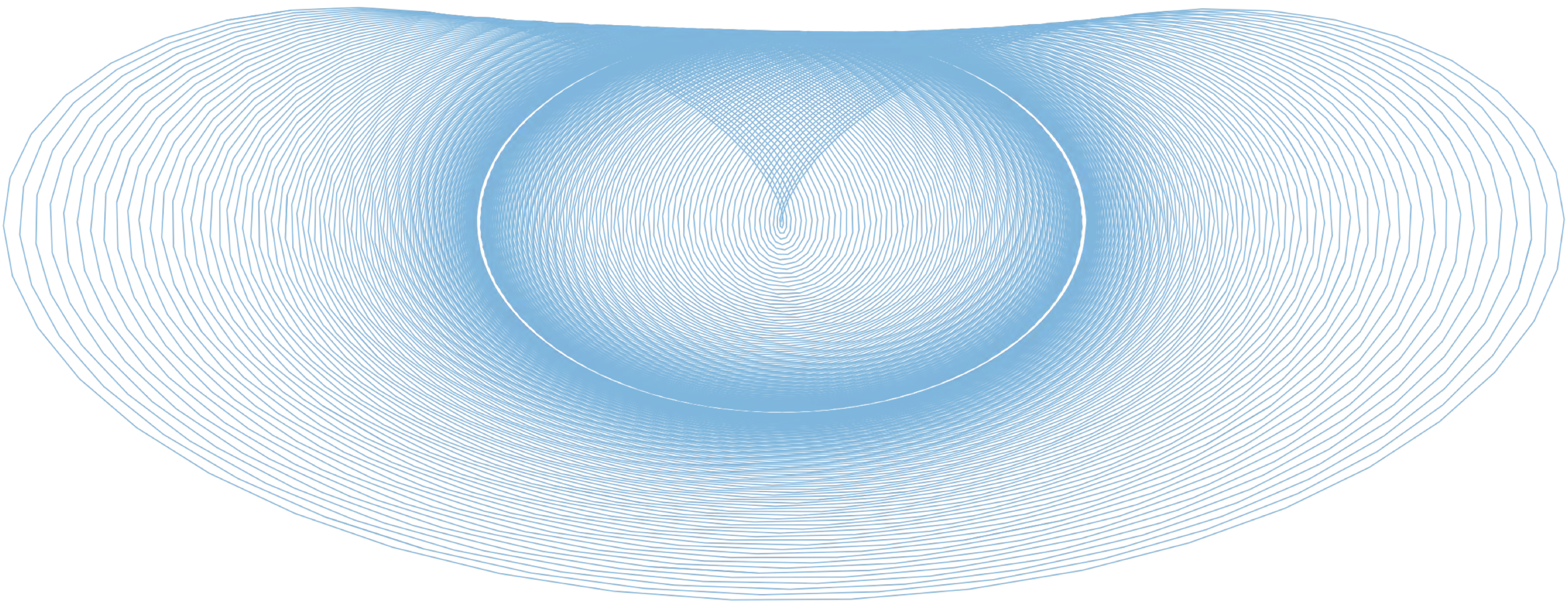
PHYS 1420 (F19)
Physics with Applications to Life Sciences



2019.10.02

Relevant reading:
Kesten & Tauck ch.5.4-5.5

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Ref. (re images):
Wolfson (2007), Knight (2017)

To the Letter

All of the eight shapes below are parts of the 26 letters shown above them, although they may have been enlarged and rotated. Figure out which letter each shape was taken from. The answers, in order, will spell a word.

abcdefghijklm
nopqrstuvwxyz

Announcements & Key Concepts (re Today)

→ Online HW #4: Due TODAY (at midnight)

→ Written HW#1 ready to be handed back --> Some common issues were…

Some relevant underlying concepts of the day…

- Drag forces
- Terminal velocity
- Circular motion (REVISTED)

Grader comments re HW1

- Read the question carefully. Answer exactly what is asked. Answer all of the parts. Reread to make sure your solution is stated in a way that clearly answers the question posed.

- Establish coordinates for your problem and stick with them: eg. for question #2 if the cliff is labeled 0, then the ground below is ±75 m (pick one) . . . also for more complicated problems label one direction x and another y and stick with this in all equations.

- Keep your directions straight: eg. for question #3 we are given horizontal velocity (Vx) and vertical acceleration is assumed (Ay) but these cannot be combined without some thought. (Be careful with kinematic and constant acceleration formulas). For simplicity work out each component separately and then combine the results.

  Vectors have both magnitude and direction. Velocity is a vector. When asked to find a velocity a complete answer must specify both completely (via components in an established coordinate system or by giving length (magnitude) and angle (direction) together in appropriate units)

- For question #6 'unit vectors' are not just normalized vectors: e.g. (i,j,k)…

- Do a sanity check: Overall does it make sense? Can a Kangaroo jump that high? Does this seem right based on the other parts of the same problem? Is this answer reasonable?…

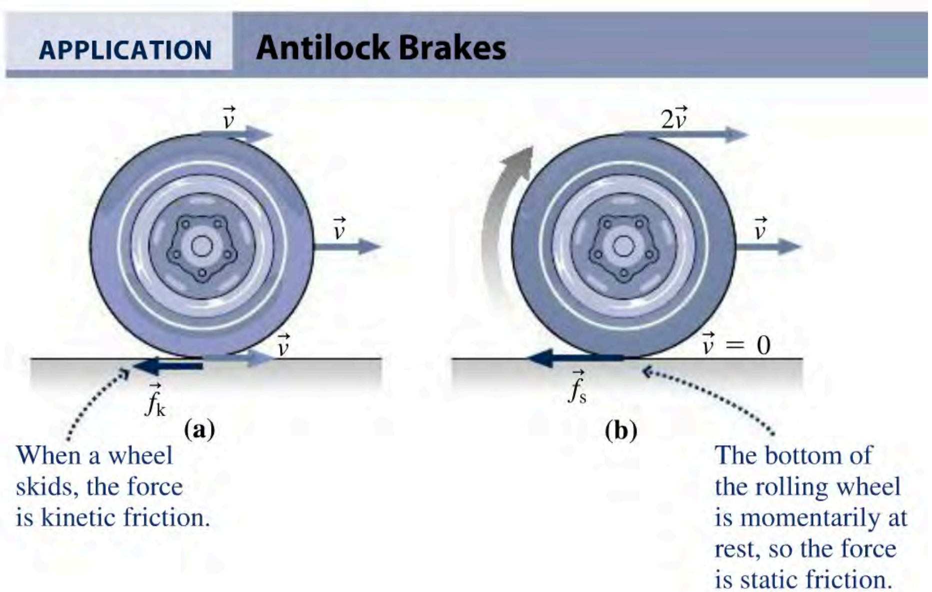
Friction: Static vs Kinetic vs "Rolling"

Note: For wheels, the notion of "rolling friction" here (as opposed to static friction) is a bit different re Kesten & Tauck (which is in ch.8!)

