PHYS 1010 6.0: Test 6, April 5, 2013

Time: 50 minutes; Calculators & formulae provided at the end = only aid.

Formula sheet is complete for final exam, but test 6 stops after beats phenomenon, i.e., does not include EM waves. Test 6 begins with EM induction, LR and LC circuits, transverse waves on a string, sound, including standing waves, Doppler effect, Beats phenomenon.

FORMULA SHEET

$v(t_f) = v(t_i) + \int_{t_i}^{t_f} a(t)\ dt \quad s(t_f) = s(t_i) + \int_{t_i}^{t_f} v(t)\ dt$

$v_f = v_i + a\Delta t \quad s_f = s_i + v_i\Delta t + \frac{1}{2}a\Delta t^2 \quad v_f^2 = v_i^2 + 2a\Delta s \qquad g = 9.8\ \mathrm{m/s^2}$

$f(t) = t \quad \frac{df}{dt} = 1 \quad F(t) = \int f(t)\ dt = \frac{t^2}{2} + C$

$f(t) = a \quad \frac{df}{dt} = 0 \quad F(t) = \int f(t)\ dt = at + C \quad F(t) =$ anti-derivative = indefinite integral

area under the curve $f(t)$ between limits t_1 and t_2: $F(t_2) - F(t_1)$

$x^2 + px + q = 0$ factored by: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

uniform circular m. $\vec{r}(t) = R(\cos\omega t\ \hat{\mathbf{i}} + \sin\omega t\ \hat{\mathbf{j}})$; $\vec{v}(t) = \frac{d\vec{r}}{dt} = ...$; $\vec{a}(t) = \frac{d\vec{v}}{dt} = ...$

$\exp' = \exp$; $\sin' = \cos$; $\cos' = -\sin$. $\quad \frac{d}{dx}[f(g(x))] = \frac{df}{dg}\frac{dg}{dx}$; $\quad (fg)' = f'g + fg'$

$m\vec{a} = \vec{F}_{\mathrm{net}}$; $\quad F_G = \frac{Gm_1m_2}{r^2}$; $g = \frac{GM_E}{R_E^2}$; $R_E = 6370$ km; $G = 6.67 \times 10^{-11}\frac{\mathrm{Nm^2}}{\mathrm{kg^2}}$; $M_E = 6.0 \times 10^{24}$kg

$f_s \le \mu_s n$; $\quad f_k = \mu_k n$; $\quad f_r = \mu_r n$; $\quad \mu_r << \mu_k < \mu_s$. $\qquad F_H = -k\Delta x = -k(x - x_0)$.

$\vec{F}_d \sim -\vec{v}$; linear: $F_d = dv$; quadratic: $F_d = 0.5\rho Av^2$; $\quad A$ = cross sectional area

$W = F\Delta x = F(\Delta r)\cos\theta$. W = area under $F_x(x)$. $\quad PE_H = \frac{k}{2}(\Delta x)^2$; $PE_g = mg\Delta y$.

$\Delta\vec{p} = \vec{J} = \int \vec{F}(t)dt$; $\Delta p_x = J_x$ = area under $F_x(t) = F_x^{\mathrm{avg}}\Delta t$; $\quad \vec{p} = m\vec{v}$; $K = \frac{m}{2}v^2$

$\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$; $K_1^{\mathrm{in}} + K_2^{\mathrm{in}} = K_1^{\mathrm{fin}} + K_2^{\mathrm{fin}}$ for elastic collisions. $\quad \vec{a}_{\mathrm{CM}} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$

$\vec{\tau} = \vec{r} \times \vec{F}$; $\quad \tau_z = rF\sin(\alpha)$ for $\vec{r}$, $\vec{F}$ in xy plane. $\quad I = \sum_i m_i r_i^2$; $\quad I\alpha_z = \tau_z$; ($\hat{k}$ = rot. axis)

$K_{\mathrm{rot}} = \frac{I}{2}\omega^2$; $L_z = I\omega_z$; $\frac{d}{dt}L_z = \tau_z$; $\vec{L} = \vec{r} \times \vec{p}$; $\frac{d}{dt}\vec{L} = \vec{\tau}$

$x(t) = A\cos(\omega t + \phi)$; $\quad \omega = \frac{2\pi}{T} = 2\pi f$; $\quad v_x(t) = ...$; $\quad v_{\max} = ...$

$m_e = 9.11 \times 10^{-31}$kg $\quad m_p = 1.67 \times 10^{-27}$kg $\quad e = 1.60 \times 10^{-19}$C $\quad K = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9\frac{\mathrm{Nm^2}}{\mathrm{C^2}}$

$\vec{F}_C = \frac{Kq_1q_2}{r^2}\hat{\mathbf{r}} \quad \vec{F}_E = q\vec{E} \quad E_{\mathrm{line}} = \frac{2K|\lambda|}{r} = \frac{2K|Q|}{Lr} \quad E_{\mathrm{plane}} = \frac{|\eta|}{2\epsilon_0} = \frac{|Q|}{2A\epsilon_0} \quad \vec{E}_{\mathrm{cap}} = \left(\frac{Q}{\epsilon_0 A}, \text{pos} \to \text{neg}\right)$

$\frac{mv^2}{2} + U_{\mathrm{el}}(s) = \frac{mv_0^2}{2} + U_{\mathrm{el}}(s_0)$, $(U \equiv PE_{\mathrm{el}})$ $\quad U_{\mathrm{el}} = qEx$ for $\vec{E} = -E\,\hat{i} \quad V_{\mathrm{el}} = U_{\mathrm{el}}/q \quad E_x = -\frac{dV_{\mathrm{el}}}{dx}$

