\le\mu_{\rm s}n$; $f_{\rm k}=\mu_{\rm k}n$; $f_{\rm r}=\mu_{\rm r}n$; $\mu_{\rm r}<<\mu_{\rm k}<\mu_{\rm s}$. $F_H=-k\Delta x=-k(x-x_0)$.

$\vec{F}_{\rm d}\sim-\vec{v}$; linear: $F_{\rm d}=dv$; quadratic: $F_{\rm d}=0.5\rho Av^2$; $A=$ cross sectional area

$W=F\Delta x=F(\Delta r)\cos\theta$. $W=$ area under $F_x(x)$. $PE_{\rm H}=\frac{k}{2}(\Delta x)^2$; $PE_g=mg\Delta y$.

$\Delta\vec{p}=\vec{J}=\int\vec{F}(t)dt$; $\Delta p_x=J_x=$ area under $F_x(t)=F_x^{\rm avg}\Delta t$; $\vec{p}=m\vec{v}$; $K=\frac{m}{2}v^2$

$\Delta\vec{p}_1+\Delta\vec{p}_2=0$; $K_1^{\rm in}+K_2^{\rm in}=K_1^{\rm fin}+K_2^{\rm fin}$ for elastic collisions. $\vec{a}_{\rm CM}=\frac{m_1\vec{a}_1+m_2\vec{a}_2}{m_1+m_2}$

$\vec{\tau}=\vec{r}\times\vec{F}$; $\tau_z=rF\sin(\alpha)$ for $\vec{r}$, $\vec{F}$ in xy plane. $I=\sum_i m_ir_i^2$; $I\alpha_z=\tau_z$; ($\hat{k}=$ rot. axis)

$K_{\rm rot}=\frac{I}{2}\omega^2$; $L_z=I\omega_z$; $\frac{d}{dt}L_z=\tau_z$; $\vec{L}=\vec{r}\times\vec{p}$; $\frac{d}{dt}\vec{L}=\vec{\tau}$

$x(t)=A\cos(\omega t+\phi)$; $\omega=\frac{2\pi}{T}=2\pi f$; $v_x(t)=...$; $v_{\max}=...$

$m_{\rm e}=9.11\times10^{-31}$kg $m_{\rm p}=1.67\times10^{-27}$kg $e=1.60\times10^{-19}$C $K=\frac{1}{4\pi\epsilon_0}=9.0\times10^9\frac{\rm Nm^2}{\rm C^2}$

$\vec{F}_{\rm C}=\frac{Kq_1q_2}{r^2}\hat{\mathbf{r}}$ $\vec{F}_{\rm E}=q\vec{E}$ $E_{\rm line}=\frac{2K|\lambda|}{r}=\frac{2K|Q|}{Lr}$ $E_{\rm plane}=\frac{|\eta|}{2\epsilon_0}=\frac{|Q|}{2A\epsilon_0}$ $\vec{E}_{\rm cap}=\left(\frac{Q}{\epsilon_0A},\text{pos}\to\text{neg}\right)$

$\frac{mv^2}{2}+U_{\rm el}(s)=\frac{mv_0^2}{2}+U_{\rm el}(s_0)$, $(U\equiv PE_{\rm el})$ $U_{\rm el}=qEx$ for $\vec{E}=-E\,\hat{\imath}$ $V_{\rm el}=U_{\rm el}/q$ $E_x=-\frac{dV_{\rm el}}{dx}$

$Q=C\Delta V_C$ farad $=\mathrm{F}=\frac{\rm C}{\rm V}$ $C=\frac{\epsilon_0A}{d}$ $\epsilon_0=8.85\times10^{-12}\frac{\rm C^2}{\rm Nm^2}$ $\mathrm{PE}_C=\frac{Q^2}{2C}$

parallel C_1,C_2: $C_{\rm eq}=C_1+C_2$ series C_1,C_2: $C_{\rm eq}^{-1}=C_1^{-1}+C_2^{-1}$

$\Delta V_{\rm loop}=\sum_i\Delta V_i=0$ $\sum I_{\rm in}=\sum I_{\rm out}$

$P=\Delta VI$ watt $=\mathrm{W}=\mathrm{VA}$ $P_R=\Delta V_RI=I^2R$

$\tau=RC$ $Q(t)=Q_0e^{-t/\tau}$ $I(t)=-\frac{dQ}{dt}=\frac{\Delta V_0}{R}e^{-t/\tau}$

