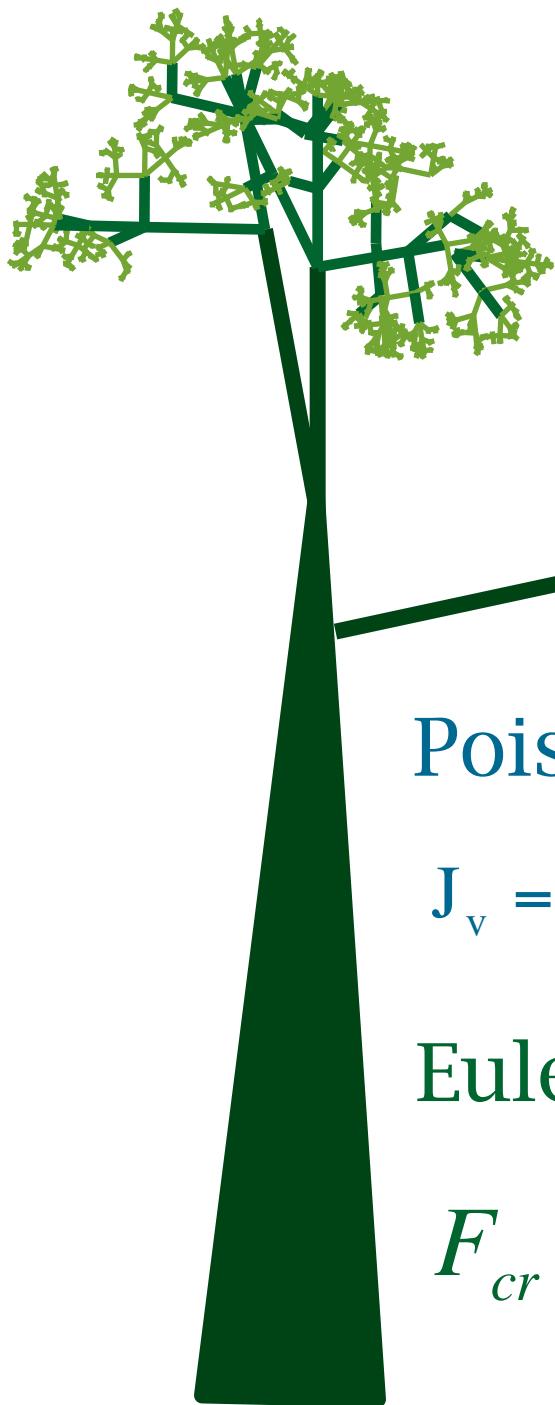


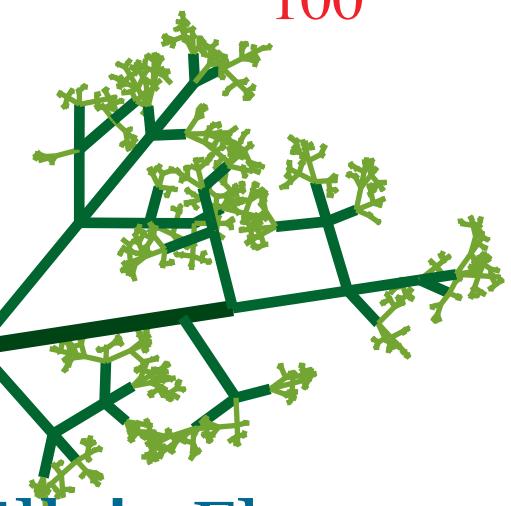
How High Can a Tree Grow?

Tension Water



Evaporative Pull

$$\Psi_{wv} = 135 \cdot \ln \frac{\%RH}{100}$$



Poiseuille's Flow

$$J_v = \left(\frac{\Delta p}{l} \right) \left(\frac{\pi}{8 \cdot \eta} \right) \cdot R^4$$

Euler's Column

$$F_{cr} = \frac{E \cdot I \cdot \pi^2}{L_{eff}^2}$$

B*io*physical
B*io*currents

How High Can a Tree Grow?