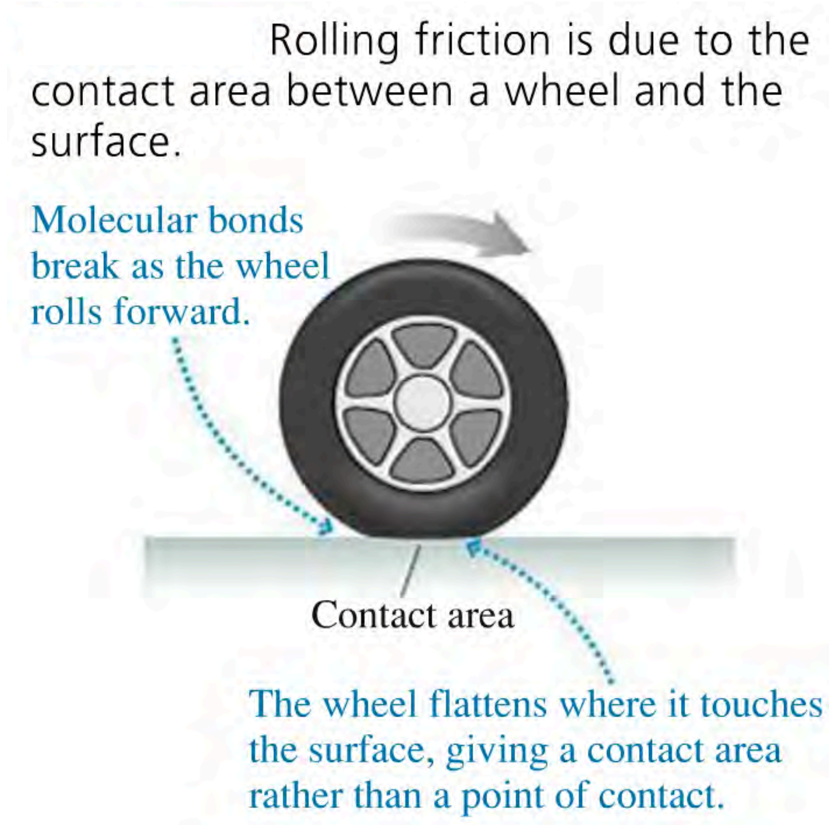
Static: $\vec{f}_s \leq (\mu_s n$, direction as necessary to prevent motion)

Kinetic: $\vec{f}_k = (\mu_k n$, direction opposite the motion)

Rolling: $\vec{f}_r = (\mu_r n$, direction opposite the motion)



Wolfson



Knight (2013)

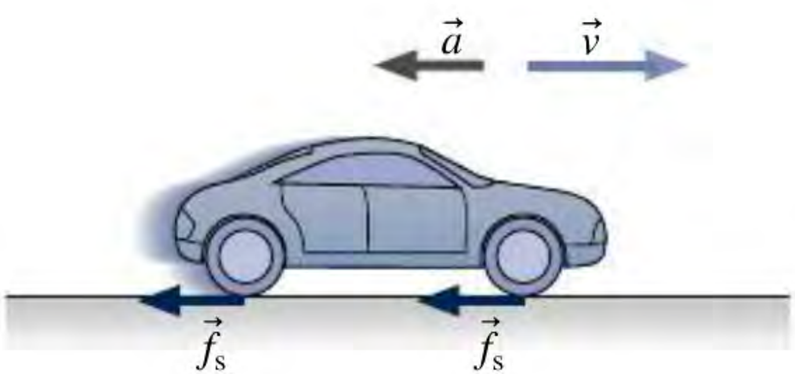
Friction: Static vs Kinetic vs "Rolling"

Coefficients of friction

Materials	Static μ_s	Kinetic μ_k	Rolling μ_r
Rubber on concrete	1.00	0.80	0.02
Steel on steel (dry)	0.80	0.60	0.002
Steel on steel (lubricated)	0.10	0.05	
Wood on wood	0.50	0.20	
Wood on snow	0.12	0.06	
Ice on ice	0.10	0.03	

Knight (2013)

→ Nonetheless, there are some key distinctions at play here!



The brakes affect only the wheels; it's friction between tires and road that stops the car. You know this if you've applied your brakes on an icy road!

Wolfson

Friction

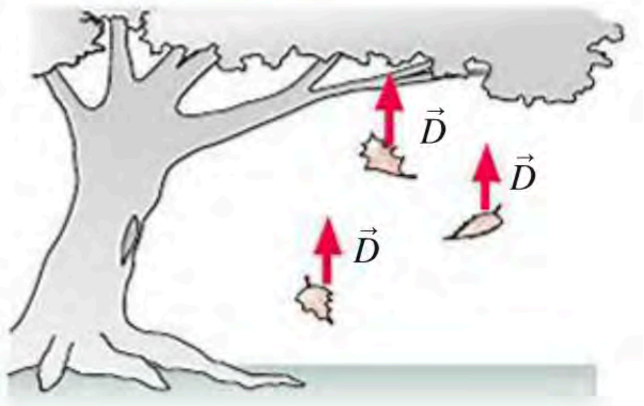
→ But is the friction between the tires and the road the entire story here?

Drag

- So how much "friction" is exerted on a speeding car or a falling object?

→ Might be thinking/remembering something a la *terminal velocity*....

Air resistance is a significant force on falling leaves. It points opposite the direction of motion.



Knight (2013)

Drag is a more complex force than ordinary friction because drag depends on the object's speed. Drag also depends on the object's shape and on the density of the medium through which it moves. Fortunately, we can use a fairly simple *model* of drag if the following three conditions are met:

- The object is moving through the air near the earth's surface.
- The object's size (diameter) is between a few millimeters and a few meters.
- The object's speed is less than a few hundred meters per second.

These conditions are usually satisfied for balls, people, cars, and many other objects in our everyday world.

Drag

These conditions are usually satisfied for balls, people, cars, and many other objects in our everyday world. Under these conditions, the drag force on an object moving with speed v can be written

$$\vec{D} = \left(\tfrac{1}{2}C\rho A v^2, \text{ direction opposite the motion}\right) \tag{6.16}$$

Notice that the drag force is proportional to the *square* of the object's speed. The symbols in Equation 6.16 are:

- A is the *cross-section area* of the object as it "faces into the wind," as illustrated in FIGURE 6.20.
- ρ is the density of the air, which is 1.2 kg/m^3 at atmospheric pressure and room temperature.
- C is the **drag coefficient.** It is smaller for aerodynamically shaped objects, larger for objects presenting a flat face to the wind. Figure 6.20 gives approximate values for a sphere and two cylinders.

→ Note here that drag (D) is a vector/force, some derived quantities are introduced (e.g., C), and a geometric parameter arises (A)

Knight (2013)

Drag

$$\vec{D} = (\tfrac{1}{2} C\rho A v^2, \text{direction opposite the motion})$$

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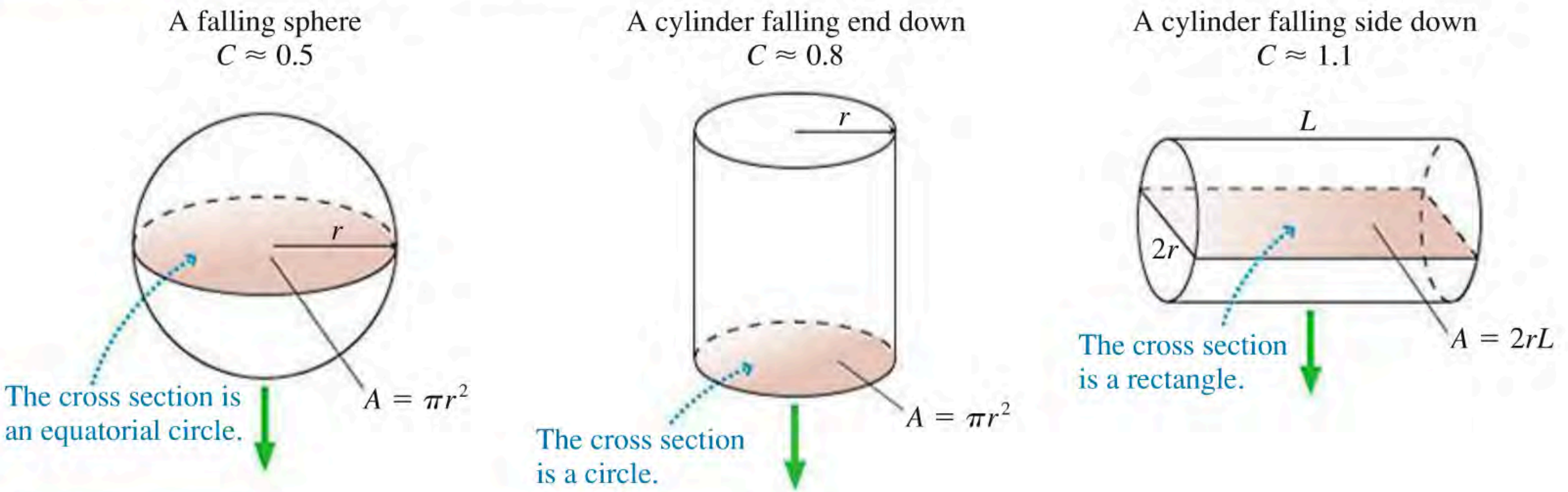
FIGURE 6.20 Cross-section areas for objects of different shape.