$\vec{B}=\frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}=\frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\vec{r}}{r^3}$ $B_{\rm wire}=\frac{\mu_0}{2\pi}\frac{I}{d}$ (use RH rule) $\frac{\mu_0}{4\pi}=10^{-7}\frac{\rm Tm}{\rm A}$ tesla $=\mathrm{T}=\frac{\rm N}{\rm Am}$

short coil, $R>>L$ (N turns): $B_{\rm coil,centre}=\frac{\mu_0NI}{2R}$ solenoid, $L>>R$: $B_{\rm sol,inside}=\frac{\mu_0NI}{L}$

mag dipole: $\vec{\mu}=(AI,\text{from south to north})$ $\vec{B}_{\rm dip}=\frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{z^3}$ on axis, far away

$\vec{F}_{{\rm on}q}=q\vec{v}\times\vec{B}$ force on current $\perp$ to $\vec{B}$: $F_{\rm wire}=ILB$

force betw. parallel wires: $F_{\rm 2wires}=\frac{\mu_0LI_1I_2}{2\pi d}$ torque on mag dipole: $\vec{\mu}$ in $\vec{B}$: $\vec{\tau}=\vec{\mu}\times\vec{B}$

bar (length L) moves w. $\vec{v}\perp\vec{B}$ gen. EMF: $\varepsilon=vLB$;

$\Phi_m=\vec{A}\cdot\vec{B}$ $\Phi_m=AB\cos\theta$ $\varepsilon=\left|\frac{d\Phi_m}{dt}\right|=\left|\vec{B}\cdot\frac{d\vec{A}}{dt}+\vec{A}\cdot\frac{d\vec{B}}{dt}\right|$

$L=\frac{\Phi_m}{I}$ henry $=\mathrm{H}=\frac{\rm Tm^2}{\rm A}$ $\varepsilon_{\rm coil}=L\left|\frac{dI}{dt}\right|$ $\Delta V_L=-L\frac{dI}{dt}$ $\mathrm{PE}_L=\frac{L}{2}I^2$

series L and R: $\tau=\frac{L}{R}$ $I(t)=I_0(1-e^{-t/\tau})$; parallel L and C: $\omega=\sqrt{\frac{1}{LC}}$ $I(t)=\omega Q_0\sin\omega t$

$\lambda f=v_{\rm w}$ sinusoid +ve $x-$dir'n: $D(x,t)=A\sin\left(2\pi(\frac{x}{\lambda}-\frac{t}{T})+\phi_0\right)=A\sin\left(kx-\omega t+\phi_0\right)$

transverse wave on a string: $v_{\rm w}=\sqrt{\frac{T}{\mu}}$ where T is tension, $\mu=M/L$

$\omega=v_{\rm w}k$ $v_{\rm sound}=343$m/s in air at T = 20°C in water: $v_{\rm sound}=1480$m/s

light in vac.: $v_{\rm w}=c=3.00\times10^8$m/s visible: $\lambda=400$nm (blue/UV); $\lambda=700$nm (red/IR)

medium: $n_{\rm glass}=1.5$; $n_{\rm water}=1.333$; speed: c/n; wavelength: $\lambda_{\rm vac}/n$; acc. phase: $\phi=\frac{2\pi n\Delta x}{\lambda_{\rm vac}}$

src speed $v_{\rm src}$: $f_+=\frac{f_0}{1-v_{\rm src}/v_{\rm w}}$; $f_-=\frac{f_0}{1+v_{\rm src}/v_{\rm w}}$; obs speed $v_{\rm obs}$: $f_+=f_0(1+\frac{v_{\rm obs}}{v_{\rm w}})$; $f_-=f_0(1-\frac{v_{\rm obs}}{v_{\rm w}})$

$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$ $\sin\alpha+\sin\beta=2\cos\frac{\alpha-\beta}{2}\ \sin\frac{\alpha+\beta}{2}$

transverse standing wave, string length L: $\lambda_n=\frac{2L}{n}$ $n=1,2,...$ f_n from $\lambda_nf_n=c_{\rm w}$