Biological Problem

Evolution and adaptation

- Colonization of land
- Competition for light

Physical aspects

- Pumping water
- Water piping
- Structural support

Biological Structure

Leaves

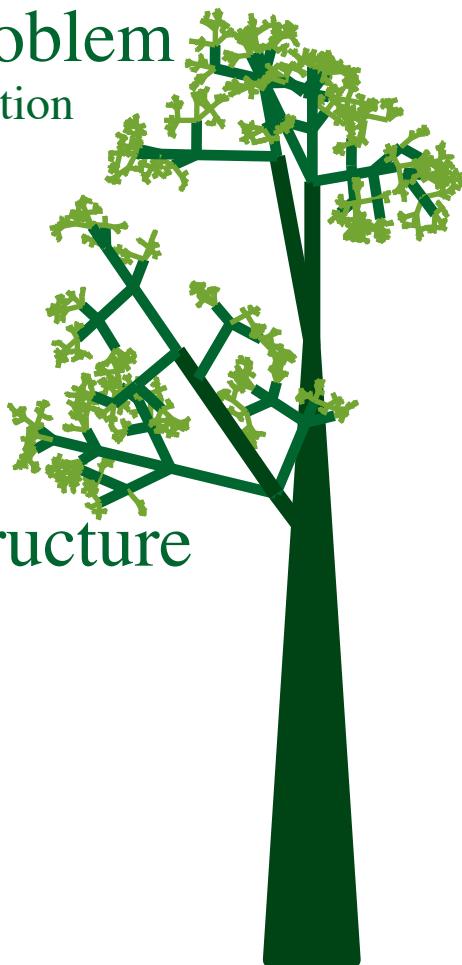
- Photosynthesis

Woody stem

- Xylem vessels
- Columnar structure

Woody roots

- Water uptake
- Structure foundation



Physical Approach

Evaporative pump (thermodynamics)

$$\Psi_{wv} = 135 \times \ln \frac{\%RH}{100}$$

Poiseuille flow (fluid dynamics)