Knight (2013)

Drag → Terminal Velocity

An object falling at terminal speed.

Terminal speed is reached when the drag force exactly balances the gravitational force: $\vec{a} = \vec{0}$.

terminal speed v_{term}

$$D = F_G$$

$$\vec{F}_{net} = \vec{0}$$

$$\frac{1}{2} C\rho A v^2 = mg$$

$$v_{term} = \sqrt{\frac{2mg}{C\rho A}}$$

Knight (2013)

<u>Terminal Velocity</u>

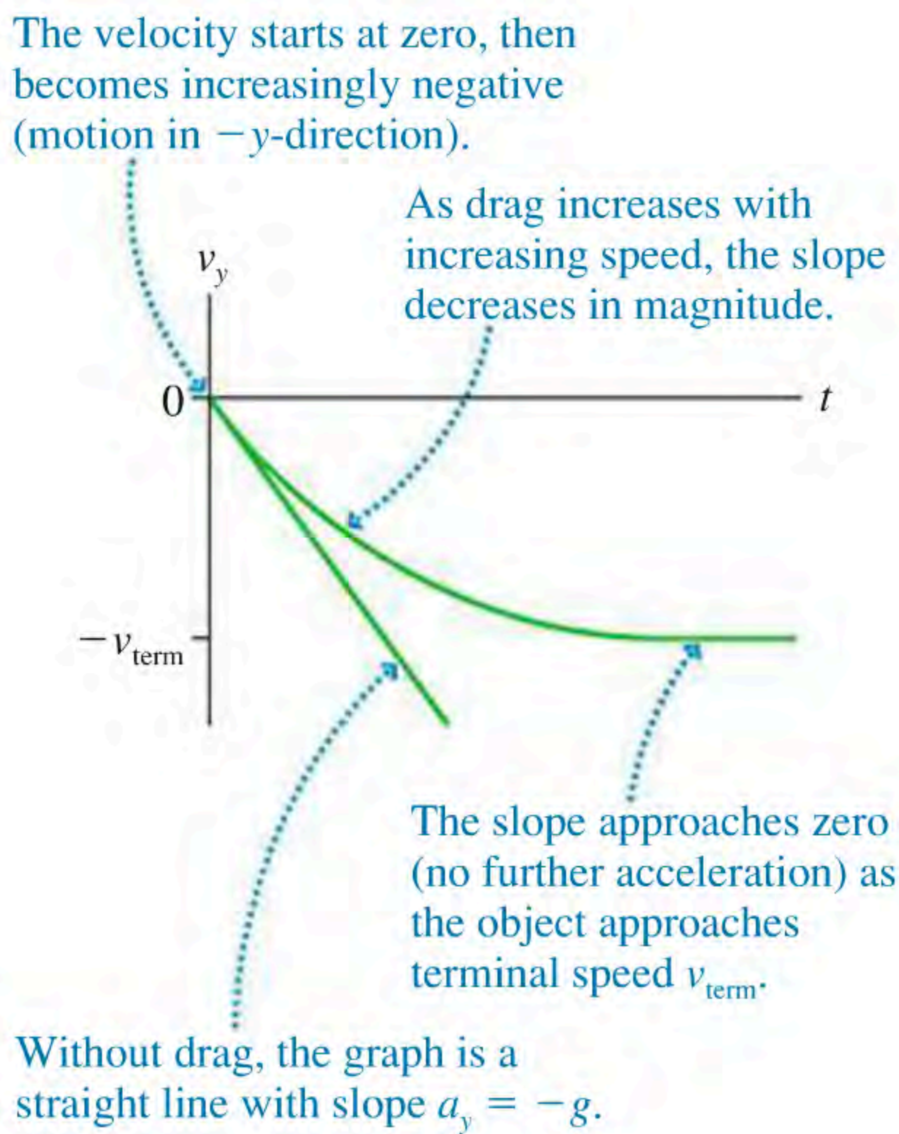
$$v_{\text{term}} = \sqrt{\frac{2mg}{C\rho A}}$$

- This equation applies to *"steady-state"* (i.e., doesn't tell you how things are changing w/ time)

$$D = F_G$$

→ Not too hard to infer relevant dynamics though...

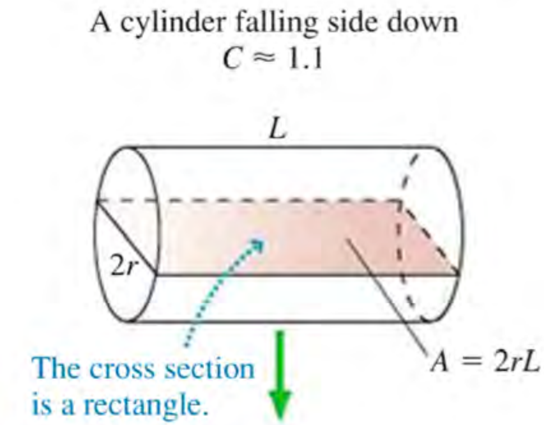
The velocity-versus-time graph of a falling object with and without drag.



Knight (2013)

Terminal Velocity

$$v_{\text{term}} = \sqrt{\frac{2mg}{C\rho A}}$$



A popular use of Equation 6.17 is to find the terminal speed of a skydiver. A skydiver is rather like the cylinder of Figure 6.20 falling "side down," for which we see that $C \approx 1.1$. A typical skydiver is 1.8 m long and 0.40 m wide ($A = 0.72\ \text{m}^2$) and has a mass of 75 kg. His terminal speed is

$$v_{\text{term}} = \sqrt{\frac{2mg}{C\rho A}} = \sqrt{\frac{2(75\ \text{kg})(9.8\ \text{m/s}^2)}{(1.1)(1.2\ \text{kg/m}^3)(0.72\ \text{m}^2)}} = 39\ \text{m/s}$$

This is roughly 90 mph. A higher speed can be reached by falling feet first or head first, which reduces the area A and the drag coefficient.

Knight (2013)

REVISIT (re 9/22)

Falling body: Terminal velocity

Assume air resistance is proportional to velocity, the Newton's 2nd Law leads to:

$$m\frac{dv}{dt} = mg - kv$$

$$\frac{dv}{dt} = -\frac{k}{m}\left(v - \frac{mg}{k}\right)$$

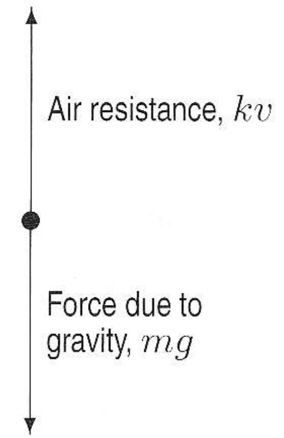


Figure 11.44: Forces acting on a falling object

Solution $$v = \frac{mg}{k}\left(1 - e^{-kt/m}\right)$$

Hughes-Hallett et al. (2005)

REVISIT (re 9/22)

