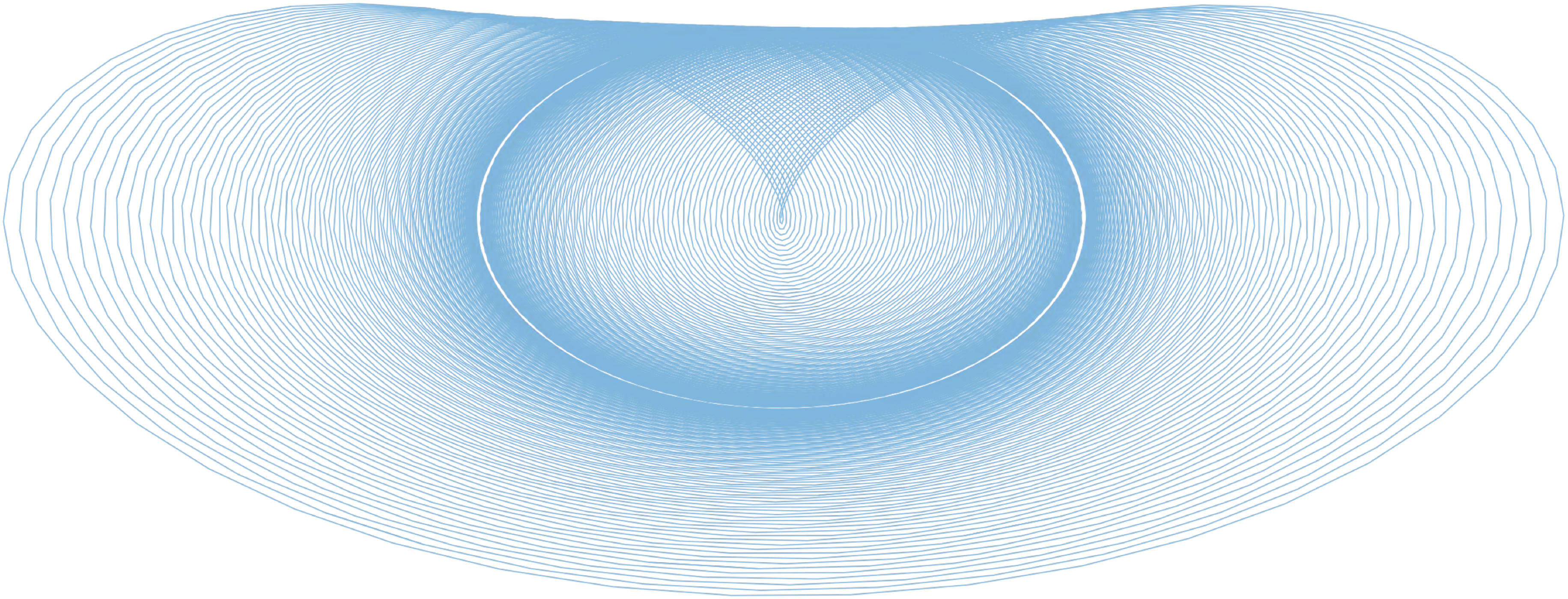
PHYS 1420 (F19)
Physics with Applications to Life Sciences



2019.09.27

Relevant reading:
Kesten & Tauck ch.4.6

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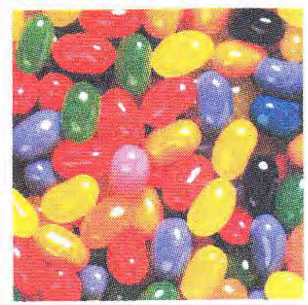
Ref. (re images):
Wolfson (2007), Knight (2017)

Piece Puzzle

Only one of these pieces fits the hole in our main picture – the others have all been altered slightly by our artist. Can you place the missing pic?

E



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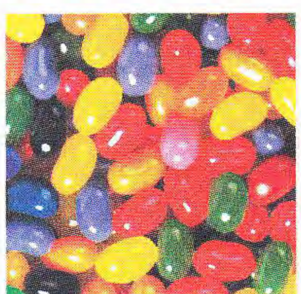
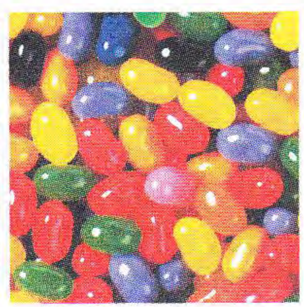


A



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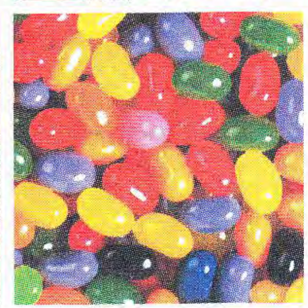


C

Announcements & Key Concepts (re Today)

→ Online HW #4 is posted and due Wednesday 10/2

→ Midterm Exam on Oct. 21

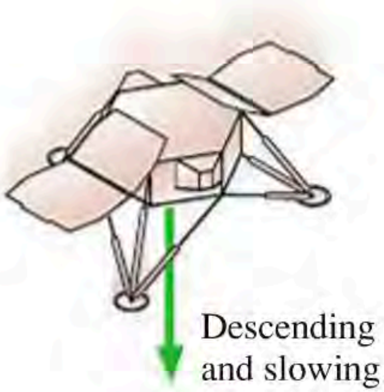
→ Read McCullough (2016) for 10/1 tutorial

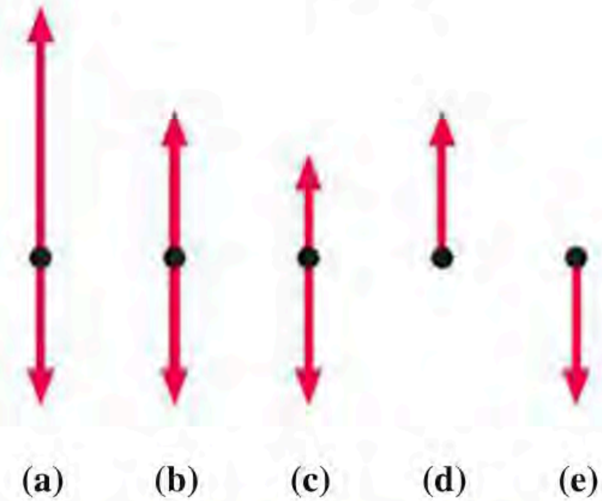
Some relevant underlying concepts of the day…

- "Easy" vs "Hard" problems….
- Newton's 3rd Law (Revisited)
- Measuring force (and biophysical applications)

Ex.

STOP TO THINK 6.1 A Martian lander is approaching the surface. It is slowing its descent by firing its rocket motor. Which is the correct free-body diagram?

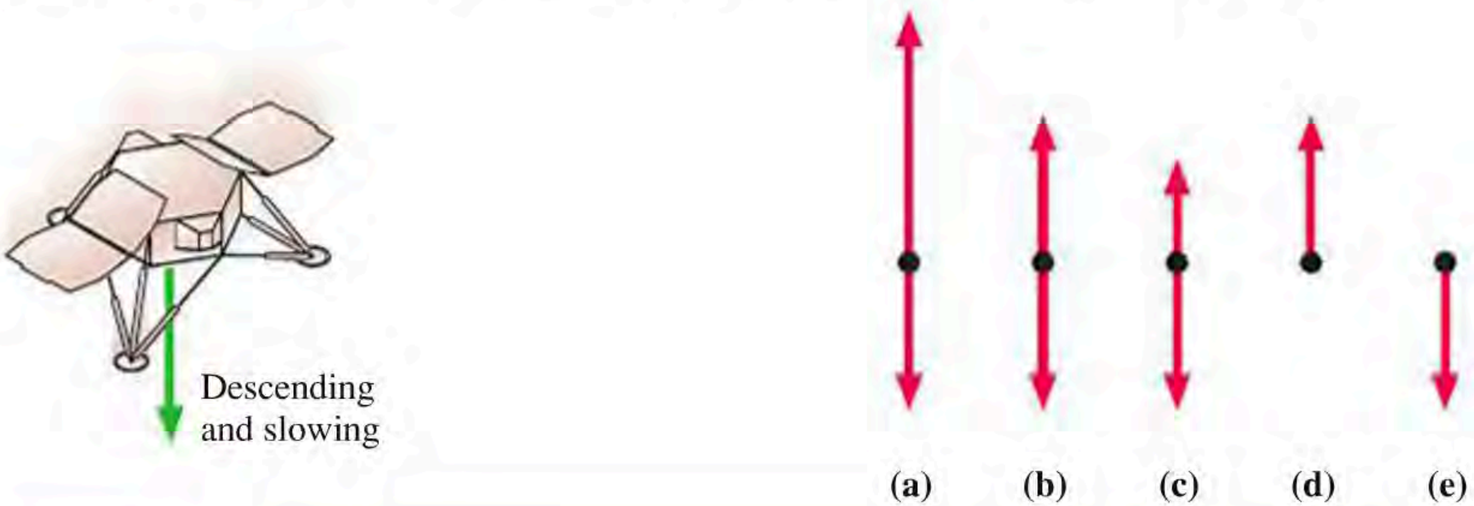




Knight (2013)

Ex. (SOL)

STOP TO THINK 6.1 A Martian lander is approaching the surface. It is slowing its descent by firing its rocket motor. Which is the correct free-body diagram?



a

Knight (2013)

Ex.

A load of mass m begins to slip without friction down the inclined face of a wedge lying on a horizontal plane surface; there is no friction either between wedge and plane. The mass of the wedge is M, the angle of inclination of the wedge's top surface with the horizontal is α. Find the acceleration of the load and of the wedge relative to the plane, the force exerted by the load on the wedge and by the wedge on the plane.

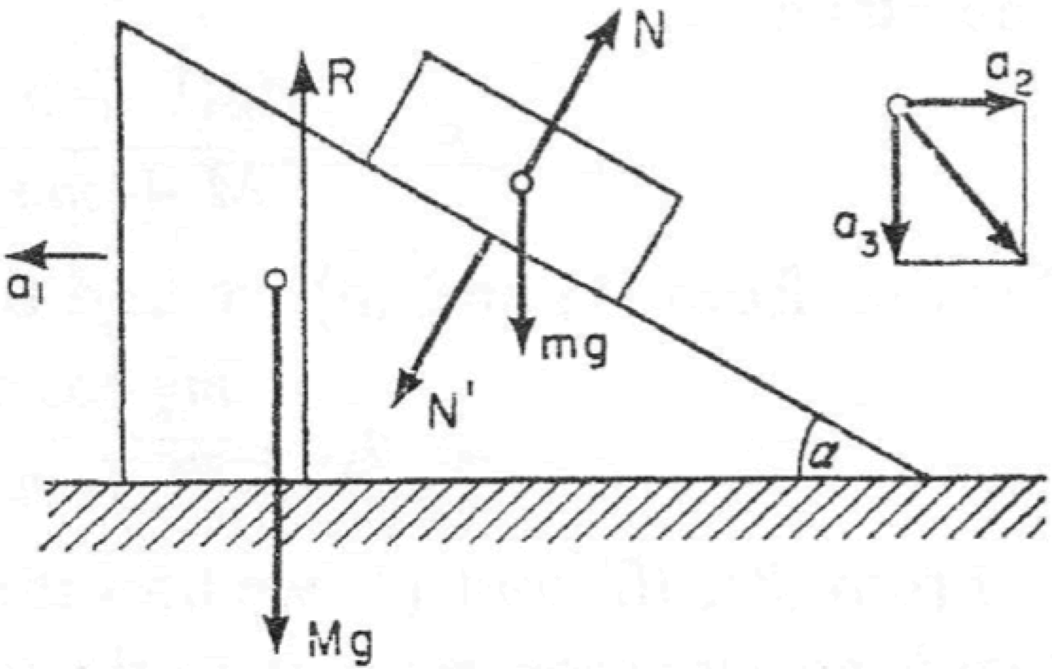
Tougher problem here since:

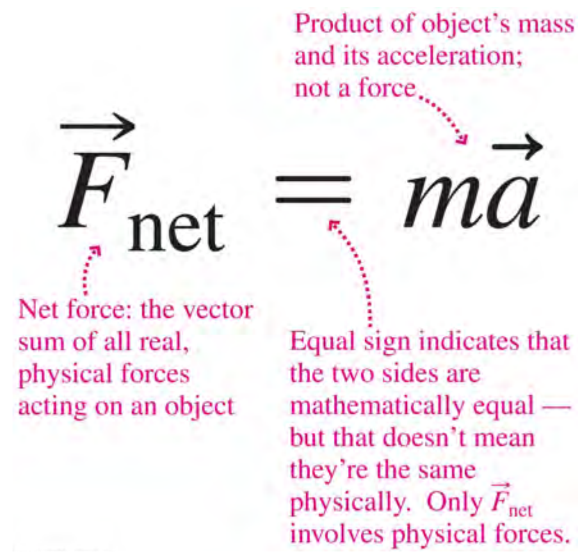
- situation is not static (as the mass on the incline moves, so will the wedge)
- there are "multiple objects" at play here

→ Putting the pieces together, realize that this is a hard problem!