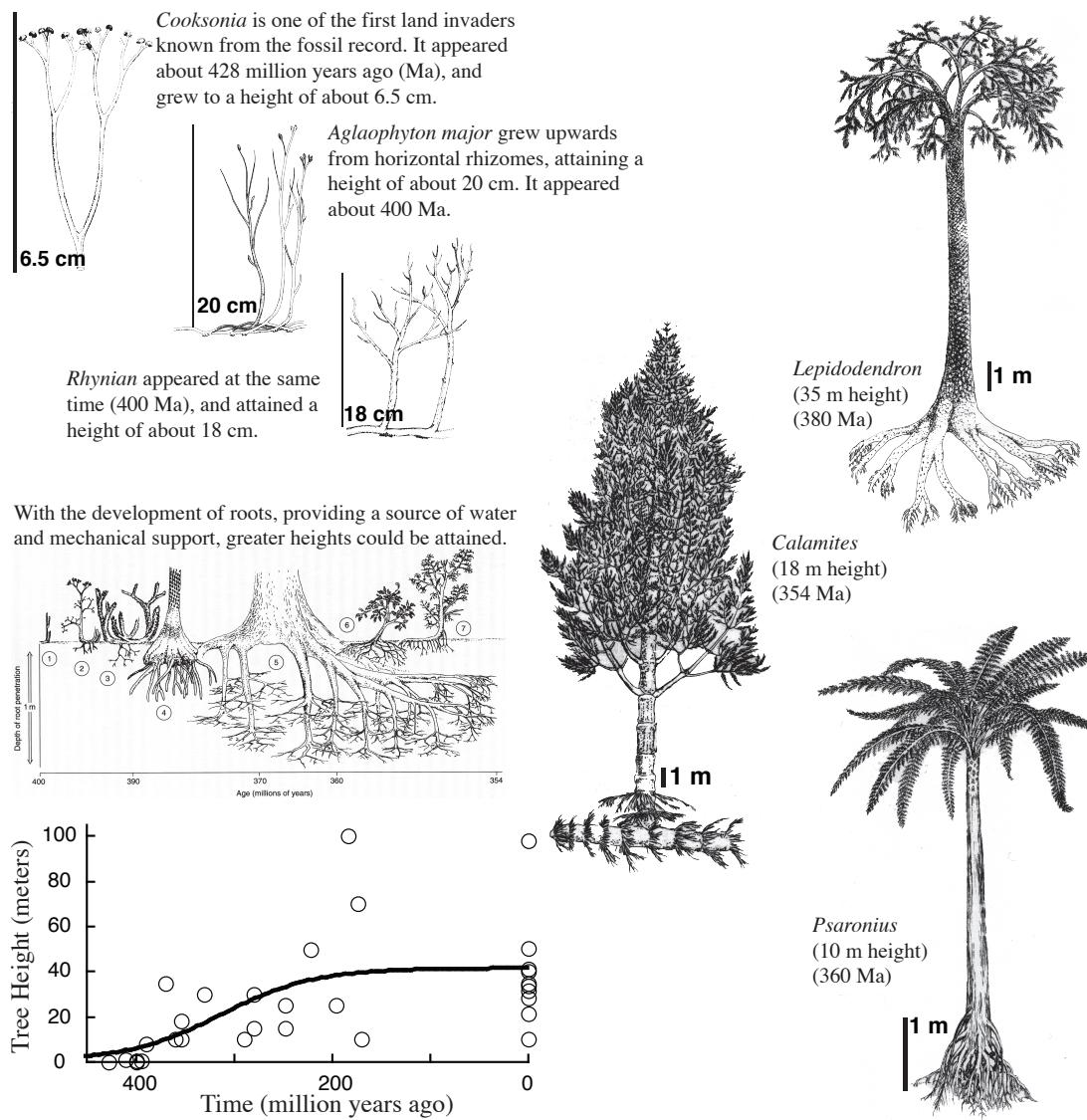
$$J_v = \left(\frac{\Delta p}{l} \right) \left(\frac{\pi}{8\eta} \right) R^4$$

Euler's column (mechanics)