Falling body: Terminal velocity

$$m\frac{dv}{dt} = mg - kv$$

$$v = \frac{mg}{k}\left(1 - e^{-kt/m}\right)$$

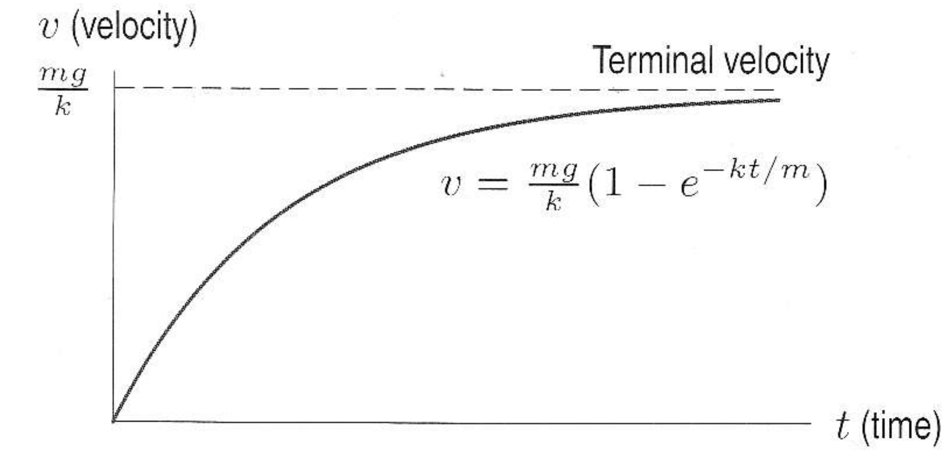
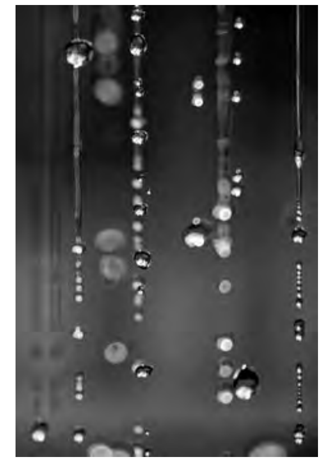


Figure 11.45: Velocity of falling dust particle assuming that air resistance is kv



e.g., speed of falling raindrop

Be careful! (re assumptions)

→ Can these two possibly be consistent w/ one another?

$$v = \frac{mg}{k}\left(1 - e^{-kt/m}\right)$$

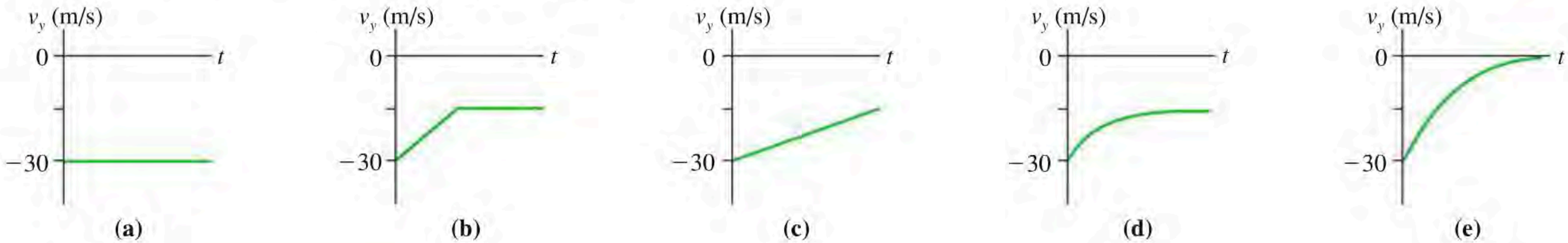
Assumed drag force proportional to v

$$v_{\text{term}} = \sqrt{\frac{2mg}{C\rho A}}$$

Assumed drag force proportional to v^2

Ex.

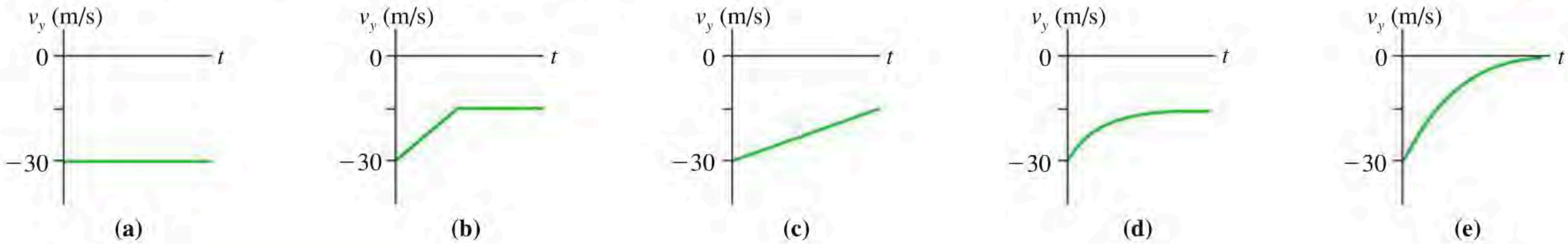
STOP TO THINK 6.4 The terminal speed of a Styrofoam ball is 15 m/s. Suppose a Styrofoam ball is shot straight down with an initial speed of 30 m/s. Which velocity graph is correct?



Knight (2013)

Ex. (SOL)

STOP TO THINK 6.4 The terminal speed of a Styrofoam ball is 15 m/s. Suppose a Styrofoam ball is shot straight down with an initial speed of 30 m/s. Which velocity graph is correct?