Shaskol'Skaya & El'Tsin (1963)

Ex. (SOL)





To get started:

- Set up a good free-body diagram
- Remember Newton's 2nd Law!
- Be careful! [e.g., $a_3 \neq mg\cos(\alpha)$]

Ex. (SOL)

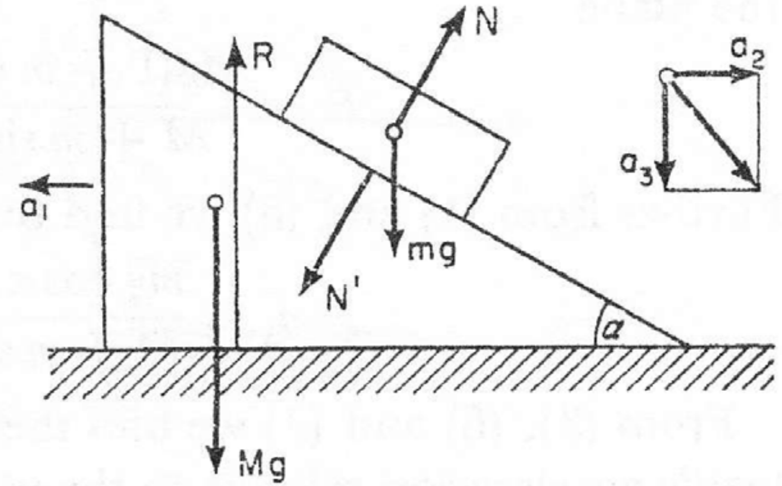


FIG. 166

Let us consider the forces acting on the load m and on the wedge M (Fig. 166). The load m is subject to: (1) its weight mg and (2) the reaction of the wedge N. The wedge is subject to (1) its own weight Mg, (2) the pressure exerted by the load N' and (3) the reaction of the plane, R. As a result of the horizontal component of the pressure exerted by the load, the wedge moves to the left relative to the plane with a horizontal acceleration a_1, which can be found from the equation

$$Ma_1 = N' \sin \alpha. \qquad (1)$$

In the vertical direction the wedge has no acceleration, therefore

$$Mg - R + N' \cos \alpha = 0 \qquad (2)$$

Shaskol'Skaya & El'Tsin (1963)

Ex. (SOL)

Let us call the horizontal component of the load's acceleration *relative to the wedge* a_2, and the vertical component a_3. Then the horizontal component of the load's acceleration *relative to the plane* will be $a_2 - a_1$, and the vertical component will be a_3. These accelerations may be found from the equations

$$m(a_2 - a_1) = N \sin\alpha \quad (3)$$

and

$$ma_3 = mg - N \cos\alpha \quad (4)$$

Plainly $N' = N$ and

$$a_3 = a_2 \tan\alpha. \quad (5)$$

From equations (4), (5), (3) and (1) we find that the pressure of the load on the wedge is

$$N = \frac{mMg\cos\alpha}{M + m\sin^2\alpha} \quad (6)$$

Now from equation (2) we can find the pressure of the wedge on the plane

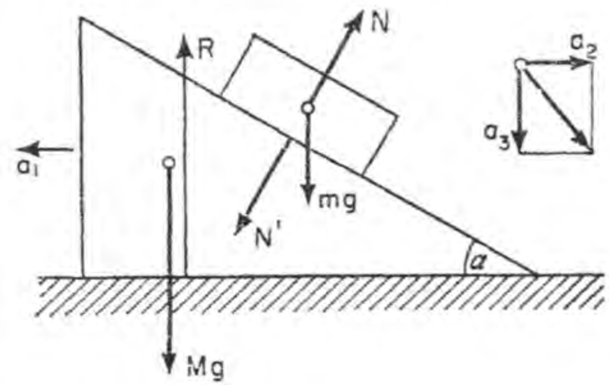
$$R = \frac{Mg(1 + m\cos^2\alpha)}{M + m\sin^2\alpha}.$$

Further from (1) and (6) we find the wedge's acceleration

$$a_1 = \frac{mg\cos\alpha\sin\alpha}{M + m\sin^2\alpha}. \quad (7)$$

$$Ma_1 = N' \sin\alpha.$$

$$Mg - R + N' \cos\alpha = 0$$



Shaskol'Skaya & El'Tsin (1963)

Ex. (SOL)

From (3), (6) and (7) we find the horizontal component of the load's acceleration relative to the wedge

$$a_2 = \frac{(M+m)\, g \cos\alpha \sin\alpha}{M + m \sin^2\alpha}, \qquad (8)$$

and the horizontal component of the load's acceleration relative to the plane

$$a_2 - a_1 = \frac{Mg \cos\alpha \sin\alpha}{M + m \sin^2\alpha}.$$

From (8) and (5) we find that the vertical component of the acceleration of the load relative to the plane

$$a_3 = \frac{(M+m)\, g \sin^2\alpha}{M + m \sin^2\alpha}.$$

Shaskol'Skaya & El'Tsin (1963)

Tips

MATHEMATICAL ASIDE Proportionality and proportional reasoning

The concept of **proportionality** arises frequently in physics. A quantity symbolized by u is *proportional* to another quantity symbolized by v if

$$u = cv$$

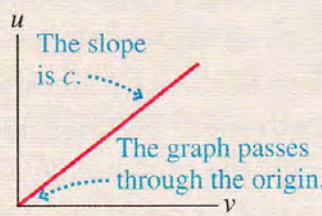
where c (which might have units) is called the **proportionality constant.** This relationship between u and v is often written

$$u \propto v$$

where the symbol $\propto$ means "is proportional to."

If v is doubled to $2v$, then u is doubled to $c(2v) = 2(cv) = 2u$. In general, if v is changed by any factor f, then u changes by the same factor. This is the essence of what we *mean* by proportionality.

A graph of u versus v is a straight line *passing through the origin* (i.e., the y-intercept is zero) with slope $= c$. Notice that proportionality is a much more specific relationship between u and v than mere linearity. The linear equation $u = cv + b$ has a straight-line graph, but it doesn't pass through the origin (unless b happens to be zero) and doubling v does not double u.



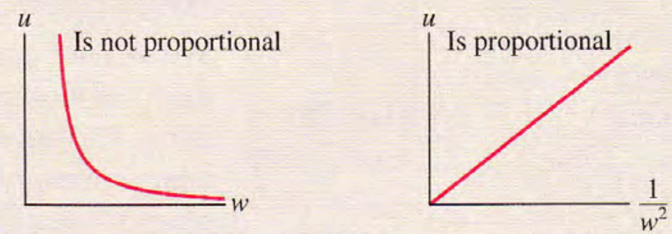
u is proportional to v.

If $u \propto v$, then $u_1 = cv_1$ and $u_2 = cv_2$. Dividing the second equation by the first, we find

$$\frac{u_2}{u_1} = \frac{v_2}{v_1}$$

By working with *ratios,* we can deduce information about u without needing to know the value of c. (This would not be true if the relationship were merely linear.) This is called **proportional reasoning.**

Proportionality is not limited to being linearly proportional. The graph on the left below shows that u is clearly not proportional to w. But a graph of u versus $1/w^2$ *is* a straight line passing through the origin, thus, in this case, u is proportional to $1/w^2$, or $u \propto 1/w^2$. We would say that "u is proportional to the inverse square of w."



u is proportional to the inverse square of w.

EXAMPLE u is proportional to the inverse square of w. By what factor does u change if w is tripled?

SOLUTION This is an opportunity for proportional reasoning; we don't need to know the proportionality constant. If u is proportional to $1/w^2$, then

$$\frac{u_2}{u_1} = \frac{1/w_2^2}{1/w_1^2} = \frac{w_1^2}{w_2^2} = \left(\frac{w_1}{w_2}\right)^2$$

Tripling w, with $w_2/w_1 = 3$, and thus $w_1/w_2 = \frac{1}{3}$, changes u to

$$u_2 = \left(\frac{w_1}{w_2}\right)^2 u_1 = \left(\frac{1}{3}\right)^2 u_1 = \frac{1}{9} u_1$$

Tripling w causes u to become $\frac{1}{9}$ of its original value.

Many *Student Workbook* and end-of-chapter homework questions will require proportional reasoning. It's an important skill to learn.

Knight (2013)

Newton's 3rd & Normal Forces

Newton's third law of motion: If object A exerts a force on object B, then object B exerts an oppositely directed force of equal magnitude on A.

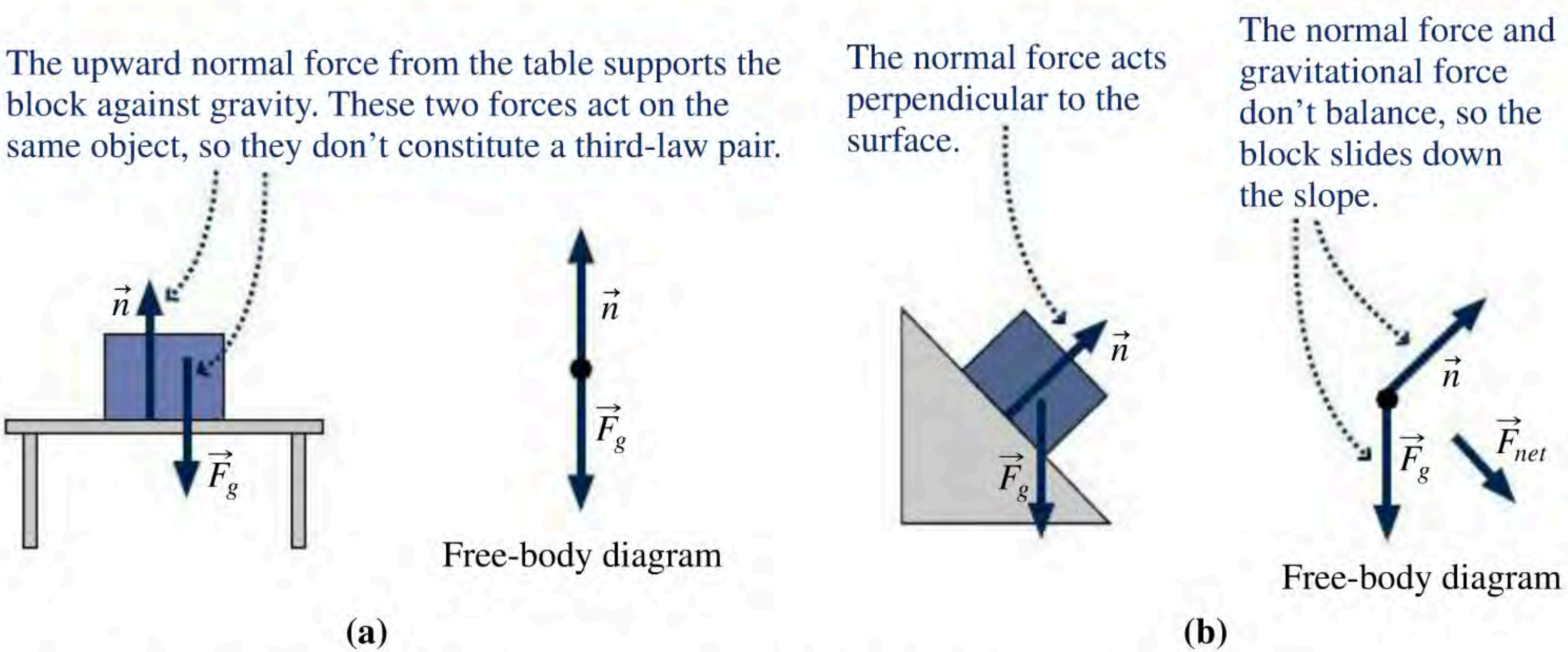


FIGURE 4.17 Normal forces. Also shown in each case is the gravitational force.