$$F_{critical} = \frac{EI\pi^2}{L_{eff}^2}$$

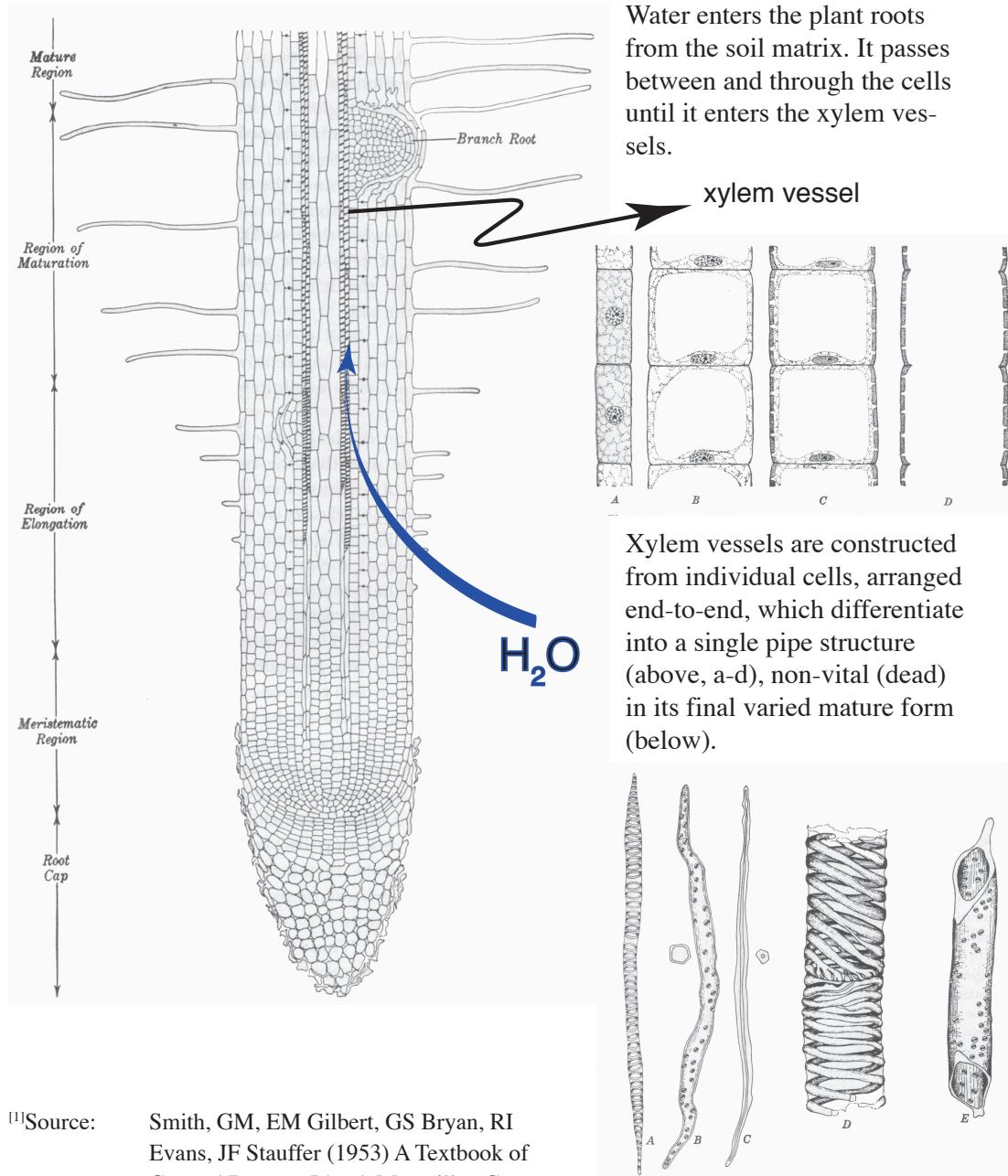
Tensile strength of water (condensed matter)

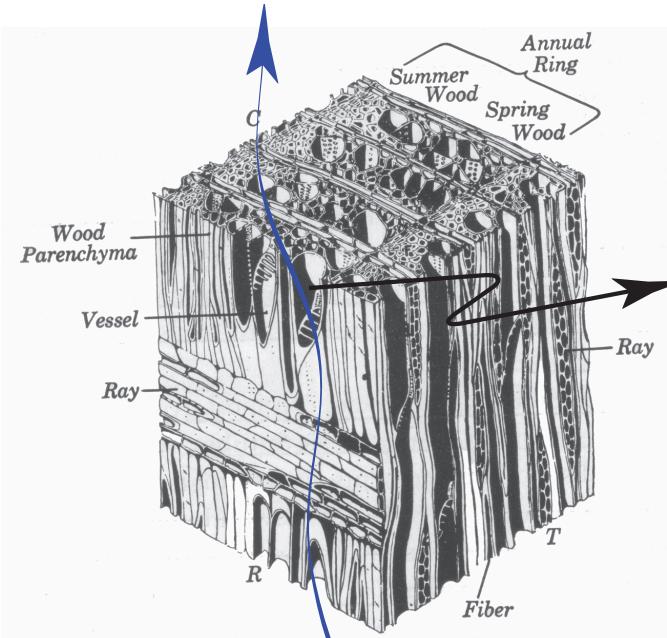
In the context of evolutionary time, the invasion of land by plants is relatively recent, only 500 million years ago. Plants had to evolve many adaptive properties to allow them to survive in a dry environment. Their life cycles were modified to protect their offspring from dessication, they developed root systems to *drink* water from the newly developed soils, they evolved an increasingly complex vasculature to move both water and nutrients long distances. And, they grew to greater and greater heights.