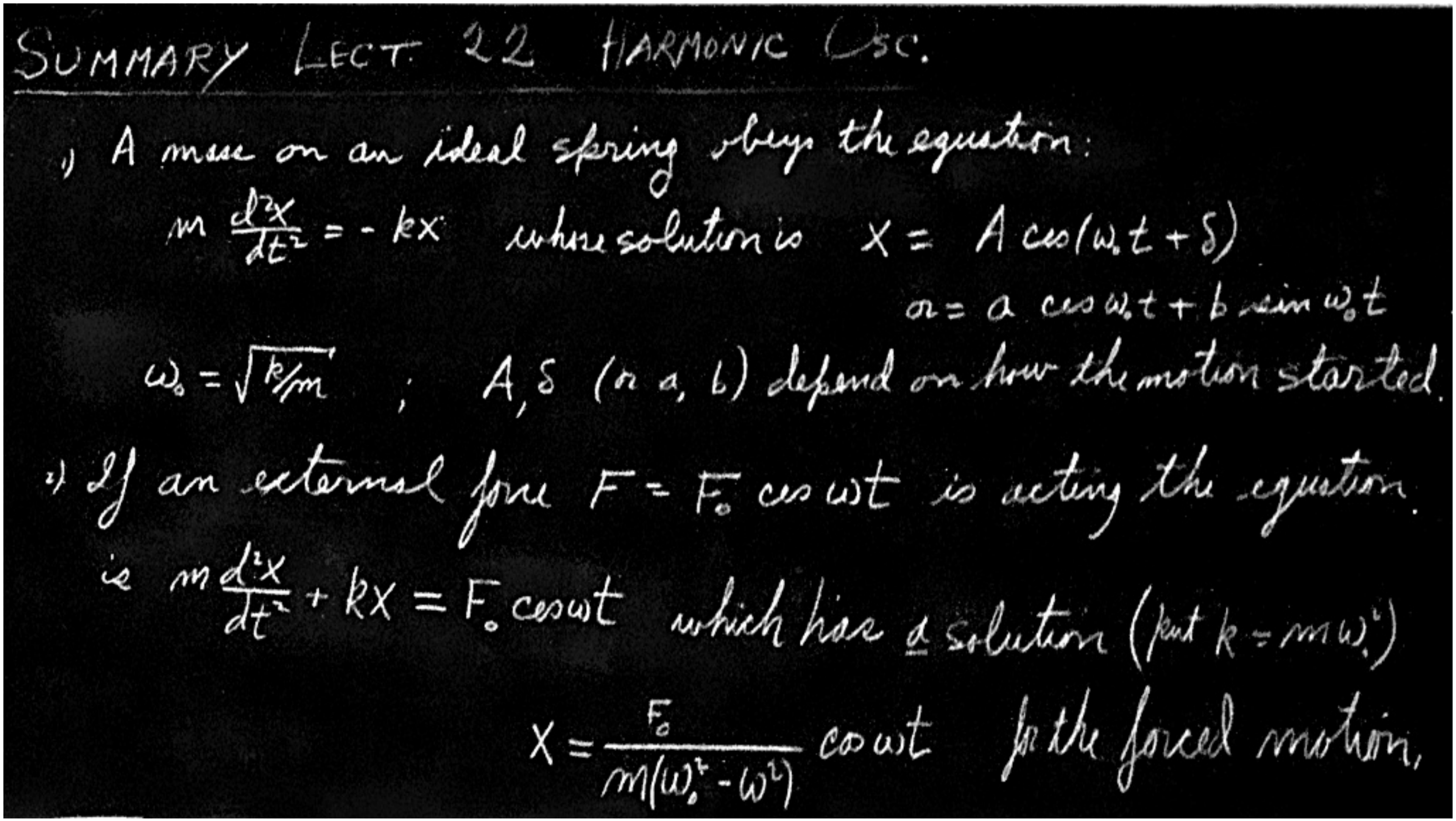


d

Knight (2013)

Looking ahead.....

- We now have most of the pieces in place for one of the most practically useful interdisciplinary examples/concepts: *Harmonic oscillator*



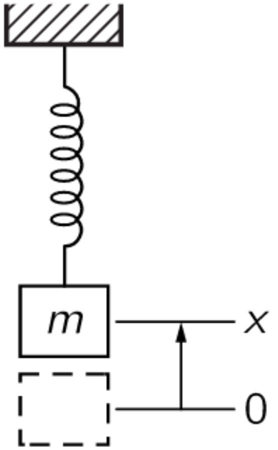
http://www.feynmanlectures.caltech.edu/I_21.html

Looking ahead.....

Wolfson Eqn.13.18

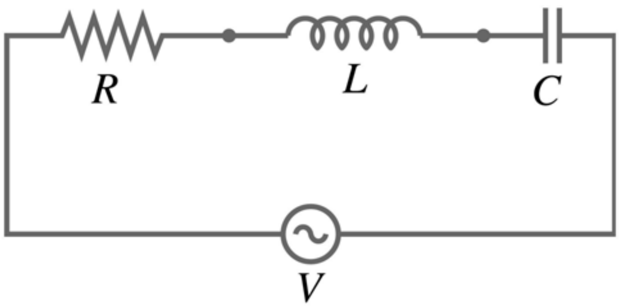
$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} + F_d$$

→ Mass-on-a-spring
(leads to oscillations)

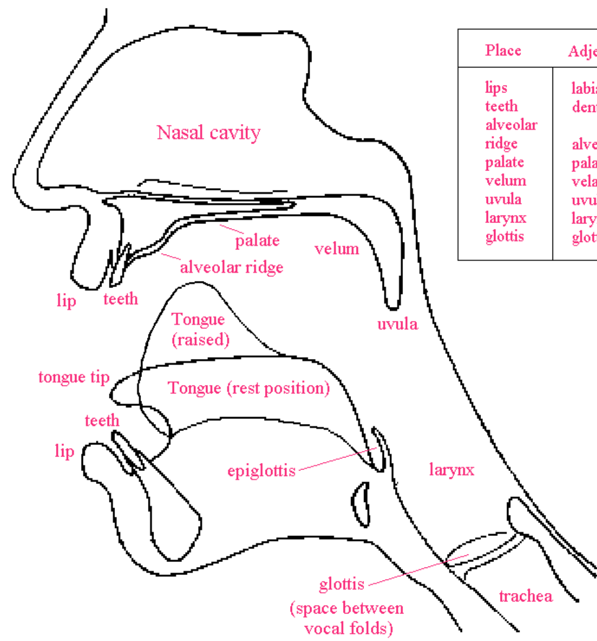


Note: Here the drag is proportional to v (not v^2)

Band-pass filter (RLC circuit)



Acoustic phonetics



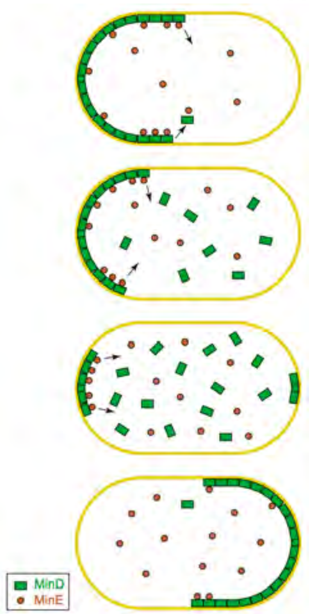
Place	Adjective
lips	labial
teeth	dental
alveolar ridge	alveolar
palate	palatal
velum	velar
uvula	uvular
larynx	laryngeal
glottis	glottal

https://www.uni-due.de/DI/REV_PhoneticsPhonology.htm

Predator-prey dynamics

Cell biology

(Kruse & Julicher, 2005)



Quantum mechanics

$$\hat{H}\,|\psi\rangle = E\,|\psi\rangle$$

→ A key concept is naturally built in to this heuristic: ***Energy*****.....**

..... but let's first return to a previously stated problem

A chain of length x and mass m is hanging over the edge of a tall building and does not touch the ground. How much work is required to lift the chain to the top of the building?

To (eventually) answer this, we'll need some more pieces:

- Definition of *work*
- Integration

$$\text{Work done} = \text{Force} \cdot \text{Distance} \qquad \text{or} \qquad W = F \cdot d.$$

In general, if force is a function $F(x)$ of position x, then in moving from $x = a$ to $x = b$,

$$\text{Work done} = \int_a^b F(x)\, dx.$$

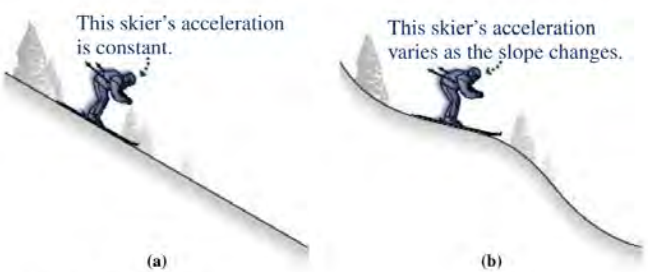


FIGURE 6.1 Two skiers.

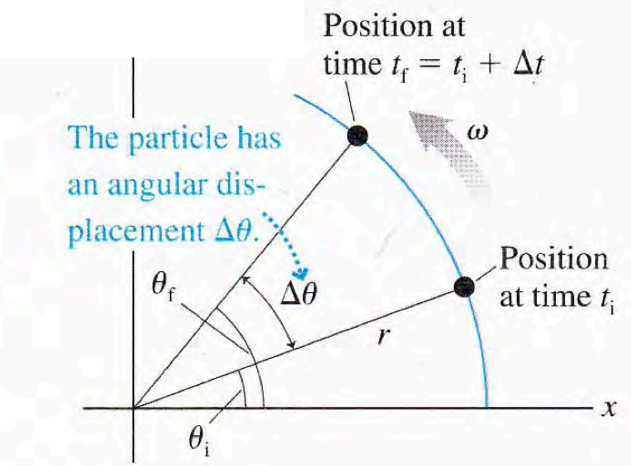
Wolfson

→ We need to further develop the notion of *integration*

Hughes-Hallet et al (2005)

Review: Uniform circular motion

FIGURE 4.27 A particle moves with angular velocity ω.