→ Explicit here is the notion of more than one object!

Note: This law (and the 1st) aren't always that intuitive (we'll return to such shortly)

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Newton's 3rd Law

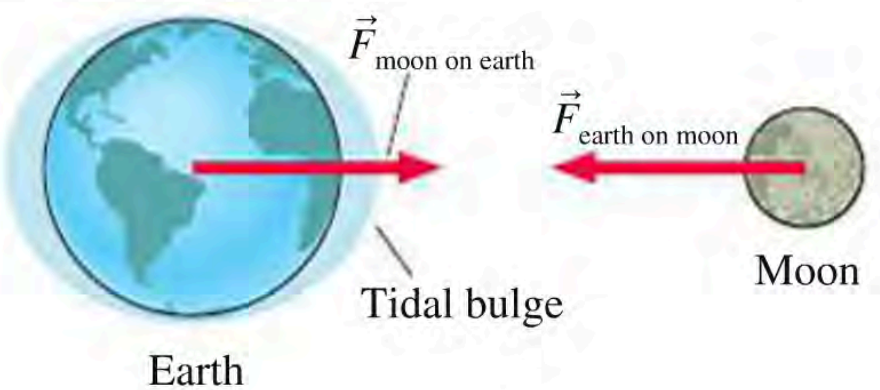
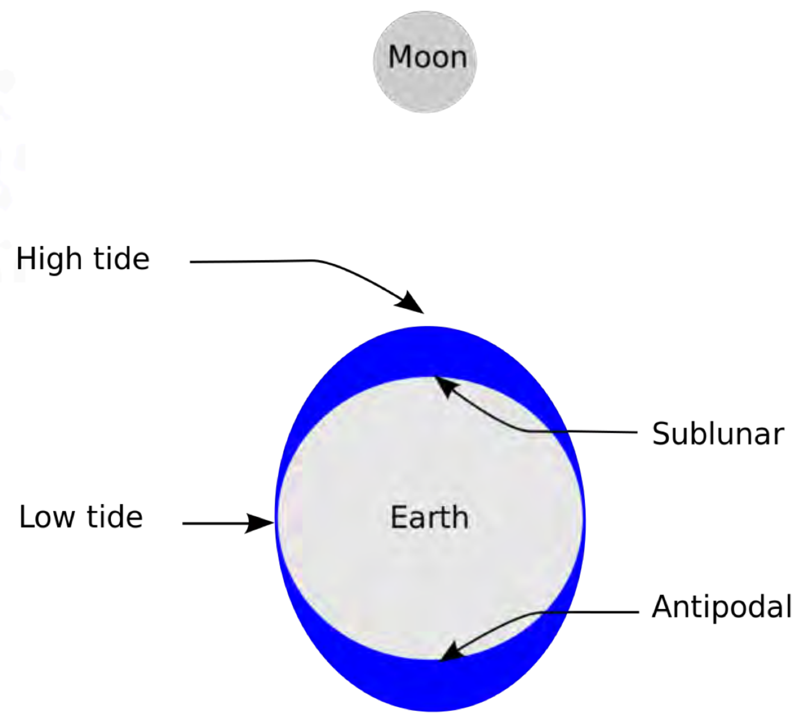


FIGURE 7.3 The ocean tides are an indication of the long-range gravitational interaction of the earth and the moon.

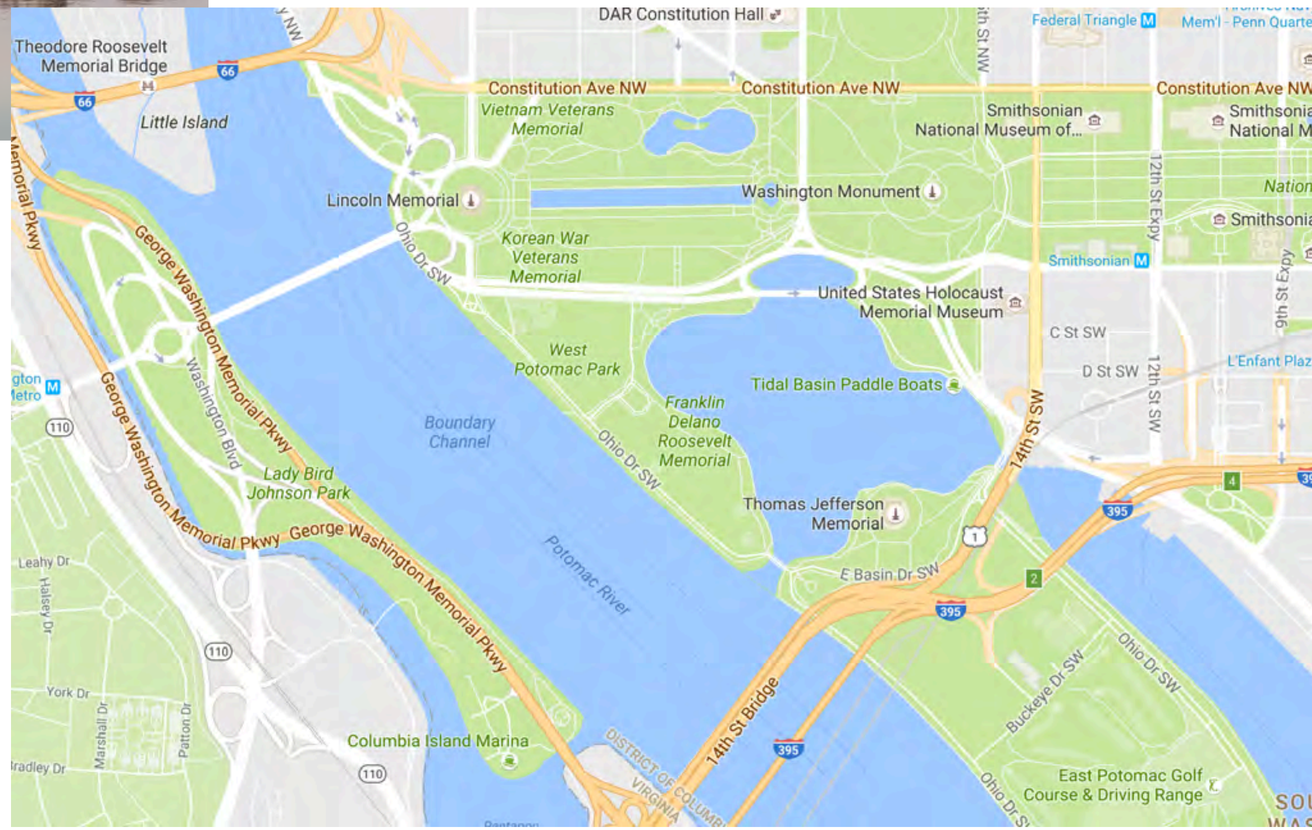


→ Moon's orbit has important sociological and ecological consequences here on earth!

https://en.wikipedia.org/wiki/Tide#/media/File:Tide_overview.svg

→ Tidal pools

Washington DC “Tidal Basin”

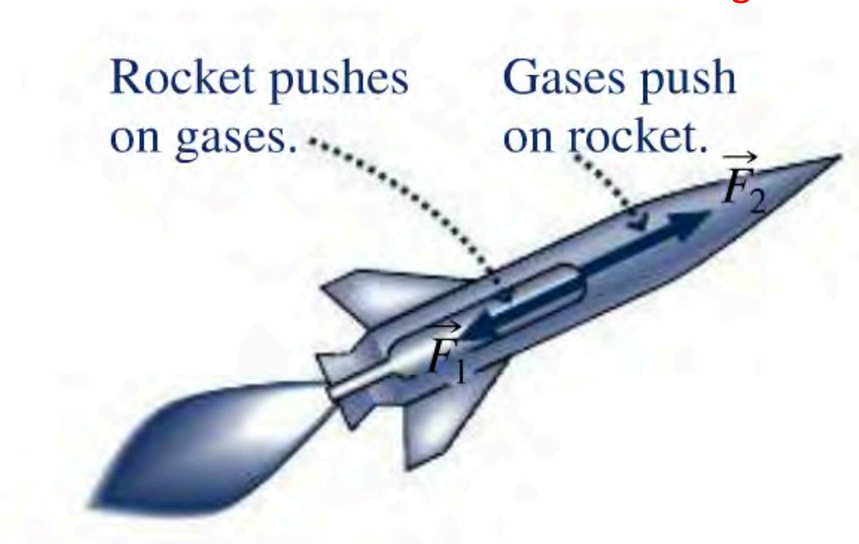


“The basin is designed to release 250 million US gallons (950,000 m^3) of water captured at high tide twice a day.”

https://en.wikipedia.org/wiki/Tidal_Basin

Newton’s 3rd Law

Fig.4.14b



“A rocket engine exerts forces that expel hot gases out of its nozzle-and the hot gases exert a force on the rocket, accelerating it forward”

→ Reduction of mass, which is exhausted (in a relatively backwards direction) plus “conservation of momentum” lead to *thrust*

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<u>Measuring Force</u>

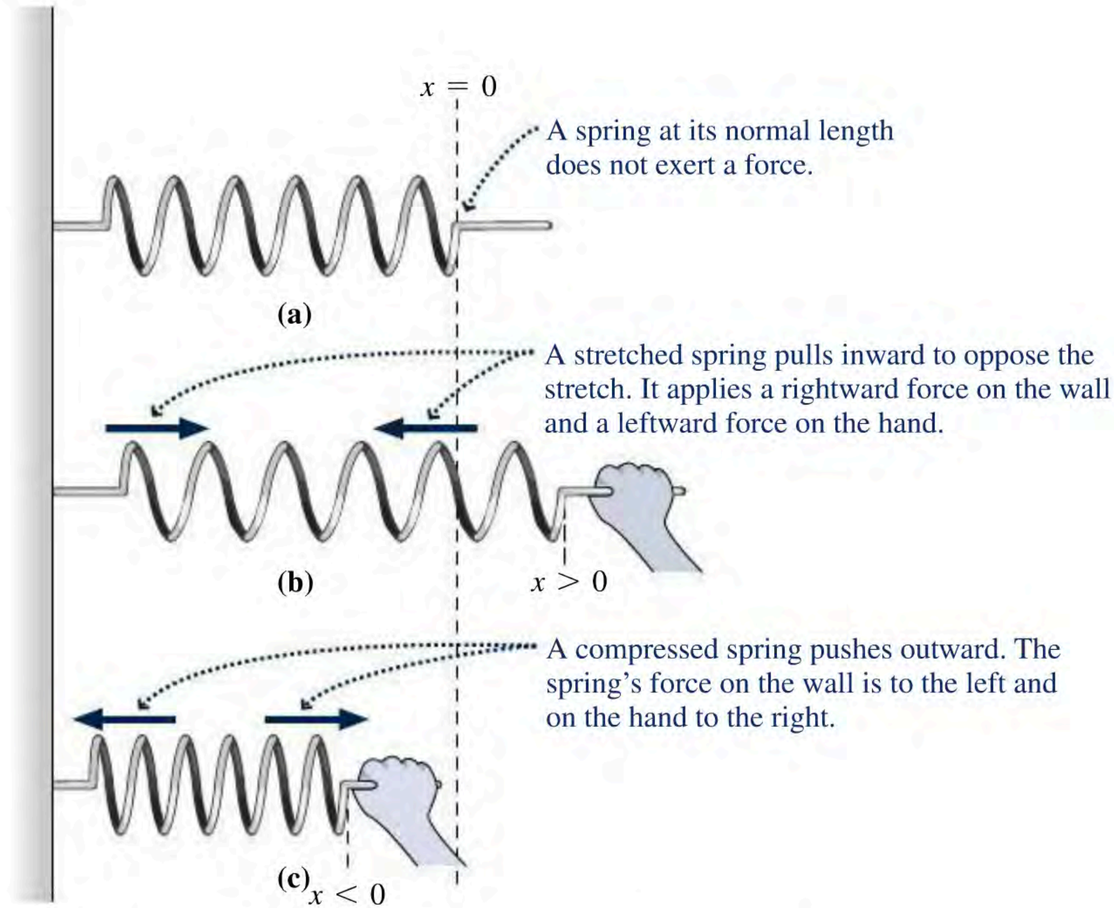

FIGURE 4.19 A spring responds to stretching or compression with an oppositely directed force.