$Q = C\Delta V_C \quad$ farad = F = $\frac{\mathrm{C}}{\mathrm{V}} \quad C = \frac{\epsilon_0 A}{d} \quad \epsilon_0 = 8.85 \times 10^{-12}\frac{\mathrm{C^2}}{\mathrm{Nm^2}} \quad \mathrm{PE}_C = \frac{Q^2}{2C}$

parallel C_1, C_2: $C_{\mathrm{eq}} = C_1 + C_2 \quad$ series C_1, C_2: $C_{\mathrm{eq}}^{-1} = C_1^{-1} + C_2^{-1}$

$\Delta V_{\mathrm{loop}} = \sum_i \Delta V_i = 0 \quad \sum I_{\mathrm{in}} = \sum I_{\mathrm{out}}$

$P = \Delta VI \quad$ watt = W = VA $\quad P_R = \Delta V_R I = I^2R$

$\tau = RC \quad Q(t) = Q_0 e^{-t/\tau} \quad I(t) = -\frac{dQ}{dt} = \frac{\Delta V_0}{R}e^{-t/\tau}$

$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\vec{r}}{r^3} \quad B_{\mathrm{wire}} = \frac{\mu_0}{2\pi}\frac{I}{d}$ (use RH rule) $\quad \frac{\mu_0}{4\pi} = 10^{-7}\frac{\mathrm{Tm}}{\mathrm{A}} \quad$ tesla = T = $\frac{\mathrm{N}}{\mathrm{Am}}$

short coil, $R >> L$ (N turns): $B_{\mathrm{coil,centre}} = \frac{\mu_0 NI}{2R} \qquad$ solenoid, $L >> R$: $B_{\mathrm{sol,inside}} = \frac{\mu_0 NI}{L}$

mag dipole: $\vec{\mu} = (AI$, from south to north$) \quad \vec{B}_{\mathrm{dip}} = \frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{z^3}$ on axis, far away

$\vec{F}_{\mathrm{on}q} = q\vec{v} \times \vec{B} \quad$ force on current $\perp$ to $\vec{B}$: $F_{\mathrm{wire}} = ILB$

force betw. parallel wires: $F_{\mathrm{2wires}} = \frac{\mu_0 L I_1 I_2}{2\pi d} \qquad$ torque on mag dipole: $\vec{\mu}$ in $\vec{B}$: $\vec{\tau} = \vec{\mu} \times \vec{B}$

bar (length L) moves w. $\vec{v} \perp \vec{B}$ gen. EMF: $\varepsilon = vLB$;

$\Phi_m = \vec{A} \cdot \vec{B}$ $\quad \Phi_m = AB\cos\theta$ $\quad \varepsilon = \left|\frac{d\Phi_m}{dt}\right| = \left|\vec{B} \cdot \frac{d\vec{A}}{dt} + \vec{A} \cdot \frac{d\vec{B}}{dt}\right|$

$L = \frac{\Phi_m}{I}$ $\quad$ henry $= \mathrm{H} = \frac{\mathrm{Tm}^2}{\mathrm{A}}$ $\quad \varepsilon_{\text{coil}} = L\left|\frac{dI}{dt}\right|$ $\quad \Delta V_L = -L\frac{dI}{dt}$ $\quad \mathrm{PE}_L = \frac{L}{2}I^2$

series L and R: $\tau = \frac{L}{R}$ $\quad I(t) = I_0(1 - e^{-t/\tau})$; parallel L and C: $\omega = \sqrt{\frac{1}{LC}}$ $\quad I(t) = \omega Q_0 \sin\omega t$

$\lambda f = v_{\mathrm{w}}$ $\quad$ sinusoid +ve $x-$dir'n: $D(x,t) = A\sin(2\pi(\frac{x}{\lambda} - \frac{t}{T}) + \phi_0) = A\sin(kx - \omega t + \phi_0)$

transverse wave on a string: $v_{\mathrm{w}} = \sqrt{\frac{T}{\mu}}$ where T is tension, $\mu = M/L$

$\omega = v_{\mathrm{w}}k$ $\quad v_{\text{sound}} = 343\text{m/s}$ in air at T = 20°C $\quad$ in water: $v_{\text{sound}} = 1480\text{m/s}$

src speed v_{src}: $f_+ = \frac{f_0}{1 - v_{\text{src}}/v_{\mathrm{w}}}$; $f_- = \frac{f_0}{1 + v_{\text{src}}/v_{\mathrm{w}}}$; $\quad$ obs speed v_{obs}: $f_+ = f_0(1 + \frac{v_{\text{obs}}}{v_{\mathrm{w}}})$; $f_- = f_0(1 - \frac{v_{\text{obs}}}{v_{\mathrm{w}}})$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\quad \sin\alpha + \sin\beta = 2\cos\frac{\alpha-\beta}{2}\ \sin\frac{\alpha+\beta}{2}$

standing wave, string length L: $\lambda_n = \frac{2L}{n}$ $\quad n = 1, 2, ...$ $\quad f_n$ from $\lambda_n f_n = c_{\mathrm{w}}$; fixed at $x = 0$: $A\sin(2\pi f_n t)\sin(2\pi x/\lambda_n)$. $\quad$ Beats: $2A\cos[(\omega_1 - \omega_2)t/2]\sin[(\omega_1 + \omega_2)t/2]$

light in vac.: $v_{\mathrm{w}} = c = 3.00 \times 10^8\text{m/s}$ $\quad$ visible: $\lambda = 400\text{nm}$ (blue/UV); $\lambda = 700\text{nm}$ (red/IR)

medium: $n_{\text{glass}} = 1.5$; $n_{\text{water}} = 1.333$; speed: c/n; wavelength: λ_{vac}/n; acc. phase: $\phi = \frac{2\pi n\Delta x}{\lambda_{\text{vac}}}$