Wolfson

$$\theta(\text{radians}) \equiv \frac{s}{r}$$

$$v_t = \frac{ds}{dt} = r\frac{d\theta}{dt}$$

$$a = \frac{v^2}{r} \qquad \text{(uniform circular motion)}$$

Polar coordinates

$$r = \sqrt{x^2 + y^2} \quad \text{and} \quad \theta = \tan^{-1} y/x$$

$$x = r\cos\theta \quad \text{and} \quad y = r\sin\theta$$

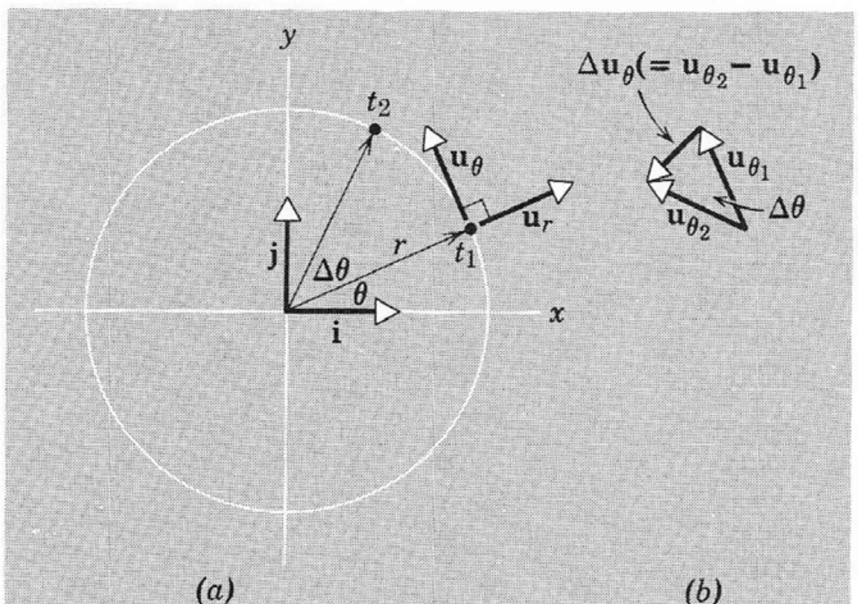


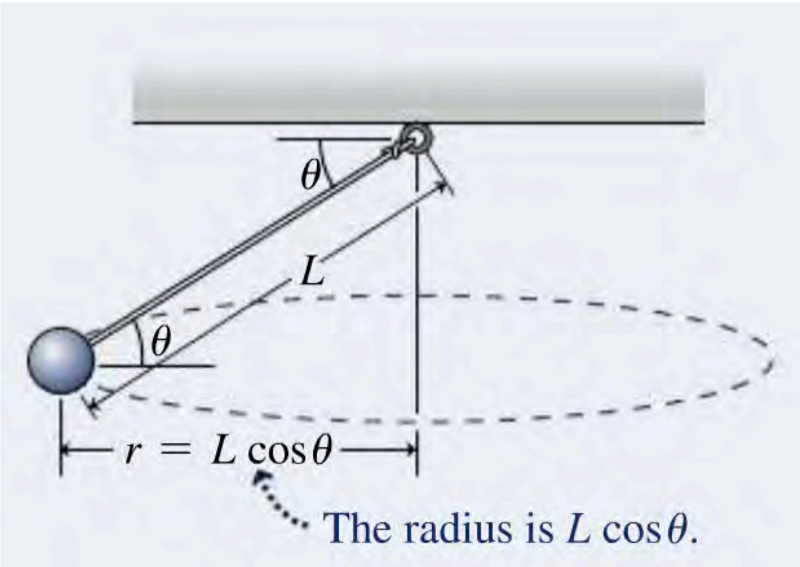
Fig. 4–8 (*a*) A particle moving counterclockwise in a circle of radius r. (*b*) The unit vectors $\mathbf{u}_{\theta_1}$ and $\mathbf{u}_{\theta_2}$ at times t_1 and t_2 respectively, and the change $\Delta\mathbf{u}_\theta$ ($= \mathbf{u}_{\theta_2} - \mathbf{u}_{\theta_1}$).

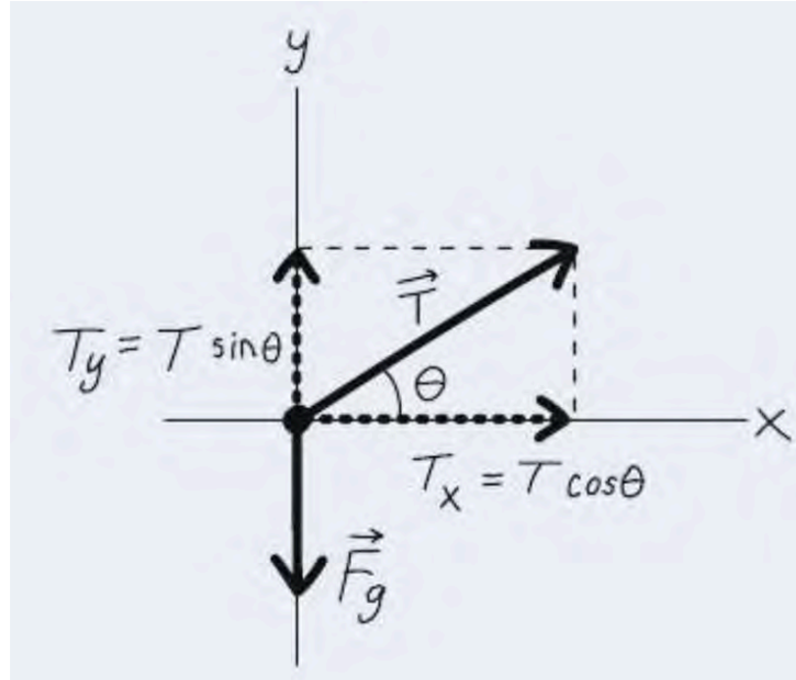
→ "Unit vectors" can readily be extended to polar coordinates

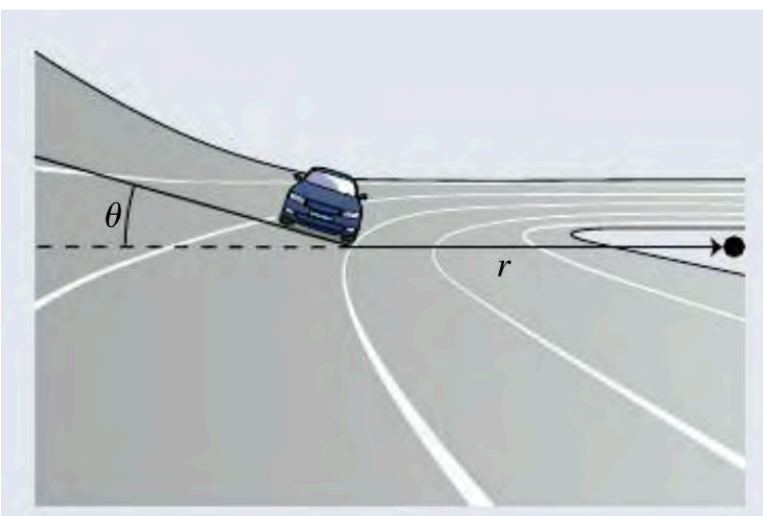
Resnick & Halliday (1966)

