# Lab 8: Faraday's Law, generators, and motors

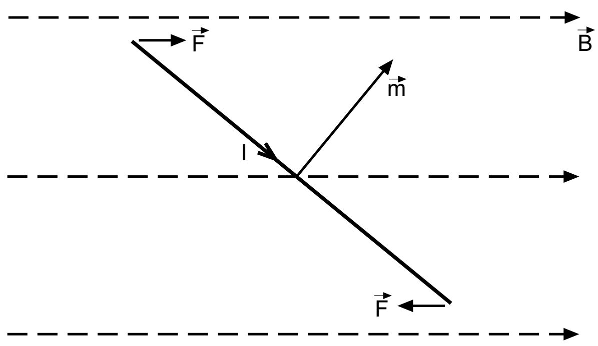


Figure 1: Top view of rectangular coil placed in uniform magnetic field. The arrows indicate the direction of current flow and forces acting on the coil, which produce a torque that rotates the coil.

## 1 Introduction

Most major power plants rely on **Faraday's law** for the conversion of mechanical energy into electrical energy. These power plants are commonly designed to derive mechanical energy from water, wind, and nuclear fission reactions, as well as from thermal energy obtained by burning coal, natural gas, or oil. In all these cases, the mechanical energy is used to rotate specially designed coils placed in a magnetic field. This arrangement is known as a **generator**. The changing magnetic flux through the generator coils produces an induced emf (**e**lectro**m**otive **f**orce), which results in a flow of induced current to electrical devices connected to the generator.

The torque acting on a current carrying loop of wire placed in a magnetic field can be engineered to rotate the wire loop. This idea finds widespread application in electric **motors** that are used universally - in computer hard drives and washing machines. The torque is the result of the force acting on a current carrying wire placed in a magnetic field.

This lab will allow you to learn the basic principles associated with Faraday's law, and learn how generators and motors work. The first section will allow you to perform several qualitative tests using simple apparatus consisting of a coil of wire, a bar magnet, and a galvanometer (a current detector). In the second section, you will analyze the predictions of Faraday's law in a quantitative manner. You will be able to control the rate of change of a magnetic field and measure the induced emf. In the final section, you will experiment with generators and motors and get a better idea of how they function.

**EXERCISES 1, 4, 5 AND 6 PERTAIN TO THE BACKGROUND CONCEPTS AND EXERCISES 2-3 AND 7-15 PERTAIN TO THE EXPERIMENTAL SECTIONS.**

## 2 Background

Consider a coil with $N$ turns, and area of cross section $A$. If the coil carries a current $I$, its magnetic dipole moment $\overrightarrow{m}$ is defined as,

$$\overrightarrow{m} = NIA\hat{n} \tag{1}$$

Here $\hat{n}$ is a unit vector drawn perpendicular to the area of the coil. You can figure out the direction of the magnetic dipole moment by using the right hand rule. If the fingers of your right hand are aligned along the direction of current in the coil, the thumb gives the direction of $\overrightarrow{m}$.

When the coil is placed in a uniform magnetic field $\overrightarrow{B}$, the net force on the coil is zero. However, there is a **torque** acting on the coil. This torque $\overrightarrow{\tau}$ is perpendicular to both $\overrightarrow{m}$ and $\overrightarrow{B}$. It tends to rotate $\overrightarrow{m}$ parallel to the direction of $\overrightarrow{B}$ (figure 1) and is expressed as,

$$\overrightarrow{\tau} = \overrightarrow{m} \times \overrightarrow{B} \tag{2}$$

The behavior of the coil is analogous to the behavior of a bar magnet. This is a consequence of the fact that the magnetic dipole moment of the magnet is associated with microscopic current loops (electrons spinning around nuclei). A vector drawn from the south pole to the north pole represents the magnetic dipole moment of the magnet. When the bar magnet is placed in a uniform magnetic field, it experiences a torque that tends to align its magnetic moment parallel to the direction of the field (figure 2).