- Springs are an intuitive means (i.e., compression/extension)

→ Implicit in this idea is the notion of **energy** (e.g., the spring stores energy as it is displaced from equilibrium). We'll revisit this idea soon...

$$F_s = -kx$$

Hooke's Law
(ideal spring)

→ k is called the *stiffness*
(or "spring constant")

- If you know k, than measuring how much compression (i.e., x) tells you something about the associated forces

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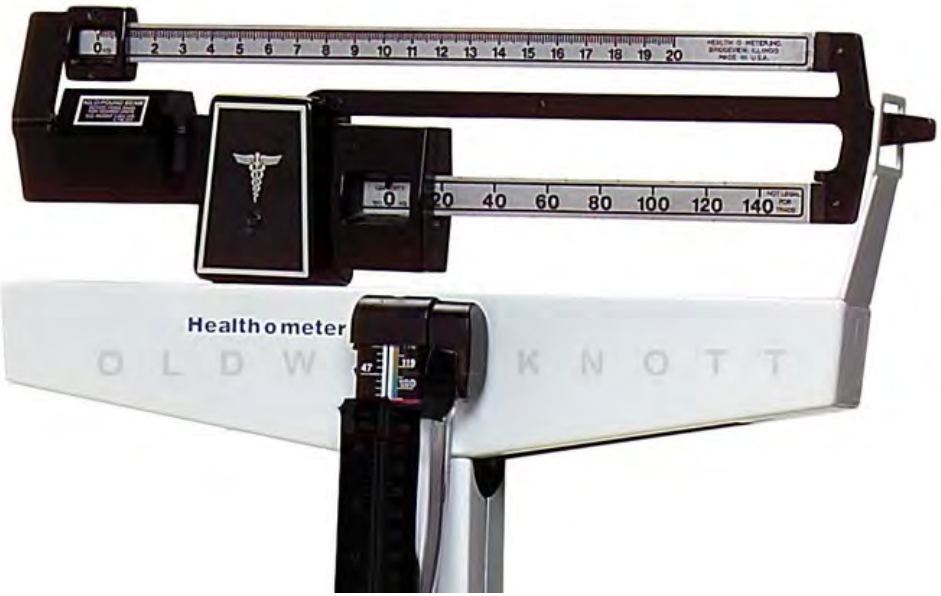
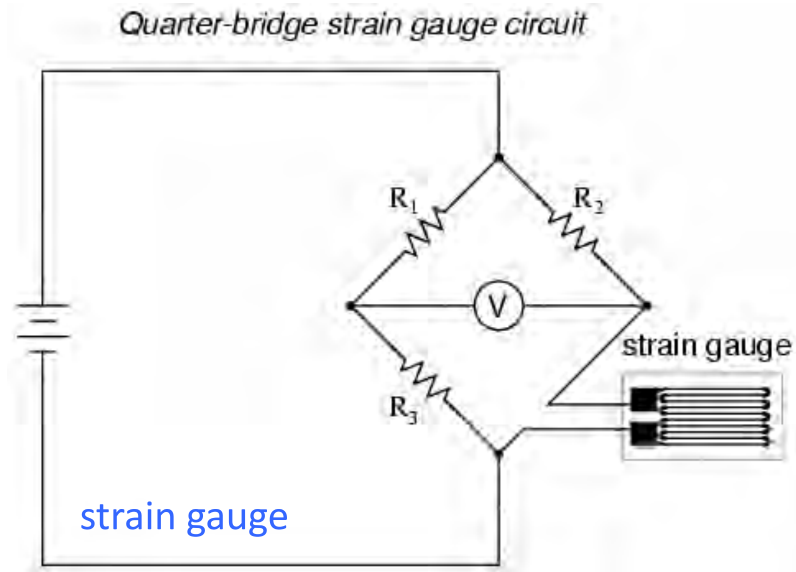
Measuring Force

This one below is a bit different in that it uses (calibrated) counterweights via a lever/balance



Measuring Force



"Engineering" aside

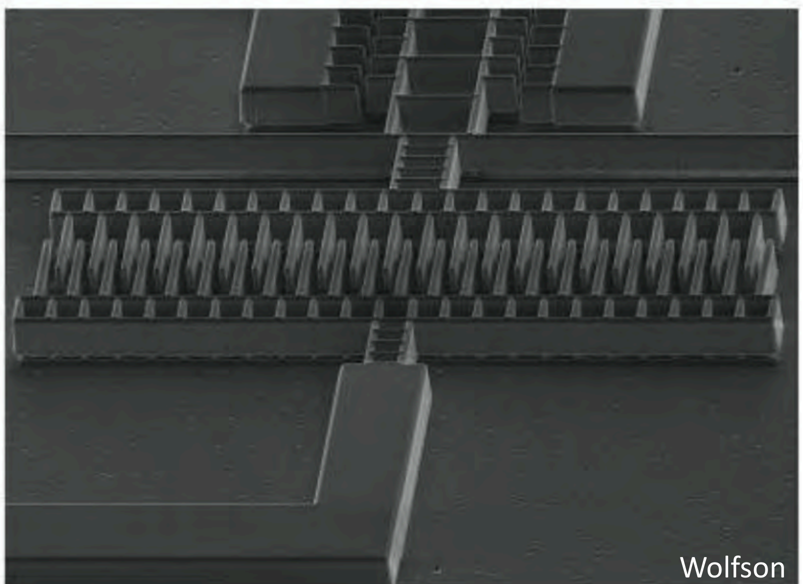
"One of the joys of writing for HowStuffWorks.com springs from encountering the surprising delicacy, beauty and complexity to be found in everyday objects. The deft engineering and intricately milled parts that go into these items are, alone, enough to inspire any technophile. Look beyond the appurtenances and appliances to the heart of any device -- particularly one used for measurement -- and you find something even more wondrous: a physical law, ingeniously harnessed to an array of specific and useful tools. Nowhere is this truer than in scales. In this article alone, I encountered Hooke's law for springs; Pascal's law for fluid pressure; Boyle's law, Charles' law and Gay-Lussac's law describing the behavior of gases; and Ohm's law for electrical resistance -- and that's to say nothing of the various unnamed laws governing stress and strain."

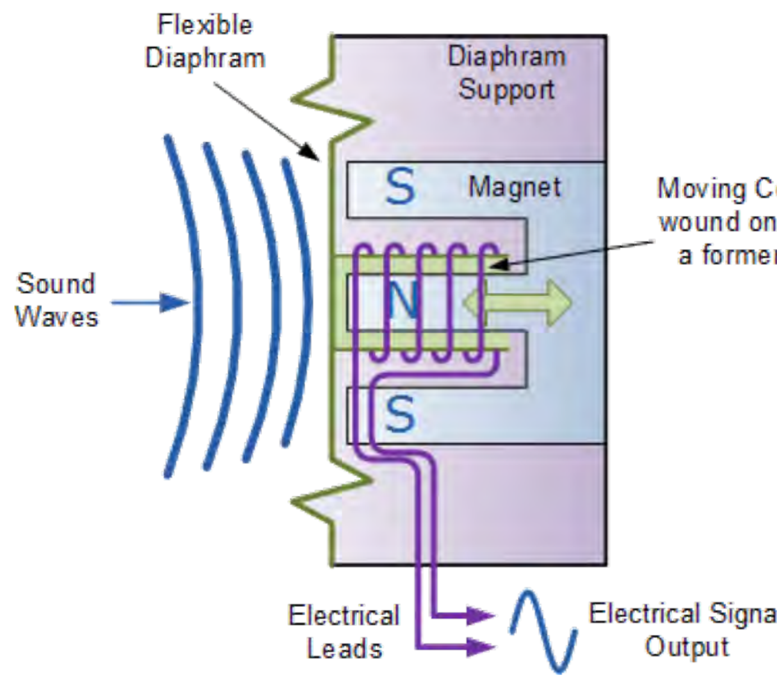
http://electronics.howstuffworks.com/gadgets/fitness/digital-scale.htm

Measuring Force

→ Notion of a "transducer"

MEMS



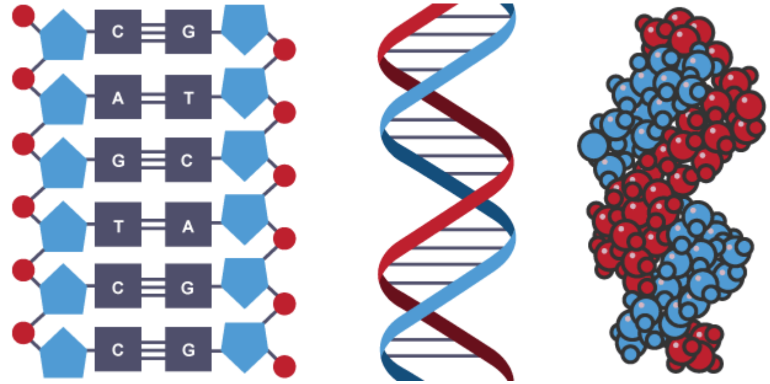


http://www.electronics-tutorials.ws/io/io_8.html

Measuring Force

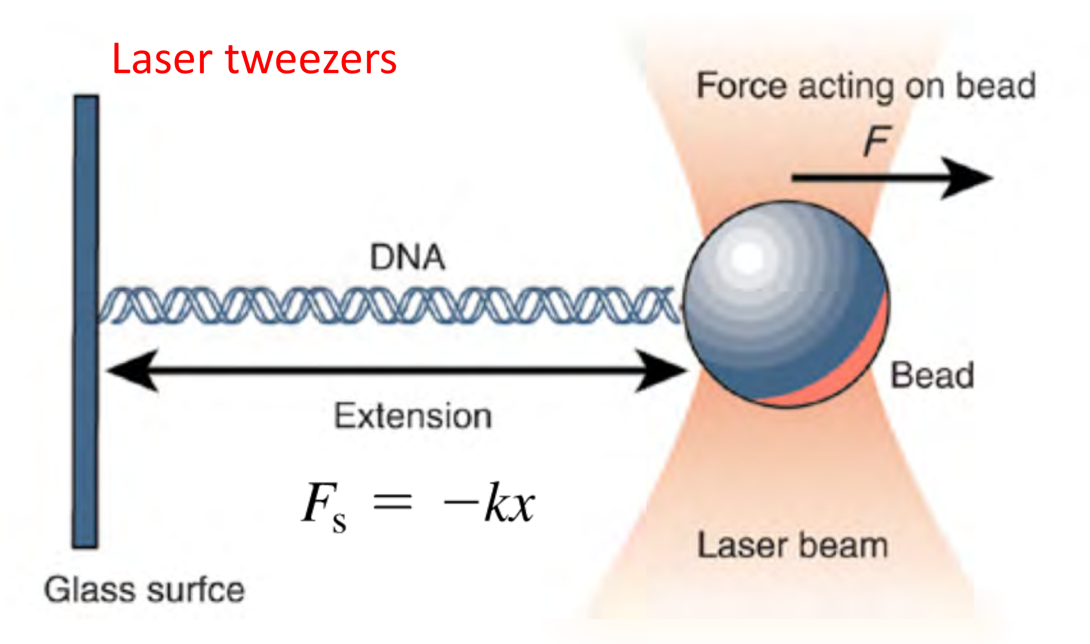
e.g., Consider: How might one measure the stiffness of DNA?

- "Springs" need not even be mechanical/electrical....