^[1]Source: Willis, K.J. and J.C. McElwain 2002. *The Evolution of Plants*. Oxford University Press.

Water is essential for biological life, which is why the ability to draw water to the maximal height of the tree is crucial for survival and may constrain the height of a tree. To explore the physical limits on elevating water, we must first explore the structure and function of the water transport system in a tree (or other vascular plant)^[1].

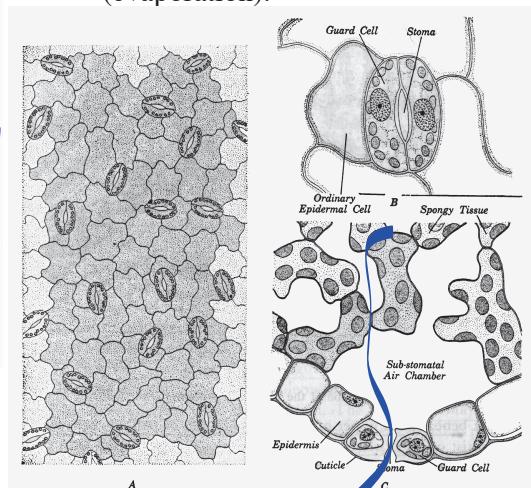
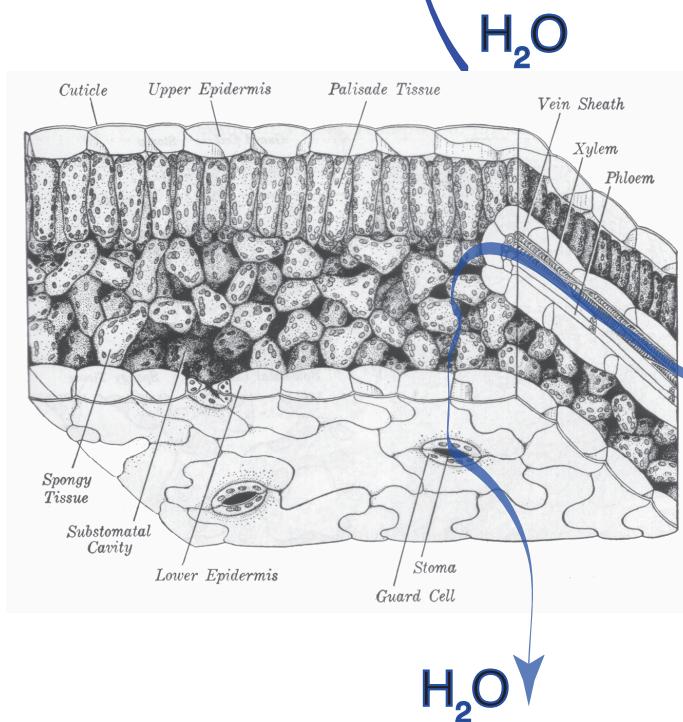




In the stem of a tree, the architecture of the vasculature can be very complex, with numerous cell types including the water-transporting xylem vessels^[11].