If a steady current is applied to the coil placed in a uniform magnetic field (due to a permanent magnet), the torque given by equation 2 initially rotates the coil's magnetic dipole moment parallel to the direction of $\overrightarrow{B}$ (in this position, the plane of the coil is perpendicular

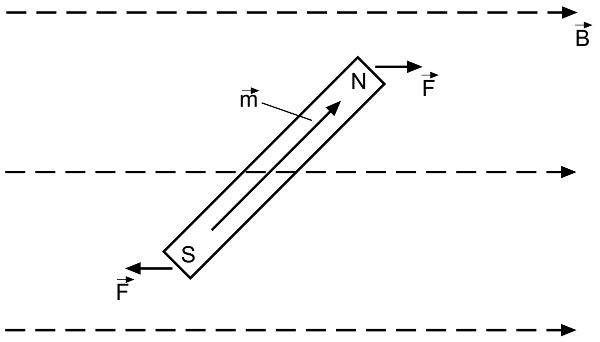


Figure 2: Forces on a bar magnet placed in a uniform magnetic field.

to $\overrightarrow{B}$). When the coil overshoots this position, the direction of the torque reverses. As a result, the coil will oscillate about this position until friction in the bearings brings it to rest. A **DC motor** is designed with split ring contacts called commutators. These ensure that the current through the coil reverses direction whenever the plane of the coil is perpendicular to the magnetic field. The torque on the coil does not change direction. This ensures that the coil will spin continuously.

The magnetic flux $\Phi_B$ through a coil placed in a magnetic field is given by,

$$\Phi_B = \int \overrightarrow{B} \cdot \overrightarrow{dA} \tag{3}$$

When a coil (with $N$ turns) rotates in a uniform magnetic field, the flux through the coil changes with time. **Faraday's law** predicts that a change in magnetic flux induces a non-conservative electric field. The negative rate of change of the magnetic flux is equal to the line integral of this induced electric field evaluated over a closed curve. The line integral is also equal to the **induced emf** $\varepsilon$. Mathematically, this result is expressed as,

$$\varepsilon = -N\frac{d\Phi_B}{dt} = \oint_C \overrightarrow{E} \cdot \overrightarrow{dl} \tag{4}$$

The induced emf causes an induced current to flow through a circuit as long as there is relative motion between the coil and the magnetic field, i.e. there is a change in magnetic flux. The negative sign in equation 4 implies that the direction of the induced current will oppose the change that causes the induced current in the first place. This is a statement of **Lenz's law**, which is a consequence of the law of conservation of energy. When you unplug a power cord from a wall outlet, the spark you may notice is due to an electric field that is induced when the magnetic field around the power cord is interrupted. The induced emf generated by a coil spinning in a magnetic field is the basis for the **generators** discussed in the introduction.

**Exercise 1a:** What is the flux through the coil in figure 1 when the plane of the coil is,

- i) Parallel to the magnetic field
- ii) Perpendicular to the magnetic field

**Exercise 1b:** Assume that the area of cross section of the coil is $A$ and that the coil is wound with $N$ turns.

- i) What is the flux through the coil when the plane of the coil is at an angle $\Theta$ with the magnetic field?
- ii) If the coil rotates at a uniform angular frequency $\omega$, show that the induced emf is, $\varepsilon = (NBA\omega)(sin\omega t)$.

**Exercise 2:** Assume that a magnet moves with a constant velocity $v$ along the axis of the circular wire loop in the direction shown in figure 3. Sketch a graph of the magnetic flux through a circular loop as a function of time. Indicate the time at which the center of the magnet passes through the center of the loop. Also sketch the corresponding induced emf as a function of time.