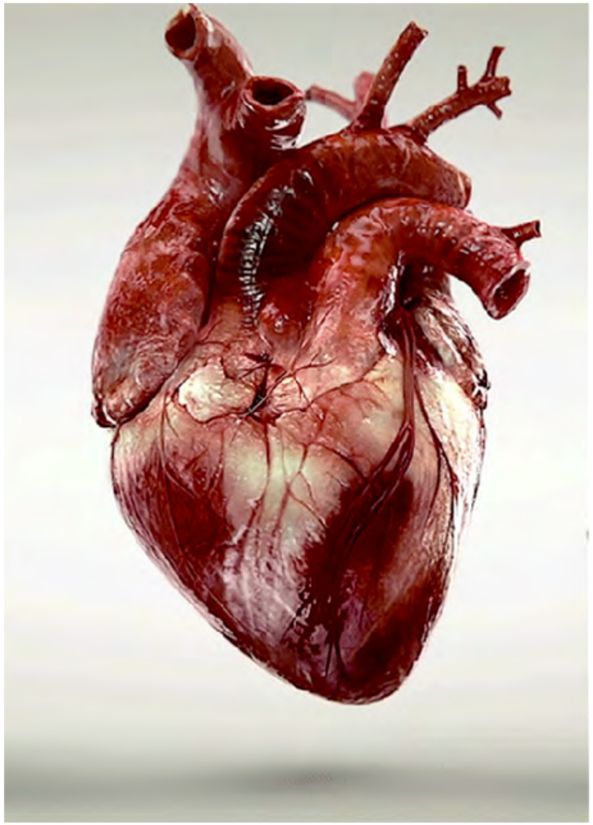
Watson & Crick

Laser tweezers



$$F_s = -kx$$

http://www.bbc.co.uk/education/guides/z36mmp3/revision
http://umdberg.pbworks.com/w/page/47555271/DNA%20spring%201%3A%20Spring%20constant%20of%20DNA



https://giphy.com/gifs/heart-beating-real-BNQlB23nW5qdq

- You don't even need "springs"!! (though calibrated measurement of force can be difficult; *impedance* is an important notion at play...)

VIDEO | SLATE IN MOTION. | SEPT. 28 2016 10:58 AM

Slate

This Device Can Predict Your Emotions Via Wi-Fi Signal

It uses your heartbeat.

By Robby Berman

22 27 5

The invention in **this video** is either very cool or very creepy, depending on your perspective. Researchers at the **Massachusetts Institute of Technology's CSAIL lab** have invented a device called "**EQ Radio**" that can, essentially, read your emotions. Specifically, it can spot joy, anger, sadness, and pleasure with 87 percent accuracy, according to CSAIL. EQ Radio doesn't need a subject's cooperation, either, since it doesn't use on-skin sensors. It uses **Wi-Fi**.

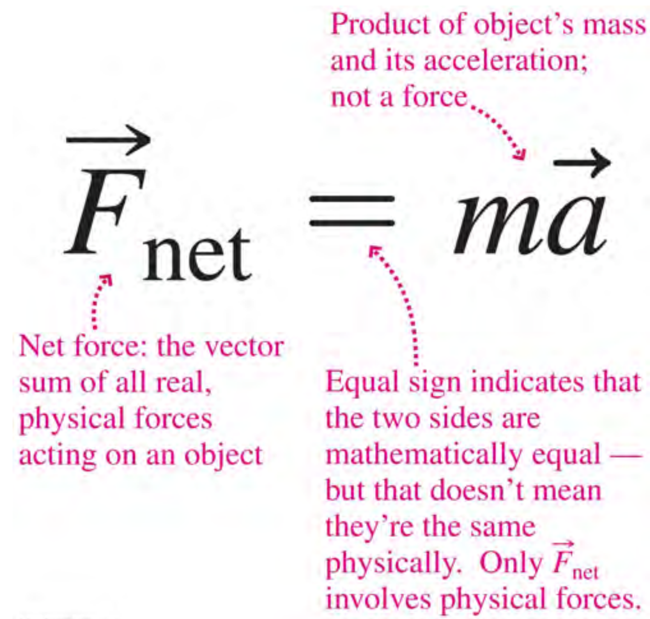
EQ Radio bounces a Wi-Fi signal off a subject and extracts the heartbeat from the returning signal using an algorithm that filters out breathing and other noise. It then analyzes the heartbeat for subtle fluctuations that ultimately give away the person's mood.

The researchers see many applications for this, of course, from more accurate film-audience surveys to advertisers' focus groups to doctors seeking a clearer view of the emotional states of patients. Police may also want to use EQ Radio during interrogations. And a smart home could adjust lighting and music to correspond to, or counteract, its owner's mood.

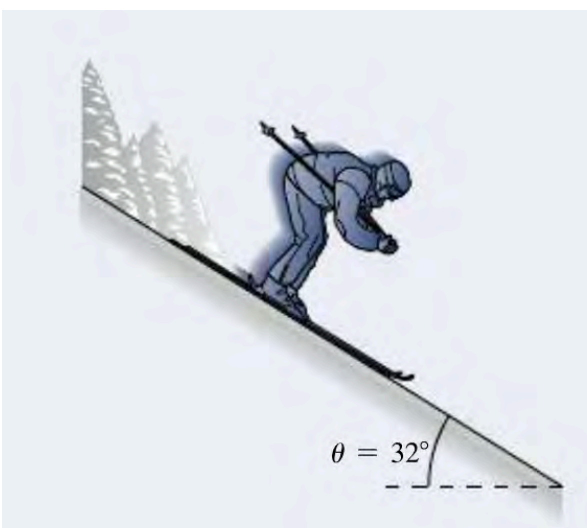
http://www.slate.com/articles/video/video/2016/09/a_new_invention_from_csail_can_read_your_hidden_emotions_wi_fi_video.html

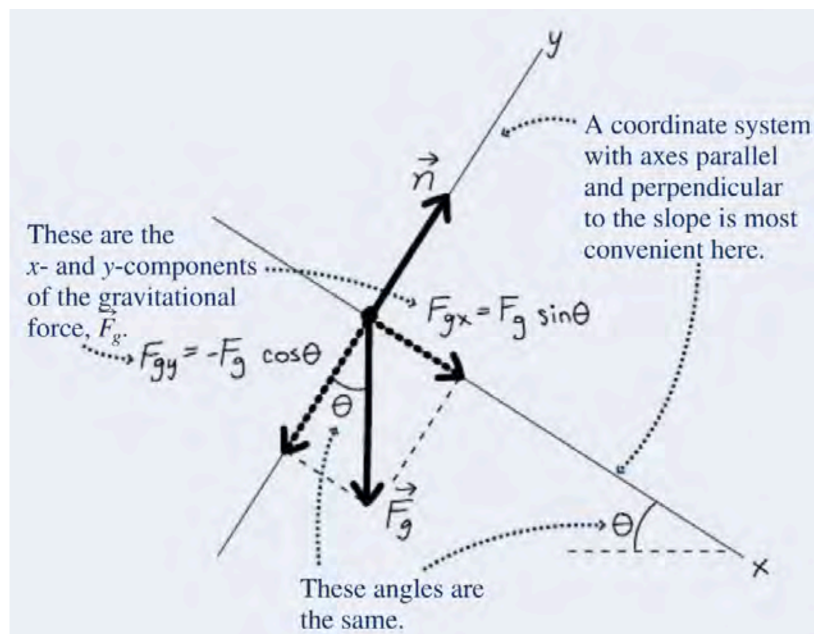
- Conceptual focus is on Newton's 2nd Law

- Best way forward is to practice!



EXAMPLE 5.1 Newton's Law in Two Dimensions: Skiing





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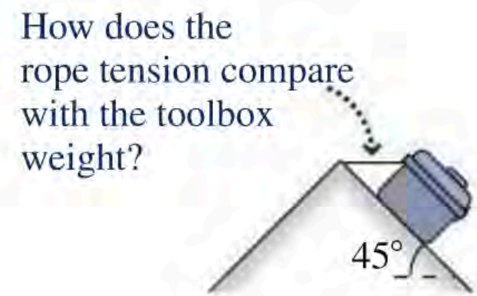
Ex.

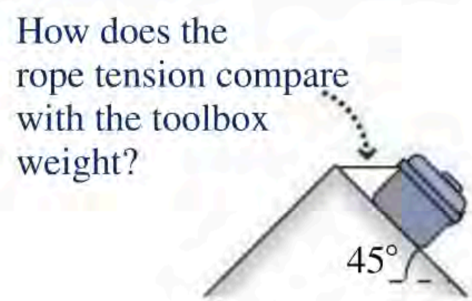
GOT IT? 5.1 A roofer's toolbox rests on an essentially frictionless metal roof with a 45° slope, secured by a horizontal rope as shown. Is the rope tension (a) greater than, (b) less than, or (c) equal to the box's weight?



Ex. (SOL)

GOT IT? 5.1 A roofer's toolbox rests on an essentially frictionless metal roof with a 45° slope, secured by a horizontal rope as shown. Is the rope tension (a) greater than, (b) less than, or (c) equal to the box's weight?

c



→ This one is not immediately intuitive per se. It's generally a good idea to draw a free-body diagram and set up the appropriate equations

EXAMPLE 5.4 Multiple Objects: Rescuing a Climber

A 73-kg climber finds himself dangling over the edge of an ice cliff, as shown in Fig. 5.7. Fortunately, he's roped to a 940-kg rock located 51 m from the edge of the cliff. Unfortunately, the ice is frictionless, and the climber accelerates downward. What's his acceleration, and how much time does he have before the rock goes over the edge? Neglect the rope's mass.

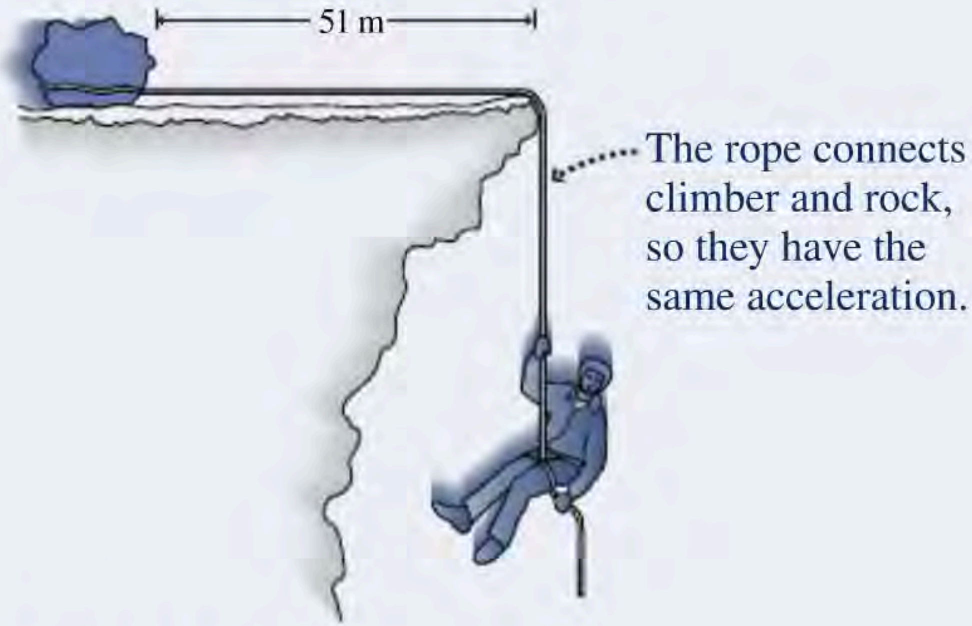
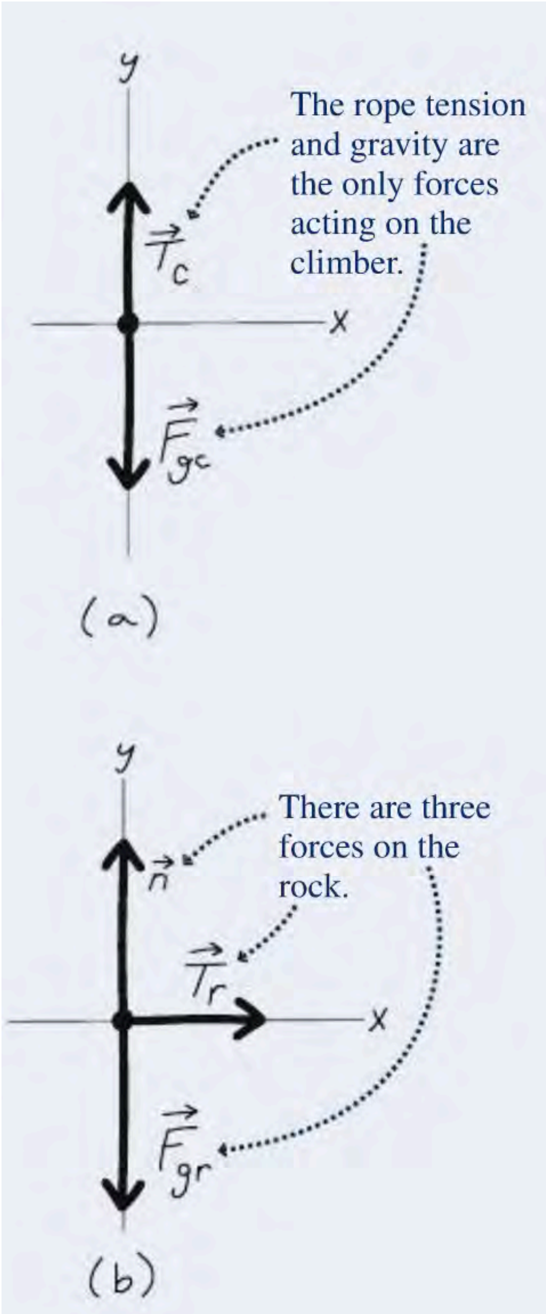