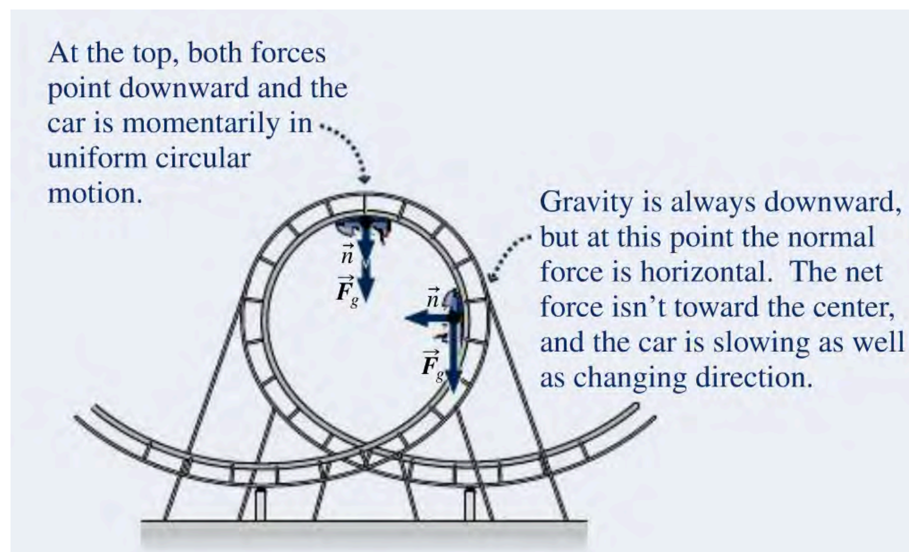
Circular Motion & Force

$$F_{\text{net}} = ma = \frac{mv^2}{r}$$





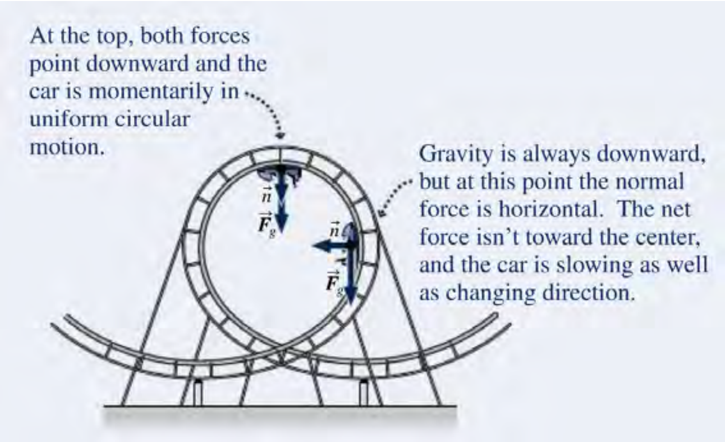




Wolfson

Circular Motion

Wolfson



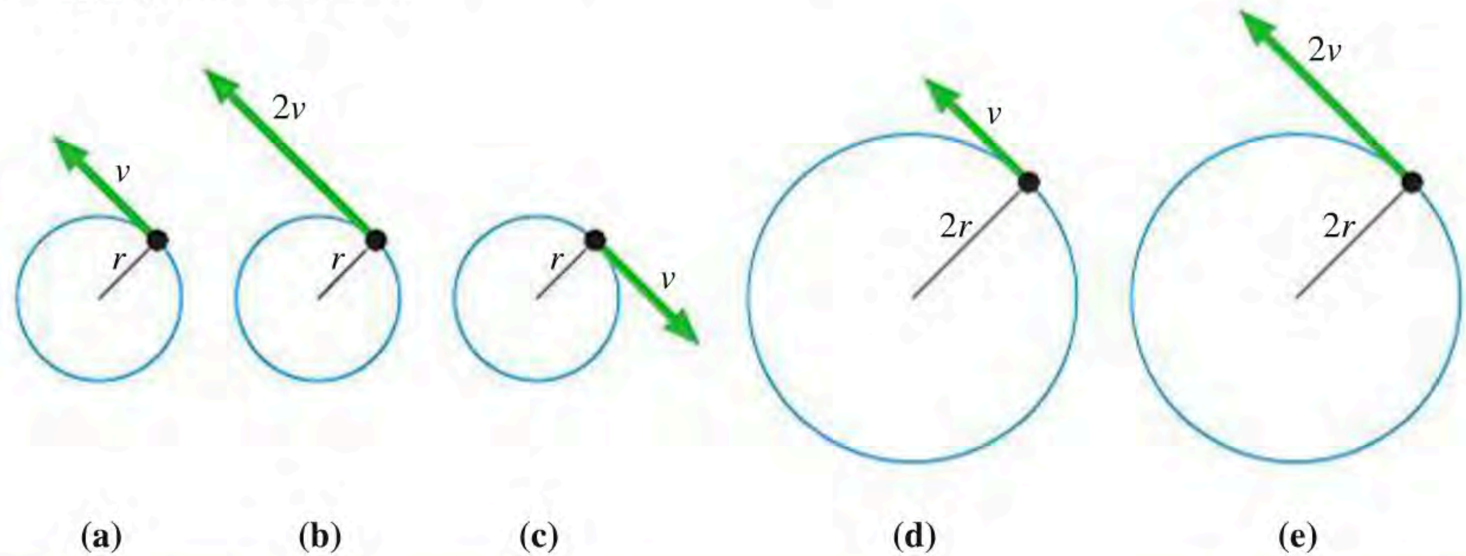
Note: This case isn't uniform circular motion per se....

Rollercoaster thrillseekers left dangling upside down 50ft up for 20 minutes after poncho gets stuck in rails

http://www.dailymail.co.uk/news/article-1271844/Flamingo-Land-rollercoaster-Thrillseekers-left-hanging-upside-50ft-up.html

Ex.

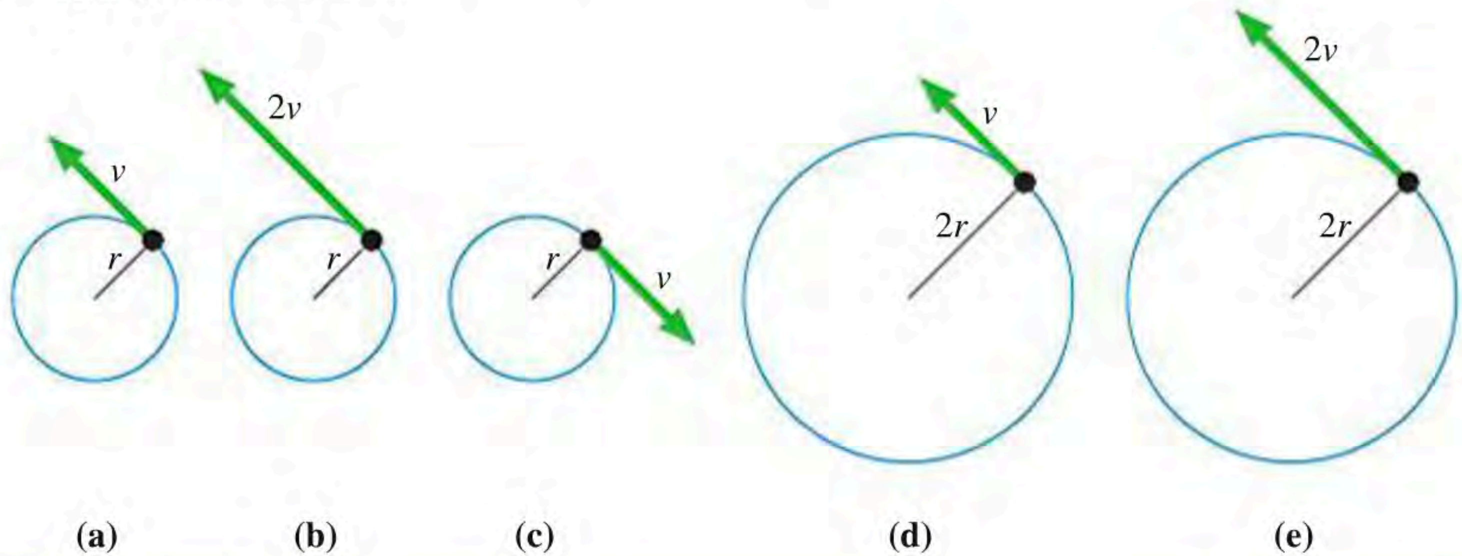
STOP TO THINK 4.6 Rank in order, from largest to smallest, the centripetal accelerations a_a to a_e of particles a to e.