## 3 Suggested Reading

Refer to chapters on the Magnetic Field and Faraday's Law,


R. Wolfson and J. Pasachoff, **Physics with Modern Physics** (3rd Edition, Addison-Wesley Longman, Don Mills ON, 1999)

D. Halliday, R. Resnick and K. S. Krane, **Physics** (Volume 2, 5th Edition, John Wiley, 2002)

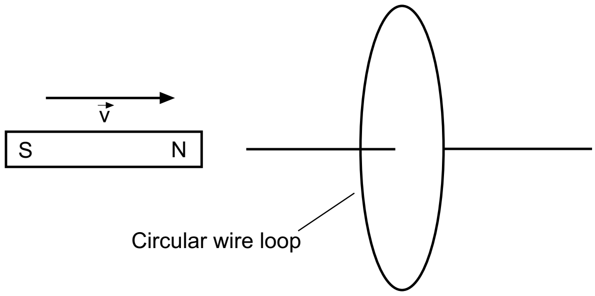


Figure 3: Induced current, exercise 2.

# 4 Apparatus

**Refer to Appendix E for photos of the apparatus**

- Bar magnet
- Two conducting wires with banana connectors
- Galvanometer
- Function generator
- 200 turn field coil with a 1.2 kΩ series resistor
- 400 turn search coil
- Oscilloscope
- Double banana plug to BNC adapter with a 10 kΩ resistor
- BNC cables
- BNC Tee connector
- BNC cable to banana cable adapter
- Low voltage power supply
- Motor
- Generator
- C-clamp

# 5 Experiment I: Magnets and Coils

Connect the 200-turn field coil to the galvanometer. The galvanometer is designed so that if current enters from the terminal labeled (+) and leaves through the terminal labeled (−), it shows a positive deflection. Make sure you note how the terminals of the galvanometer are connected to the terminals of the field coil. Note the direction of the windings on the field coil. Draw an accurate diagram of this simple circuit in your lab notebook.

**Exercise 3a:** Use the right-hand rule as well as Lenz's law to predict which way the needle will deflect when you insert the N-pole of the bar magnet into the field coil. Current flow (defined by convention, as the direction of flow of positive charge carriers) into the positive terminal of the galvanometer causes the needle to deflect to the right.

**Exercise 3b:** Insert the N-pole of the magnet into the field coil and check your prediction. Is your prediction correct? If not, find and correct the error in your reasoning.

**Exercise 3c:** Predict and check the direction of deflection of the needle on the galvanometer when you pull the N-pole of the magnet away from the field coil.

**Exercise 3d:** Insert the S-pole of the magnet into the field coil. Explain the outcome using a diagram.

**Exercise 3e:** Move the magnet through the field coil at different speeds and explain your observations.

**Exercise 3f:** Predict the outcome if the bar magnet is stationary and the field coil is moved toward the north pole of the magnet.

**Exercise 3g:** What will be the result of the experiment in part (a) if the negative sign in equation 4 is a positive sign? Perform exercises 3a-3f and explain your results. Your answers to exercises 3a-3g should be summarized with relevant figures, observations, and comments.

**Exercise 4:** For exercise 3a, draw a diagram showing the direction of the magnetic moment of the magnet as well as the induced magnetic moment.

# 6 Experiment II: Testing Faraday's Law

It is possible to test the predictions of equation 4 using two coils in closed circuits. If one of the coils (called the primary coil) is connected to an alternating current (AC) source, then an induced current will be produced in the secondary coil (search coil). In this experiment, alternating current from a waveform generator produces a changing magnetic field in the primary coil. The current induced in the search coil can be studied using an oscilloscope. For the main experiment, the search coil will be used to measure the induced electric field at the center of the primary coil.

The magnetic field at the center of the primary coil can be derived using the Biot-Savart law. It is given by,

$$B = \frac{\mu_0 N_1 I}{2R_1} \tag{5}$$

Here, $N_1$ is the number of turns in the primary coil, $R_1$ is the radius of the coil, $I$ is the current, and $\mu_0$ is the permeability of free space ($\mu_0 = 4\pi \times 10^{-7}$ N/$A^2$).