xylem vessel

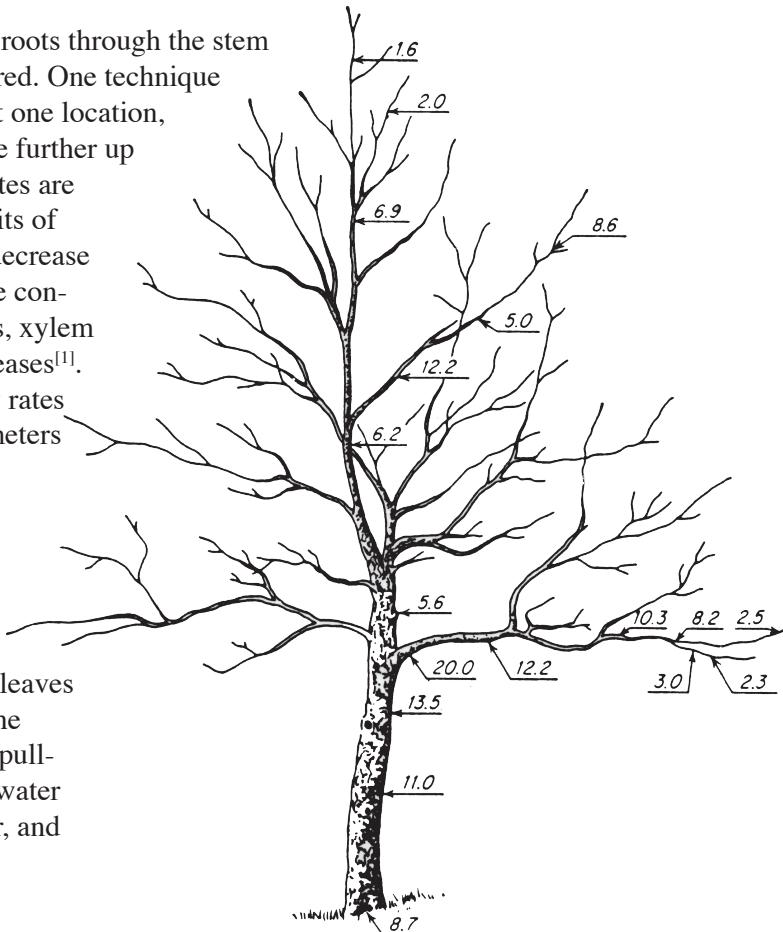
The size of the xylem vessels varies, some can be quite large, about $150 \mu\text{m}$ in diameter (range $20\text{--}300 \mu\text{m}$ in dicotyledonous trees, about $50 \mu\text{m}$ in conifers). Nevertheless, this is still a small tube in the context of hydrodynamics. In fact, microfluidic.



^[11]Source: Smith, GM, EM Gilbert, GS Bryan, RI Evans, JF Stauffer (1953) A Textbook of General Botany. 5th ed. Macmillan Co.

The flow of water from the roots through the stem and branches can be measured. One technique is to apply a pulse of heat at one location, and monitor the temperature further up the stem or branch. Flow rates are shown for an oak tree in units of meters per hour. The rates decrease towards the top, because the conducting surface ratio (that is, xylem cross area to leaf area) increases^[1]. In a variety of species, flow rates vary from about 0.1 to 60 meters per hour.

The rates of flow are strongly affected by the atmospheric relative humidity, since it is evaporation of water at the leaves that ‘pulls’ water up from the soil. The energetics of the ‘pulling’ force are given by the water potential of the water vapor, and its dependence on relative humidity^[2]:



$$\Psi_{wv} = \frac{RT}{V_w} \ln\left(\frac{\% \text{ relative humidity}}{100}\right) + \rho_w gh$$

where R is the gas constant (8.314 m³ Pa mol⁻¹ °K⁻¹), T is the temperature (°K), V_w is the partial molal volume of water (1.805•10⁻⁵

m³ mol⁻¹ at 20°C [293°K]). At 20°C, the term RT/V_w is 135 MPa. The second term is the gravitational potential: ρ_w is the density of water (998.2 kg m⁻³ at 20°C), g is the gravitational constant (9.807 m sec⁻²) and h is the height. For a tree 100 m high, ρ_wgh is 978 kPa. Even at a relatively high relative humidity (95%), the water potential is about -6 MPa, more than sufficient to ‘pull’ water from the soil, providing the flow through the xylem vessels (the hydraulic tubes of the tree) is not limiting.

^[1]Source: Kramer, PJ (1983) Water Relations of Plants. Academic Press. pp. 274-275

^[2]Source: Nobel, PS (1991) Physicochemical and Environmental Plant Physiology. Academic Press. pp. 91-95.