PHYSICS 1012 Exam Equation Sheet

"*Never memorize something you can look up.*" – Albert Einstein

Constants and Conversion Factors

$$
\begin{aligned}
1\text{ m} = 10^6\ \mu\text{m} = 10^9\text{ nm} &= 10^{10}\ \mathring{A} = 10^{12}\text{ pm} = 10^{15}\text{ fm} \\
1\text{ GeV} = 10^3\text{ MeV} &= 10^6\text{ keV} = 10^9\text{ eV} = 1.6022\times10^{-10}\text{ J} \\
e &= 1.6022\times10^{-19}\text{ C} \\
\varepsilon_0 &= 8.85418781762\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2 \\
1/4\pi\varepsilon_0 &= 8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2 \\
\mu_0 &= 4\pi\times10^{-7}\text{ T}\cdot\text{m/A} \\
c &= 2.99792458\times10^8\text{ m/s}\sim3.0\times10^8\text{ m/s} \\
k_B &= 1.380649\times10^{-23}\text{ J/°K} \\
N_A &= 6.02214076\times10^{23}\text{ per mole} \\
R &= 8.3144\text{ J/°K}\cdot\text{mol} \\
\sigma &= 5.67\times10^{-8}\text{ J/s}\cdot\text{m}^2\cdot{}^\circ\text{K}^4 \\
m_e\text{ (the electron mass)} &= 9.11\times10^{-31}\text{ kg} \\
M_p\text{ (the proton mass)} &= 1836m_e
\end{aligned}
$$

Work:

$$W = \Delta U = -\int \vec{F}\cdot d\vec{s} \qquad \Delta U = Q\Delta V$$

Coulomb's Law:

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|\vec{r}|^2}\hat{r}$$

The Electric Field:

$$\vec{F}(x,y,z) = Q\vec{E}(x,y,z)$$

Energy density stored in an electric field: $u_E = \frac{1}{2}\varepsilon_0|\vec{E}|^2$

For a point charge;

$$|\vec{E}| = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

For an infinite uniformly charged line;

$$|\vec{E}| = \frac{\lambda}{2\pi\varepsilon_0\ell}$$

For an infinite uniformly charged sheet;

$$|\vec{E}| = \frac{\sigma}{2\varepsilon_0}$$

At a distance ℓ away along the bisector of an electric dipole ($\vec{p} = q\vec{d}$);

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{-\vec{p}}{[\ell^2 + (d/2)^2]^{3/2}}$$

For a dipole $\vec{p}$ **in** an external field $\vec{E}$; $\quad \vec{\tau} = \vec{p} \times \vec{E}$

Gauss' Law:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

Electric Potential:

$$\Delta V = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{s}$$

At a distance r from a point charge q with $V(r = \infty) = 0$;

$$V(r) = \frac{q}{4\pi\varepsilon_0 r}$$

Electrical Properties of Materials:

$$i = \frac{dq}{dt} = \int \vec{j} \cdot d\vec{A} \qquad \vec{j} = -en\vec{v}_d \qquad |\vec{j}| = \frac{i}{A}$$

For ohmic materials:

$$R = \rho\frac{L}{A} \qquad \Delta V = iR$$

DC Circuits: $P = \Delta V i = i^2 R = \Delta V^2 / R$

Kirchoff's Laws

Current Law: At a junction $\sum i_{\text{in}} = \sum i_{\text{out}}$

Voltage Law: For a closed loop $\sum \Delta V = 0$

The equivalent resistance R for N resistors:

- On the same current path
$$R = \sum_{i=1}^{N} R_i$$
- Having the same potential difference across them
$$\frac{1}{R} = \sum_{i=1}^{N} \frac{1}{R_i}$$

Capacitors: $Q = C\Delta V \qquad U = Q^2/2C$

Parallel-plate capacitor:

$$C = \varepsilon_0 \frac{A}{d}$$

With a dielectric filling the space between plates $C' = \kappa_e C$

The equivalent capacitance C for N capacitors:

- On the same current path
$$\frac{1}{C} = \sum_{i=1}^{N} \frac{1}{C_i}$$
- Having the same potential difference across them
$$C = \sum_{i=1}^{N} C_i$$

For a charging capacitor:

$$\Delta V(t - t_0) = \Delta V(t_0) \left(1 - e^{-(t-t_0)/\tau}\right)$$

$$\tau = RC$$

Magnetic Force:

On a moving charge at the instant it is at the location (x, y, z)

$$\vec{F}(x, y, z) = q\vec{v}(x, y, z) \times \vec{B}(x, y, z)$$

On a current-carrying wire;

$$\vec{F} = i\vec{L} \times \vec{B}$$

Torque on a current-carrying loop of N turns: $\vec{\tau} = Ni\vec{A} \times \vec{B}$

Magnetic Field:

Due to a moving charge:

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{|\vec{r}|^2}$$

Around a long, thin, straight current-carrying wire:

$$|\vec{B}| = \frac{\mu_0 i}{2\pi d}$$

Along the axis of a thin, current-carrying ring of radius R:

$$|\vec{B}| = \frac{\mu_0 i R^2}{2(R^2 + d^2)^{3/2}}$$

Inside an ideal solenoid:

$$|\vec{B}| = \mu_0 n i$$

Magnetic Dipole Moment:

$$\vec{\mu} = i\vec{A} \qquad \vec{\tau} = \vec{\mu} \times \vec{B} \qquad U = -\vec{\mu} \cdot \vec{B}$$

Ampère's Law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enclosed}}$$

Magnetic Flux and Faraday's Law of Induction:

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Induced Electric Field:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

Inductors:

$$\Delta V_L = L\frac{di}{dt} \qquad\qquad U = \frac{1}{2}Li^2$$

AC Circuits and Complex Impedance:

$$\Delta V = V_0 \cos \omega t \qquad \Longrightarrow \qquad \Delta V = V_0 e^{j\omega t}$$

$$\begin{aligned} \Delta V &= iZ \\ Z_R &= R \\ Z_C &= \frac{1}{j\omega C} \\ Z_L &= j\omega L \end{aligned}$$

$$A_V = \left|\frac{\Delta V_{out}}{\Delta V_{in}}\right|$$

Optics

Reflection and Refraction

Reflection: $\theta_r = \theta_i$

Index of Refraction: $n = c/v \qquad c = f\lambda \quad v = f\lambda_n \quad \lambda_n = \lambda/n$

Dispersion: n is a function of λ

Snell's Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Critical angle:

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_2}\right) \text{ for } n_1 > n_2$$

Double Slit Interference

Constructive:

$$\Delta l = d\sin\theta = m\lambda \quad \text{with} \quad m = 0, \pm1, \pm2, \pm3, \ldots$$

Destructive:

$$\Delta l = d\sin\theta = \left(m + \frac{1}{2}\right)\lambda \quad \text{with} \quad m = 0, \pm1, \pm2, \pm3, \ldots$$

Distance from central maximum to m^{th} bright fringe:

$$y_m = \frac{m\lambda D}{d}$$

Diffraction

Single Slit Destructive Interference:

$$a\sin\theta = m\lambda \text{ with } m = \pm1, \pm2, \pm3, \ldots$$

Rayleigh Criterion for Circular Aperatures: $\theta = 1.22\frac{\lambda}{D}$

Bragg Equation: $m\lambda = 2d\sin\theta$ with $m = 1, 2, 3, \ldots$

Thermodynamics

Temperature Scales – Celsius to Kelvin: $T_K = T_C + 273.15°$

Pressure conversion: 1 atm = 101.3 kPa

Heat

Heat and Temperature: $Q = mc\Delta T$

Phase Change: $Q = mL$

Conductive Heat Transfer:

$$P = \frac{dQ}{dt} = -\kappa A \frac{dT}{dx}$$

Radiative Heat Transfer:

$$P = e\sigma A T^4$$

Gas Kinetics

Ideal Gas Law:

$$pV = Nk_BT = nRT$$

Temperature and Kinetic Energy:

$$< K >= \frac{1}{2}m < v^2 >= \frac{3}{2}k_BT$$

RMS speed:

$$v_{RMS} = \sqrt{< v^2 >} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3RT}{M}}$$

Internal Energy:

$$E_{\text{int}} = \frac{3}{2}nRT$$

First Law of Thermodynamics

Work and gases:

$$W = \int_{V_1}^{V_2} p dV$$

The First Law of Thermodynamics:

$$\begin{aligned} \Delta E_{\text{int}} &= Q - W \\ \Delta E_{\text{int}} &= Q = nC_V \Delta T \\ Q &= nC_p \Delta T \\ C_V &= \frac{d}{2} R \\ C_p &= C_V + R \end{aligned}$$

Thermodynamic Processes:

Isothermal ($\Delta T = 0$): $W = nRT \ln\left(\frac{V_2}{V_1}\right)$

Isobaric ($\Delta p = 0$): $W = p(V_2 - V_1)$

Isochoric ($\Delta V = 0$): $W = 0$

Adiabatic ($Q = 0$): $pV^\gamma = \text{constant}$

$TV^{\gamma-1} = \text{constant}$

$W = \frac{p_2 V_2 - p_1 V_1}{1-\gamma}$

Entropy:

$$\Delta S = \int_1^2 \frac{dQ}{T}$$

$$\text{(at constant } T) \quad \Delta S = \frac{\Delta Q}{T} = nR \ln\left(\frac{V_2}{V_1}\right)$$

Table 1: Specific Heats and Thermal Conductivities of Some Common Materials (Temperature range 0°C to 100°C)

Substance	c (J/kg°K)	κ (W/m°K)
Air		0.026
Water	4184	0.61
Copper	386	401
Steel	502	46
Glass	753	0.8
Wood	1400	0.11

Table 2: Heats of Transformation (at Atmospheric Pressure)

Substance	Melting Point (°K)	L_f (kJ/kg)	Boiling Point (°K)	L_v (kJ/kg)
Copper	1357	205	2840	4726
Lead	601	24.7	2013	858
Oxygen	54.8	13.8	90.2	213
Water	273.15	334	373.15	2257

Table 3: Index of Refraction

Substance	n
Air	1.000293
Water	1.333
Ice (at 0°C)	1.309
Glass, crown	1.52
Glass, flint	1.66
Diamond	2.419

Volumes and Areas:

Area of a circle of radius R and area of the side of a cylinder of length ℓ and radius R;

$$A = \pi R^2 \qquad\qquad A = 2\pi R\ell$$

Volume and surface area of a sphere of radius R;

$$V = \frac{4}{3}\pi R^3 \qquad\qquad A = 4\pi R^2$$

Vectors, Trigonometry, Derivatives, Complex Numbers

We will use Cartesian coordinates throughout.

Vectors

$$\begin{aligned}
\vec{A} &= |\vec{A}|\hat{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \\
|\vec{A}| &= \sqrt{A_x^2 + A_y^2 + A_z^2} \\
\hat{A} &= \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{\sqrt{A_x^2 + A_y^2 + A_z^2}} \\
\vec{C} &= C_x\hat{i} + C_y\hat{j} + C_z\hat{k} \\
&= \vec{A} + \vec{B} \\
&= (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k} \\
\vec{A}\cdot\vec{B} &= |\vec{A}||\vec{B}|\cos\theta \\
&= (A_xB_x + A_yB_y + A_zB_z) \\
\vec{A}\times\vec{B} &= |\vec{A}||\vec{B}|\sin\theta\hat{n} \\
&\quad \text{where } \hat{n} \text{ is the unit direction vector normal to the } \vec{A}\vec{B} \text{ plane} \\
&= (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}
\end{aligned}$$

Derivatives

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x} A\cos bx &= -Ab\sin bx \\
\frac{\mathrm{d}}{\mathrm{d}x} A\sin bx &= Ab\cos bx \\
\frac{\mathrm{d}Ax^{\pm b}}{\mathrm{d}x} &= \pm Abx^{\pm b-1} \\
\frac{\mathrm{d}Ae^{\pm bx}}{\mathrm{d}x} &= \pm Abe^{\pm bx}
\end{aligned}$$

Trigonometry

$$\begin{aligned}
\sin^2\theta + \cos^2\theta &= 1 \\
\sin 2\theta &= 2\sin\theta\cos\theta \\
\cos 2\theta &= \cos^2\theta - \sin^2\theta \\
\sin(\theta \pm \phi) &= \sin\theta\cos\phi \pm \cos\theta\sin\phi \\
\cos(\theta \pm \phi) &= \cos\theta\cos\phi \mp \sin\theta\sin\phi \\
\sin\theta \pm \sin\phi &= 2\sin\left[\frac{1}{2}(\theta \pm \phi)\right]\cos\left[\frac{1}{2}(\theta \mp \phi)\right] \\
\cos\theta + \cos\phi &= 2\cos\left[\frac{1}{2}(\theta + \phi)\right]\cos\left[\frac{1}{2}(\theta - \phi)\right] \\
\cos\theta - \cos\phi &= -2\sin\left[\frac{1}{2}(\theta + \phi)\right]\sin\left[\frac{1}{2}(\theta - \phi)\right]
\end{aligned}$$

Complex Numbers

$$\begin{aligned}
j &= \sqrt{-1} \\
Z &= a \pm bj \\
&= |Z|(\cos\theta \pm j\sin\theta) \\
&= |Z|e^{\pm j\theta} \\
|Z| &= \sqrt{a^2 + b^2} \\
\theta &= \pm\tan^{-1}\left(\frac{b}{a}\right)
\end{aligned}$$