**Exercise 5:** Let $N_2$ and $R_2$ be the number of turns, and the radius, respectively, of the search coil. If the voltage through the primary coil changes at a rate $\frac{dV(t)}{dt}$ , and if $R$ is the resistance of the primary coil, use equation 4 to show that the emf produced in the search coil placed at the center of the primary coil is given by,

$$\varepsilon = -(\mu_0 N_1 N_2)(\frac{\pi R_2^2}{2R_1})[\frac{1}{R}\frac{dV(t)}{dt}] \tag{6}$$

In this experiment, you will use a function generator connected to the primary coil to produce a magnetic field that varies as a function of time. Since the primary coil has a very small resistance ($\sim 1\Omega$), the function generator will be forced to supply a large current through the primary coil if its amplitude is adjusted to a convenient value ($\sim 5V$). This could damage the generator. To avoid this situation, a comparatively large current limiting resistor (R=1.2 k$\Omega$) is connected in series with the coil. The rate of change of the current through the coil can then be obtained using Ohm's law if the rate of change of voltage produced by the generator is measured using the oscilloscope.

**Exercise 6:** Assume that the current in the primary coil is a triangular waveform with period $T$. Sketch the anticipated shape of the induced emf in the search coil. Your sketch must show the triangular waveform and the corresponding induced emf. The period $T$ should also be marked on the figure.

Find the BNC-banana connector used to plug the 400-turn search coil into the oscilloscope. Notice that one

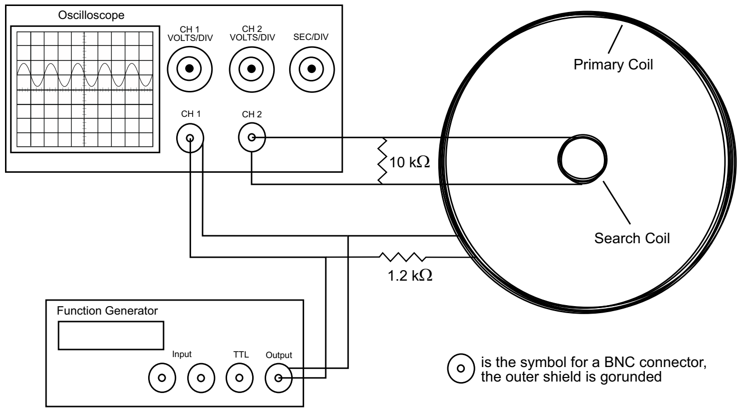


Figure 4: Experimental setup

of its terminals or jacks has a tab, which indicates that the jack is connected to the outer shield of the BNC connector, while the other jack is connected to the inner pin of the BNC connector.

The oscilloscope is designed to read a positive voltage if the inner pin is at a higher potential than the outer shield. In other words, it reads a negative voltage if the inner pin is at a lower potential than the outer shield. You need this information to test the negative sign in Faraday's law. If you did not follow the sign convention for the BNC connector, you may end up concluding that the negative sign is of no consequence, when in fact it is vital.

**Exercise 7:** Explain the consequences if the negative sign is missing in Faraday's law.

Set up the apparatus as shown in figure 4. Attach a tee connector to the output of the waveform generator. Connect the output of the generator (one arm of the tee) to channel 1 of the oscilloscope using a coaxial BNC cable. In this manner, the outer shield of the BNC connector on the oscilloscope is connected to the outer shield of the BNC connector on the function generator. The outer shields of the BNC connectors are grounded. Set the trigger source on the oscilloscope to channel 1 and observe the output waveform. Make sure that channel 1 of the oscilloscope is DC coupled. **Adjust the generator to produce a triangular waveform with a frequency of ~500 Hz and a peak to peak amplitude of ~5 V.** Make sure that you understand how to interpret the amplitude and the time base settings on the oscilloscope. Using a BNC-banana connector, connect the generator to the primary coil using the second arm of the tee connector. The primary coil should be clamped horizontally on the table. Make sure that a 1.2 k$\Omega$ resistor is connected in series with the generator and the primary coil. Next, clamp the search coil at the center of the primary coil. Connect the terminals of the search coil to channel 2 of the oscilloscope. This channel should also be DC coupled. Ensure that a 10 k$\Omega$ resistor is connected across the terminals of the search coil. This resistor is necessary to damp out oscillations in the induced emf generated in the secondary coil. Adjust the settings for channel 2 until the signal from the search coil can be observed clearly.