FIGURE 5.7 A climber in trouble.



$$\text{climber:} \quad \vec{T}_\text{c} + \vec{F}_\text{gc} = m_\text{c}\vec{a}_\text{c}$$
$$\text{rock:} \quad \vec{T}_\text{r} + \vec{F}_\text{gr} + \vec{n} = m_\text{r}\vec{a}_\text{r}$$

Note: These "vector equations" are essentially 2-D

$$\text{climber, } y\text{:} \quad T - m_\text{c}g = -m_\text{c}a$$
$$\text{rock, } x\text{:} \quad T = m_\text{r}a$$
$$\text{rock, } y\text{:} \quad n - m_\text{r}g = 0$$

→ Now broken up into a set(s) of 1-D eqns.

$$a = \frac{m_\text{c}g}{m_\text{c} + m_\text{r}}$$

→ This is a good problem to ensure that the "answer" makes sense (e.g., what if $m_\text{r}=0$?)

$$t = \sqrt{\frac{2x}{a}}$$

Looking ahead: What if we hadn't ignored the rope's mass?

Looking ahead: What if we hadn't ignored the rope's mass?

A related problem...

A chain of length x and mass m is hanging over the edge of a tall building and does not touch the ground. How much work is required to lift the chain to the top of the building?

To (eventually) answer this, we'll need some more pieces:

- Definition of *work*
- Integration

Ex.

A biologist is studying the growth of rats on the Space Station. To determine a rat's mass, she puts it in a 320-g cage, attaches a spring scale, and pulls so that the scale reads 0.46 N. If rat and cage accelerate at 0.40 m/s^2, what's the rat's mass?

Ex. (SOL)

INTERPRET This problem involves applying Newton's second law to find the mass of the rat, given the mass of the cage, the acceleration of the cage + rat system, and the force applied to the cage + rat system.

DEVELOP Because this is a one-dimensional, unidirectional problem, we can dispense with vector notation. In this scenario, for constant mass, Newton's second law (Equation 4.3) reads $F_{net} = ma$, where the $m = m_r + m_c$ is the mass of the rat m_r plus the mass of the cage $m_c = 0.320$ kg, and $F_{net} = 0.46$ N. Use this formula to find the mass of the cage and the rat, then take the difference to find the mass of the rat.

EVALUATE Inserting the given quantities into Newton's second law gives

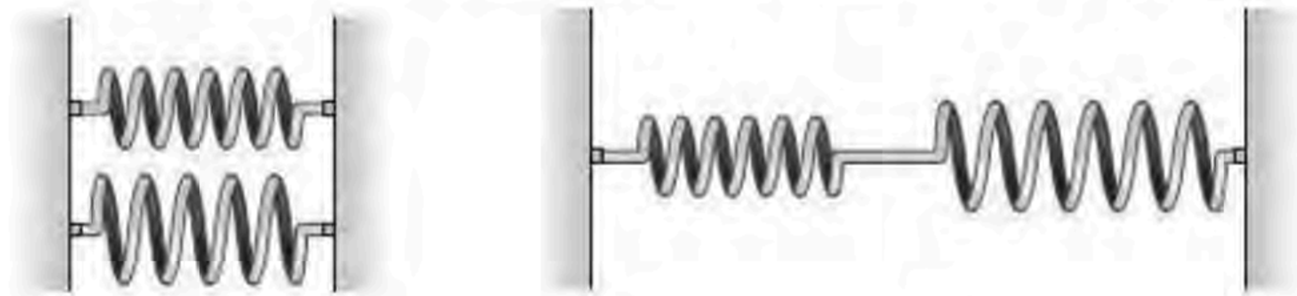
$$F_{net} = ma = (m_r + m_c)a$$

$$m_r = \frac{F_{net}}{a} - m_c = \frac{0.46\ \text{N}}{0.40\ \text{m/s}^2} - 0.32\ \text{kg} = 0.83\ \text{kg} = 830\ \text{g}$$

ASSESS A mass of 830 g corresponds to 1.8 lbs on the surface of the Earth. This is a fairly large rat!

Ex.

Two springs have the same unstretched length but different spring constants, k_1 and k_2. (a) If they're connected side by side and stretched a distance x, as shown in Fig. 4.25*a*, show that the force exerted by the combination is $(k_1 + k_2)x$. (b) If they're connected end to end (Fig. 4.25*b*) and the combination is stretched a distance x, show that they exert a force $k_1k_2x/(k_1 + k_2)$.



(a) (b)

FIGURE 4.25 Problem 62

Ex. (SOL)

INTERPRET This problem involves using Hooke's law to compute the total force exerted by two springs (of spring constants k_1 and k_2) that are connected side-by-side or end-to-end.

DEVELOP For two springs connected side-by-side (in "parallel"), $F_{\text{Tot}} = F_1 + F_2$ and $x = x_1 = x_2$ where F_{Tot} and x are the (magnitude of the) force and the stretch of the spring combination, and subscripts 1 and 2 refer to the individual springs. When the springs are connected end-to-end (in "series"), the tension is the same in both springs, so $F_{\text{Tot}} = F_1 = F_2$ (true for "massless" springs), whereas the total stretch of the two springs is the sum of the stretch of each individual spring; $x = x_1 + x_2$.

EVALUATE **(a)** For the "parallel" combination, Hooke's law gives $F_1 = k_1 x_1$ and $F_2 = k_2 x_2$. Therefore, the total force is

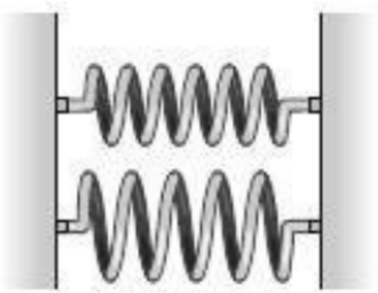
$$F_{\text{Tot}} = k_1 x_1 + k_2 x_2 = (k_1 + k_2)x.$$

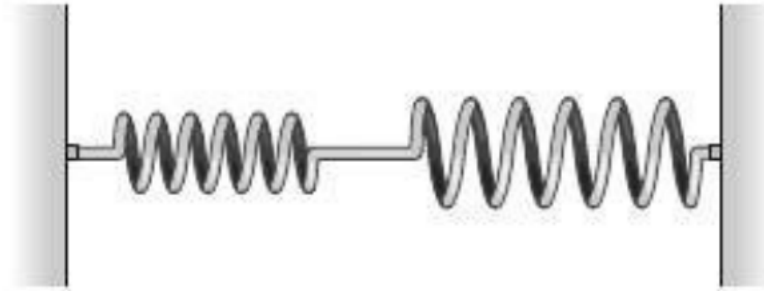
(b) For the "series" combination, Hooke's law gives

$$x = x_1 + x_2 = \frac{F_1}{k_1} + \frac{F_2}{k_2} = F_{\text{Tot}}\left(\frac{1}{k_1} + \frac{1}{k_2}\right) = F_{\text{Tot}}\left(\frac{k_1 + k_2}{k_1 k_2}\right)$$

$$F_{\text{Tot}} = \left(\frac{k_1 k_2}{k_1 + k_2}\right)x$$

ASSESS For a system with many springs, we may define an effective spring constant as $k_{\text{eff}} = F_{\text{Tot}}/x$. In the parallel case, we have $k_p = k_1 + k_2$, whereas in the series case, $k_s = k_1 k_2/(k_1 + k_2)$. Common experience tells us that the parallel combination is stiffer than the series combination, and thus requires a greater amount of force to stretch by the same amount. One can readily see this by considering the simple case where $k_1 = k_2 = k$. The above formulae give $k_p = 2k$ and $k_s = k/2$.





→ This problem is very closely related to that of capacitors in parallel/series (we'll eventually get there via electric circuits)

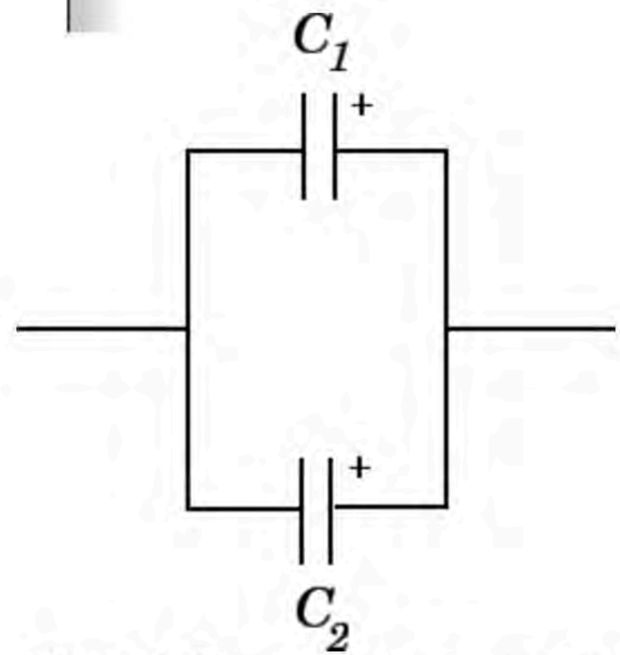


Figure 15: *Two capacitors connected in parallel.*

http://farside.ph.utexas.edu/teaching/302l/lectures/node46.html

Figure 5-5a shows a weight W hung by strings. Consider the knot at the junction of the three strings to be "the body." The body remains at rest under the action of the three forces shown in Fig. 5–5b. Suppose we are given the magnitude of one of these forces. How can we find the magnitude of the other forces?

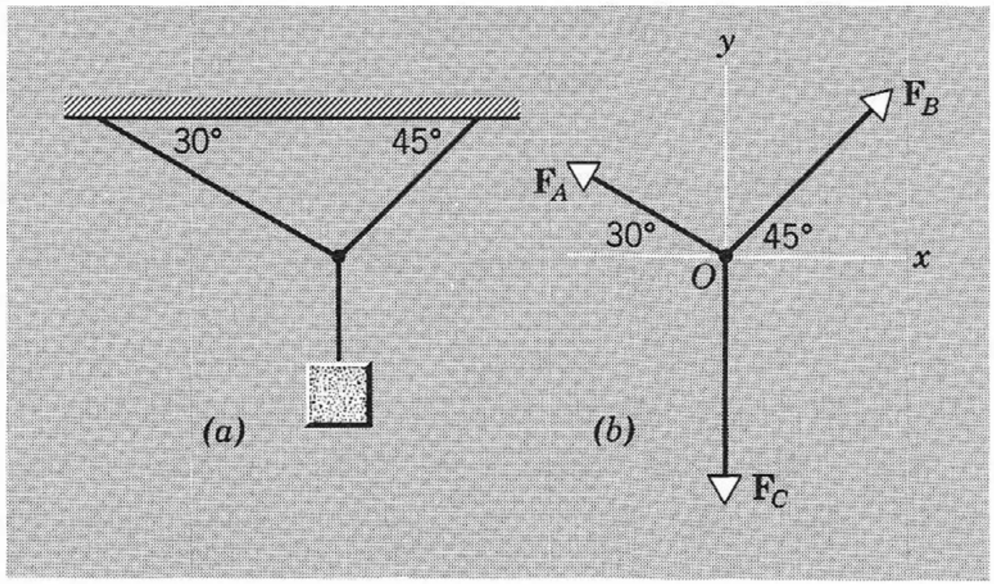


Fig. 5–5 Example 3. (a) A mass is suspended by strings. (b) A free-body diagram showing all the forces acting on the knot. The strings are assumed to be weightless.

Resnick & Halliday (1966)

Ex.

Can a man, standing against a wall so that his right shoulder and right leg are in contact with the wall (Fig. 27), raise his left leg and in so doing not lose equilibrium?

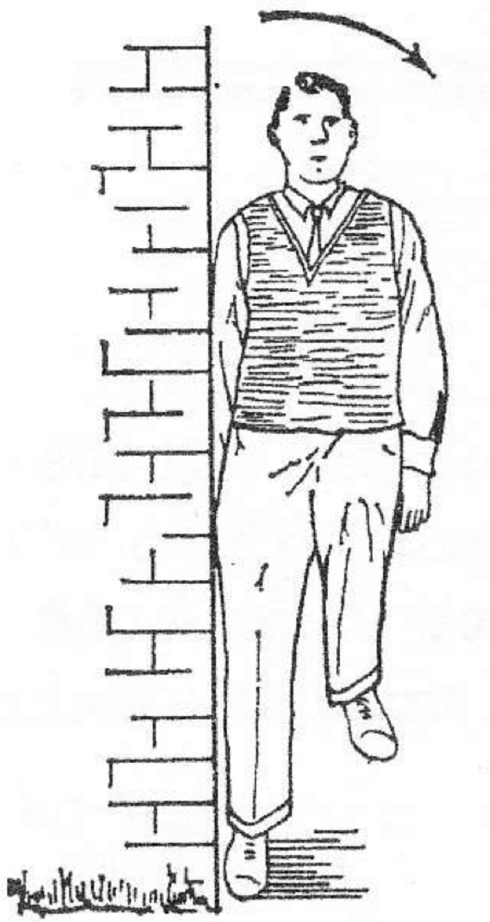


FIG. 27

Shaskol'Skaya & El'Tsin (1963)

Ex. (SOL)

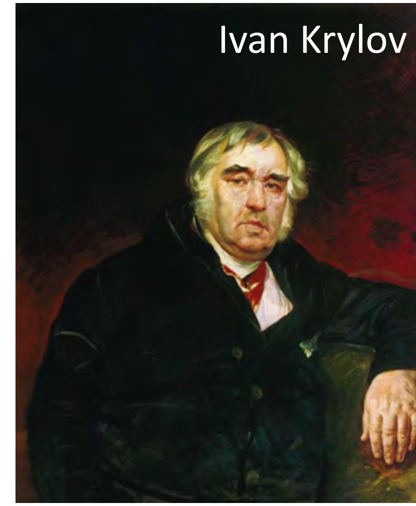
No. A man can raise his left leg and not lose balance only in the case when the vertical line passing through his centre of gravity passes also through the sole of his right foot. In the position described, this cannot be so.

→ Draw a free-body diagram to convince yourself this is the case!

Shaskol'Skaya & El'Tsin (1963)

Ex.

In what cases could the heroes of Krylov's famous fable, the swan, the pike and the crab, not have moved the cart in fact, assuming that they are all of equal strength and that there is no friction between cart and ground?*



* In this fable, a swan, a pike and a crab pull at a cart in three opposing directions. The cart does not move.

Shaskol'Skaya & El'Tsin (1963)

http://www.bartleby.com/380/poem/885.html

Ex. (SOL)

1st case: all three equal forces act in the same plane and make an angle of 120° with each other, their resultant equalling zero (Fig. 173a). 2nd case: the pike and the crab pull in directly opposite directions while the swan pulls vertically upwards (Fig. 173b); then the swan's strength would be less than the weight of the cart (though the last condition is not given in the fable).

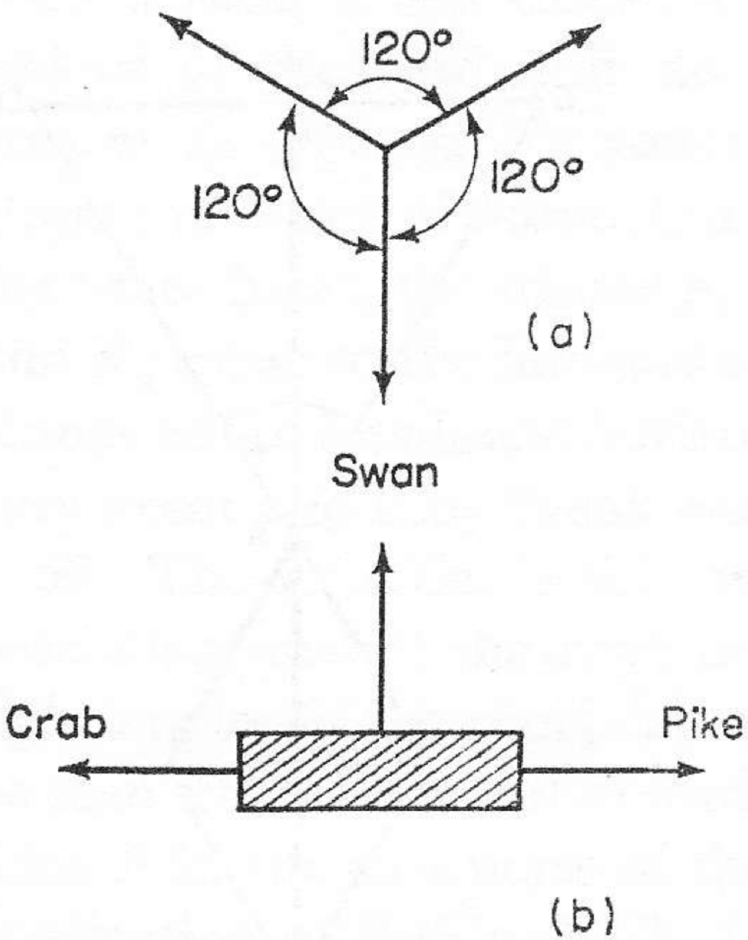


FIG. 173

Shaskol'Skaya & El'Tsin (1963)

Ex.

A lamp hangs from a bracket whose three arms each have one end fixed in the wall, the other ends meeting at a point. The two upper arms form an isosceles triangle with an angle of 60° between the arms. The plane of this triangle is at right angles to the third arm, which makes an angle of 30° with the wall. The bulb and shade weigh 1 kg. Find the stresses in the arms (Fig. 28).

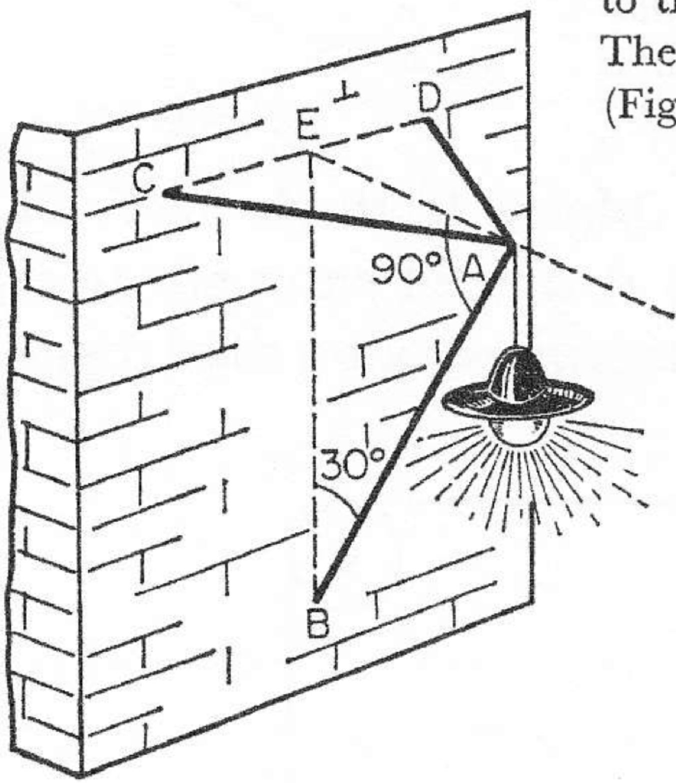


FIG. 28

Shaskol'Skaya & El'Tsin (1963)

Ex.

A load is attached to two strings *AB* and *AC* of equal length and suspended from them (Fig. 29). In what case will the strings break most easily, when they hang down almost vertically, or when they are stretched almost horizontally? Neglect the weight of the strings.

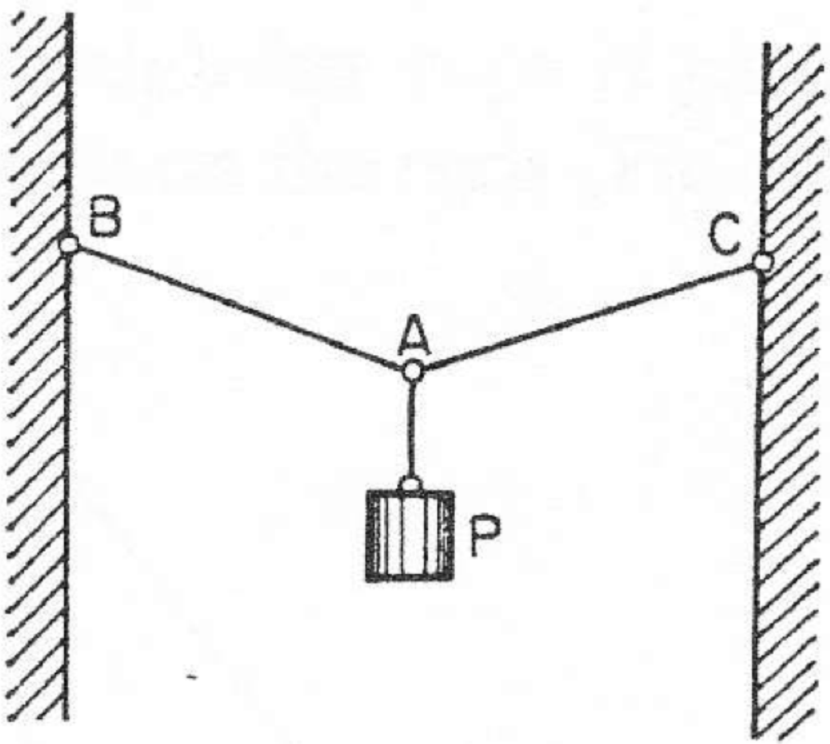


FIG. 29

Shaskol'Skaya & El'Tsin (1963)

Ex.

We wish to analyze the motion of a block on a smooth incline.

(*a*) *Static case.* Figure 5–6*a* shows a block of mass m kept at rest on a smooth plane, inclined at an angle θ with the horizontal, by means of a string attached to the vertical wall. The forces acting *on* the block are shown in Fig. 5–6*b*. $\mathbf{F}_1$ is the force exerted *on* the block by the string; $m\mathbf{g}$ is the force exerted *on* the block by the earth, that is, its weight; and $\mathbf{F}_2$ is the force exerted *on* the block by the inclined surface. $\mathbf{F}_2$, called the normal force, is normal to the surface of contact because there is no frictional force between the surfaces.* If there were a frictional force, $\mathbf{F}_2$ would have a component parallel to the incline. Because we wish to analyze the motion of *the block*, we choose *ALL the forces acting ON the block*. The student will note that the block will exert forces on other bodies in its environment (the string, the earth, the surface of the incline) in accordance with the action-reaction principle; these forces, however, are not needed to determine the motion of the block because they do not act on the block.