**Exercise 8:** Is the shape of this signal consistent with your expectations? Sketch the triangular waveform and the induced emf. Show the period of the triangular waveform on the sketch. Mark the zero of both waveforms on the sketch. Investigate the shape of the signal by interchanging the connections to the search coil.

**Exercise 9:** Record the rate of change of voltage $\frac{dV(t)}{dt}$ (this is the slope of the waveform) due to the primary coil displayed on channel 1, and the induced emf due to the search coil displayed on channel 2. **Make sure the induced emf is measured with respect to its zero.** Vary $\frac{dV(t)}{dt}$ (by changing the amplitude or frequency or both of these parameters), and measure the corresponding emf produced in the secondary coil. Also note the errors associated with the measured quantities. Tabulate your results.

**CAUTION:** Do not increase the frequency of the triangular waveform greater than 2 kHz.

**Exercise 10:** For one of the values of $\frac{dV(t)}{dt}$ in your table, calculate the expected induced emf using equation 6. Does the calculated result agree with your measured value in the table? This exercise will serve as a preliminary check of your data. If you find a significant or absurd discrepancy between the calculated and expected values, please consult the TA.

**Exercise 11a:** Plot a graph of the induced emf as a function of $\frac{dV(t)}{dt}$. What is the slope of the resulting graph? What quantity does the slope represent? Does the slope agree with the calculated value? What is the percentage difference between the calculated value and the value obtained from the graph?

**Exercise 11b:** Make an estimate of the errors involved in all your measurements. Is the discrepancy between the calculated value and the value obtained from the graph consistent with your estimate?

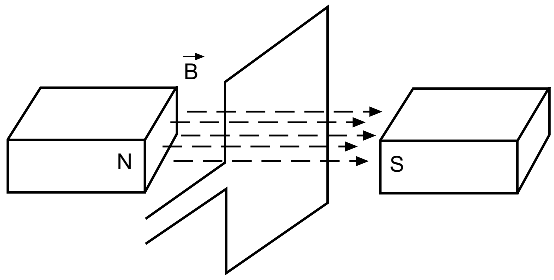


Figure 5: Generator/motor setup

**Exercise 12:** Move the search coil along the axis of the primary coil. Describe your results qualitatively. State your expectations briefly. Are your results consistent with your expectations?

**Exercise 13:** Position the search coil at the middle of the primary coil. Tilt the search coil with respect to the plane of the primary coil. Describe your results qualitatively. State your expectations. Are your results consistent with your expectations?

## 7 Experiment III: Motors and Generators

A generator consists of a system of coils in a magnetic field (figure 5). When mechanical energy is supplied to rotate the coil(s), the change in magnetic flux induces an emf in the coil (refer to exercise 1). A DC motor consists of a similar arrangement. When current flows through the coil, the torque on the coil will cause it to rotate as discussed earlier.

Connect the output of the generator to the light bulb. Rotate the crankshaft connected to the coil at a regular rate. Increase the rate of rotation until you see the light bulb glow. You can also view the output of the generator on an oscilloscope.

**Exercise 14:** Make sure that the oscilloscope is AC coupled. Draw a sketch of the signal. State your expectations. Is the signal consistent with your expectations?

**Exercise 15:** Connect the DC power source to the motor. Before turning the power on, make sure that the voltage knob on the power supply is turned all the way to zero. Examine the windings on the coil. Draw a simple diagram such as figure 5 and indicate the direction of the current flow. Predict the direction in which the motor will turn (you have to know the direction of $B$ to make this prediction). Turn on the power supply and slowly increase the voltage. Spin the coil until the motor starts spinning without assistance. If the motor does not spin, increase the voltage setting on the power supply. Is your prediction correct? If it is incorrect, try to understand the reason for the error. How can you reverse the direction of rotation of the motor?

**Your lab report should include:**

Answers to exercises 1-15 with relevant data tables, graphs, figures and qualitative comments.

Refer to Appendix D for Maple worksheets.