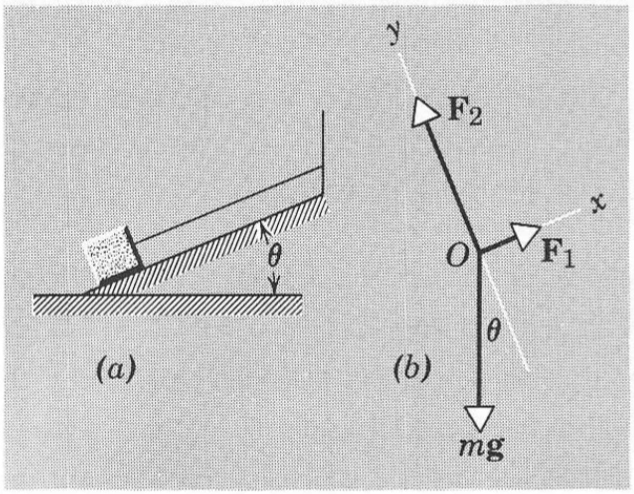


Fig. 5–6 Example 4. (*a*) A block is held on a smooth inclined plane by a string. (*b*) A free-body diagram showing all the forces acting on the block.

Suppose θ and m are given. How do we find F_1 and F_2? Since the block is unaccelerated, we obtain

$$\mathbf{F}_1 + \mathbf{F}_2 + m\mathbf{g} = 0.$$

It is convenient to choose the x-axis of our reference frame to be along the incline and the y-axis to be normal to the incline (Fig. 5–6*b*). With this choice of coordinates, only one force, $m\mathbf{g}$, must be resolved into components in solving the problem. The two scalar equations obtained by resolving $m\mathbf{g}$ along the x- and y-axes are

$$F_1 - mg \sin\theta = 0, \quad \text{and} \quad F_2 - mg\cos\theta = 0,$$

from which F_1 and F_2 can be obtained if θ and m are given.

(*b*) *Dynamic case.* Now suppose that the string is cut. Then the force $\mathbf{F}_1$, the pull of the string on the block, will be removed. The resultant force on the block will no longer be zero, and the block will accelerate. What is its acceleration?

From Eq. 5–2 we have $F_x = ma_x$ and $F_y = ma_y$. Using these relations we obtain

$$F_2 - mg\cos\theta = ma_y = 0,$$

and

$$-mg\sin\theta = ma_x,$$

which yield

$$a_y = 0, \quad a_x = -g\sin\theta.$$

The acceleration is directed down the incline with a magnitude of $g \sin\theta$.

Resnick & Halliday (1966)

Ex.

Consider a block of mass m pulled along a smooth horizontal surface by a horizontal force **P**, as shown in Fig. 5–7. **N** is the normal force exerted on the block by the frictionless surface and **W** is the weight of the block.

(*a*) If the block has a mass of 2.0 kg, what is the normal force?

From the second law of motion with $a_y = 0$, we obtain

$$F_y = ma_y \quad \text{or} \quad N - W = 0.$$

Hence, $N = W = mg = (2.0 \text{ kg})(9.8 \text{ meters/sec}^2) = 20$ nt.

(*b*) What force P is required to give the block a horizontal velocity of 4.0 meters/sec in 2.0 sec starting from rest?

The acceleration a_x follows from

$$a_x = \frac{v_x - v_{x0}}{t} = \frac{4.0 \text{ meters/sec} - 0}{2.0 \text{ sec}} = 2.0 \text{ meters/sec}^2.$$

From the second law, $F_x = ma_x$ or $P = ma_x$. The force P is then

$$P = ma_x = (2.0 \text{ kg})(2.0 \text{ meters/sec}^2) = 4.0 \text{ nt}.$$

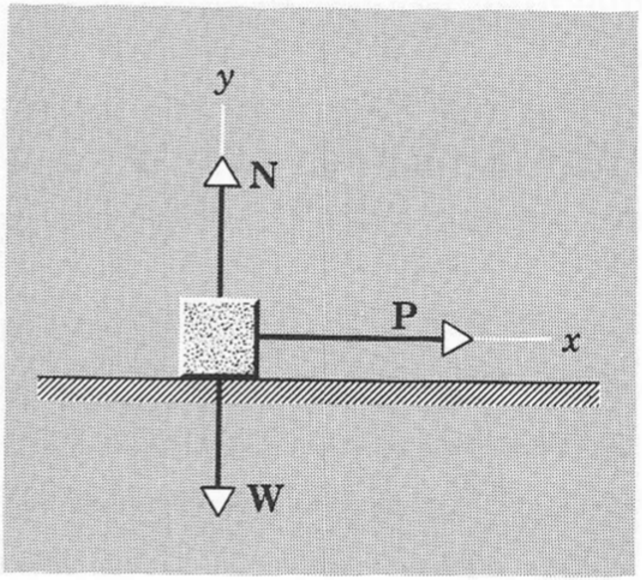


Fig. 5–7 Example 5. A block is being pulled along a smooth table. The forces acting on the block are shown.

Resnick & Halliday (1966)

Ex.

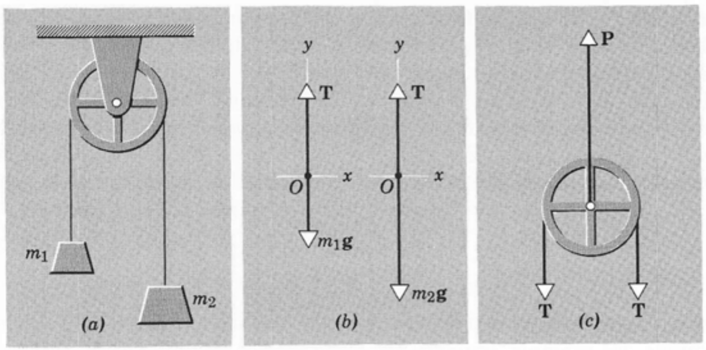


Fig. 5–9 Example 7. (*a*) Two unequal masses are suspended by a string from a pulley (Atwood's machine). (*b*) Free-body diagrams for m_1 and m_2. (*c*) Free-body diagram for the pulley, assumed massless.

Consider two unequal masses connected by a string which passes over a frictionless and massless pulley, as shown in Fig. 5–9*a*. Let m_2 be greater than m_1. Find the tension in the string and the acceleration of the masses.

We consider an *upward* acceleration *positive*. If the acceleration of m_1 is a, the acceleration of m_2 must be $-a$. The forces acting on m_1 and on m_2 are shown in Fig. 5–9*b* in which T represents the tension in the string.

The equation of motion for m_1 is

$$T - m_1 g = m_1 a$$

and for m_2 is

$$T - m_2 g = -m_2 a.$$

Combining these equations, we obtain

$$a = \frac{m_2 - m_1}{m_2 + m_1} g, \tag{5–12}$$

and

$$T = \frac{2m_1 m_2}{m_1 + m_2} g.$$

For example, if $m_2 = 2.0$ slugs and $m_1 = 1.0$ slug,

$$a = (32/3.0) \text{ ft/sec}^2 = g/3,$$

$$T = (\tfrac{4}{3})(32) \text{ slug-ft/sec}^2 = 43 \text{ lb}.$$

Notice that the magnitude of T is always intermediate between the weight of the mass m_1 (32 lb in our example) and the weight of the mass m_2 (64 lb in our example). This is to be expected, since T must exceed $m_1 g$ to give m_1 an upward acceleration, and $m_2 g$ must exceed T to give m_2 a downward acceleration. In the special case when $m_1 = m_2$, we obtain $a = 0$ and $T = m_1 g = m_2 g$, which is the static result to be expected.

Figure 5–9*c* shows the forces acting on the massless pulley. If we treat the pulley as a particle, all the forces can be taken to act through its center. P is the upward pull of the support on the pulley and T is the downward pull of each segment of the string on the pulley. Since the pulley has no translational motion,

$$P = T + T = 2T.$$

If we were to drop our assumption of a massless pulley and assign a mass m to it, we would then be required to include a downward force mg on the support. Also, as we shall see later, the rotational motion of the pulley results in a different tension in each segment of the string. Friction in the bearings also affects the rotational motion of the pulley and the tension in the strings.

Resnick & Halliday (1966)

Ex.

Consider an elevator moving vertically with an acceleration **a**. We wish to find the force exerted by a passenger on the floor of the elevator.

Acceleration will be taken *positive upward* and *negative downward*. Thus positive acceleration in this case means that the elevator is either moving upward with increasing speed or is moving downward with decreasing speed. Negative acceleration means that the elevator is moving upward with decreasing speed or downward with increasing speed.

From Newton's third law the force exerted by the passenger on the floor will always be equal in magnitude but opposite in direction to the force exerted by the floor on the passenger. We can therefore calculate either the action force or the reaction force. When the forces acting on the passenger are used, we solve for the latter force. When the forces acting on the floor are used, we solve for the former force.

The situation is shown in Fig. 5–10: The passenger's true weight is **W** and the force exerted on him by the floor, called **P**, is his *apparent* weight in the accelerating elevator. The resultant force acting on him is **P** + **W**. Forces will be taken as positive when directed upward. From the second law of motion we have

$$F = ma,$$

or

$$P - W = ma, \tag{5–13}$$

where m is the mass of the passenger and a is his (and the elevator's) acceleration.

Suppose, for example, that the passenger weighs 160 lb and the acceleration is 2.0 ft/sec² upward. We have

$$m = \frac{W}{g} = \frac{160 \text{ lb}}{32 \text{ ft/sec}^2} = 5.0 \text{ slugs},$$

and from Eq. 5–13,

$$P - 160 \text{ lb} = (5.0 \text{ slugs})(2.0 \text{ ft/sec}^2)$$

or

$$P = \text{apparent weight} = 170 \text{ lb}.$$

If we were to measure this force directly by having the passenger stand on a spring scale fixed to the elevator floor (or suspended from the ceiling), we would find the scale reading to be 170 lb for a man whose weight is 160 lb. The passenger feels himself pressing down on the floor with greater force (the floor is pressing upward on him with greater force) than when he and the elevator are at rest. Everyone experiences this feeling when an elevator starts upward from rest.

Resnick & Halliday (1966)

If the acceleration were taken as 2.0 ft/sec² downward, then $a = -2.0$ ft/sec² and $P = 150$ lb for the passenger. The passenger who weighs 160 lb feels himself pressing down on the floor with less force than when he and the elevator are at rest.

If the elevator cable were to break and the elevator were to fall freely with an acceleration $a = -g$, then P would equal $W + (W/g)(-g) = 0$. Then the passenger and floor would exert no forces on each other. The passenger's apparent weight, as indicated by the spring scale on the floor, would be zero. ◀

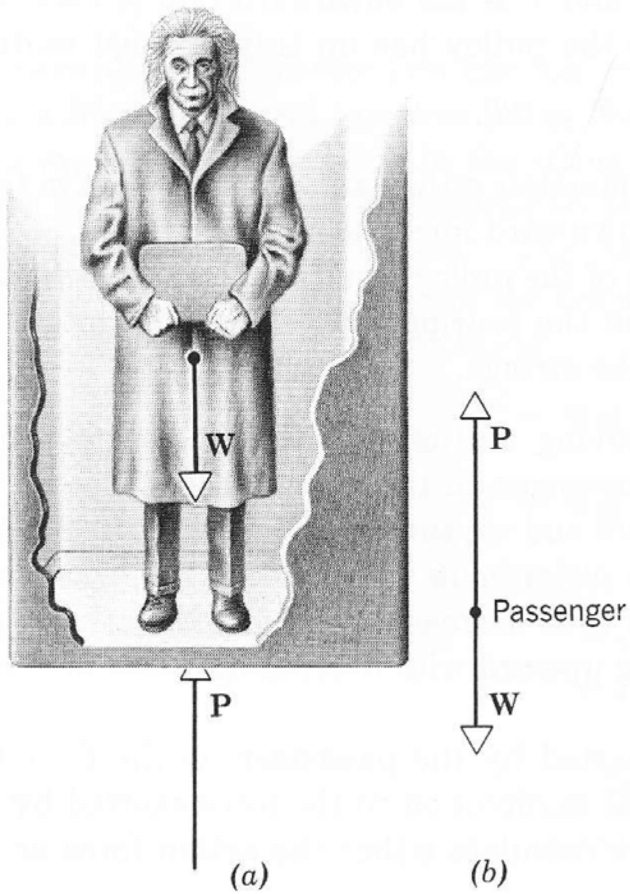


Fig. 5–10 Example 8. (*a*) A passenger stands on the floor of an elevator. (*b*) A free-body diagram for the passenger.

Resnick & Halliday (1966)