Knight (2013)

Ex. (SOL)

STOP TO THINK 4.6 Rank in order, from largest to smallest, the centripetal accelerations a_a to a_e of particles a to e.



b > e > a = c > d

$$F_{net} = ma = \frac{mv^2}{r}$$

Note: Changing sign of v doesn't affect a

Knight (2013)

Circular Motion

- 1-D kinematics translates directly to circular motion (in polar coords.)

TABLE 4.1 Rotational and linear kinematics for constant acceleration

Rotational kinematics	Linear kinematics
$\omega_f = \omega_i + \alpha\,\Delta t$	$v_{fs} = v_{is} + a_s\,\Delta t$
$\theta_f = \theta_i + \omega_i\,\Delta t + \frac{1}{2}\alpha(\Delta t)^2$	$s_f = s_i + v_{is}\,\Delta t + \frac{1}{2}a_s(\Delta t)^2$
$\omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta$	$v_{fs}^2 = v_{is}^2 + 2a_s\,\Delta s$

Knight (2013)

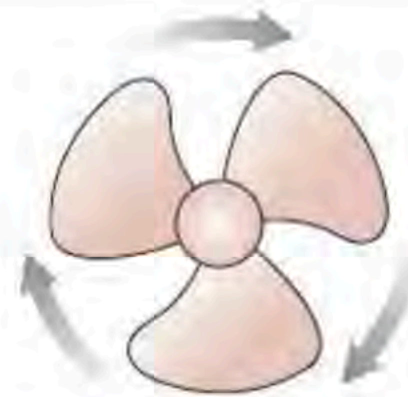
Note:

- You solved several "differential equations" to get these (linear) formulae (see 9/11ff notes)
- Newton's 2nd Law is a differential equation

Ex.

STOP TO THINK 4.7 The fan blade is slowing down. What are the signs of ω and α?

a. ω is positive and α is positive.
b. ω is positive and α is negative.
c. ω is negative and α is positive.
d. ω is negative and α is negative.



Knight (2013)

Ex. (SOL)

STOP TO THINK 4.7 The fan blade is slowing down. What are the signs of ω and α?

a. ω is positive and α is positive.
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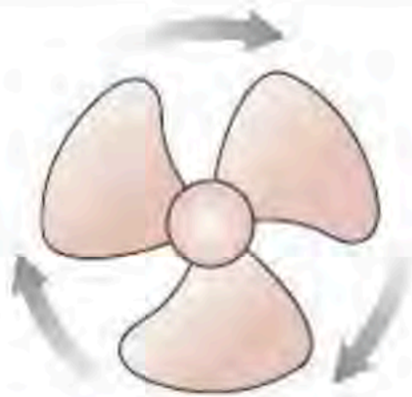
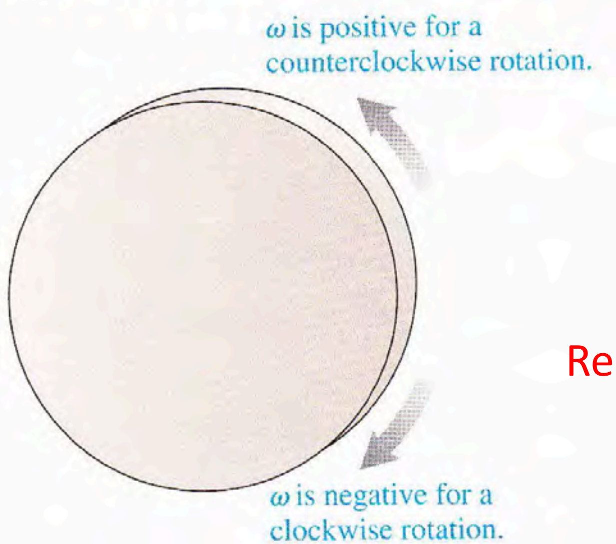


c

FIGURE 4.28 Positive and negative angular velocities.



Remember the chosen convention!

Knight (2013)

Additional Problems

(some w/ solutions, some w/o)

Ex.

Carol pushes a 50 kg wood box across a wood floor at a steady speed of 2.0 m/s. How much force does Carol exert on the box? If she stops pushing, how far will the box slide before coming to rest?

Knight (2013)

Ex. (SOL)

Carol pushes a 50 kg wood box across a wood floor at a steady speed of 2.0 m/s. How much force does Carol exert on the box? If she stops pushing, how far will the box slide before coming to rest?

Carol was pushing at 2 m/s $\approx$ 4 mph, which is fairly fast. The box slides 1.0 m, which is slightly over 3 feet. That sounds reasonable.

Knight (2013)

Ex.

A 50 kg steel file cabinet is in the back of a dump truck. The truck's bed, also made of steel, is slowly tilted. What is the size of the static friction force on the cabinet when the bed is tilted 20°? At what angle will the file cabinet begin to slide?

Knight (2013)

Ex. (SOL)

A 50 kg steel file cabinet is in the back of a dump truck. The truck's bed, also made of steel, is slowly tilted. What is the size of the static friction force on the cabinet when the bed is tilted 20°? At what angle will the file cabinet begin to slide?

$$f_s = mg\sin\theta = (50\text{ kg})(9.80\text{ m/s}^2)\sin 20^\circ$$
$$= 170\text{ N}$$

39°

Knight (2013)

Ex.

Which raindrops fall faster, big ones or little ones? Why?

Shaskol'Skaya & El'Tsin (1963)

Ex.

The profile of a typical 1500 kg passenger car, as seen from the front, is 1.6 m wide and 1.4 m high. Aerodynamic body shaping gives a drag coefficient of 0.35. At what speed does the magnitude of the drag equal the magnitude of the rolling friction?

Knight (2013)

Ex. (SOL)

The profile of a typical 1500 kg passenger car, as seen from the front, is 1.6 m wide and 1.4 m high. Aerodynamic body shaping gives a drag coefficient of 0.35. At what speed does the magnitude of the drag equal the magnitude of the rolling friction?

SOLVE Drag is less than friction at low speeds, where air resistance is negligible. But drag increases as v increases, so there will be a speed at which the two forces are equal in size. Above this speed, drag is more important than rolling friction.

There's no motion and no acceleration in the vertical direction, so we can see from the free-body diagram that $n = F_G = mg$. Thus $f_r = \mu_r mg$. Equating friction and drag, we have

$$\tfrac{1}{2} C\rho A v^2 = \mu_r mg$$

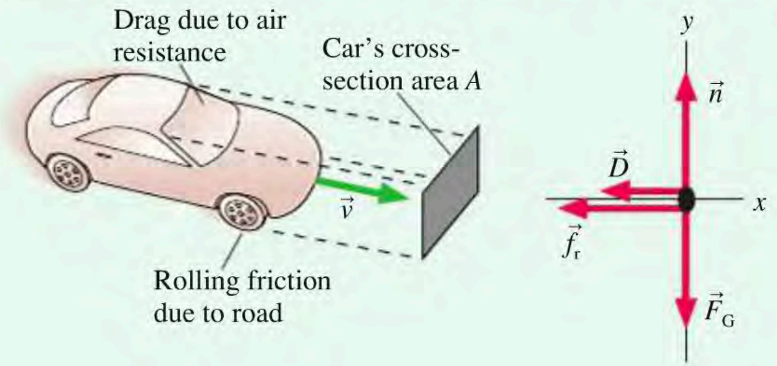
Solving for v, we find

$$v = \sqrt{\frac{2\mu_r mg}{C\rho A}} = \sqrt{\frac{2(0.02)(1500\text{ kg})(9.80\text{ m/s}^2)}{(0.35)(1.2\text{ kg/m}^3)\,(1.4\text{ m} \times 1.6\text{ m})}} = 25\text{ m/s}$$

where the value of μ_r for rubber on concrete was taken from Table 6.1.

ASSESS 25 m/s is approximately 50 mph, a reasonable result. This calculation shows that we can reasonably ignore air resistance for car speeds less than 30 or 40 mph. Calculations that neglect drag will be increasingly inaccurate as speeds go above 50 mph.

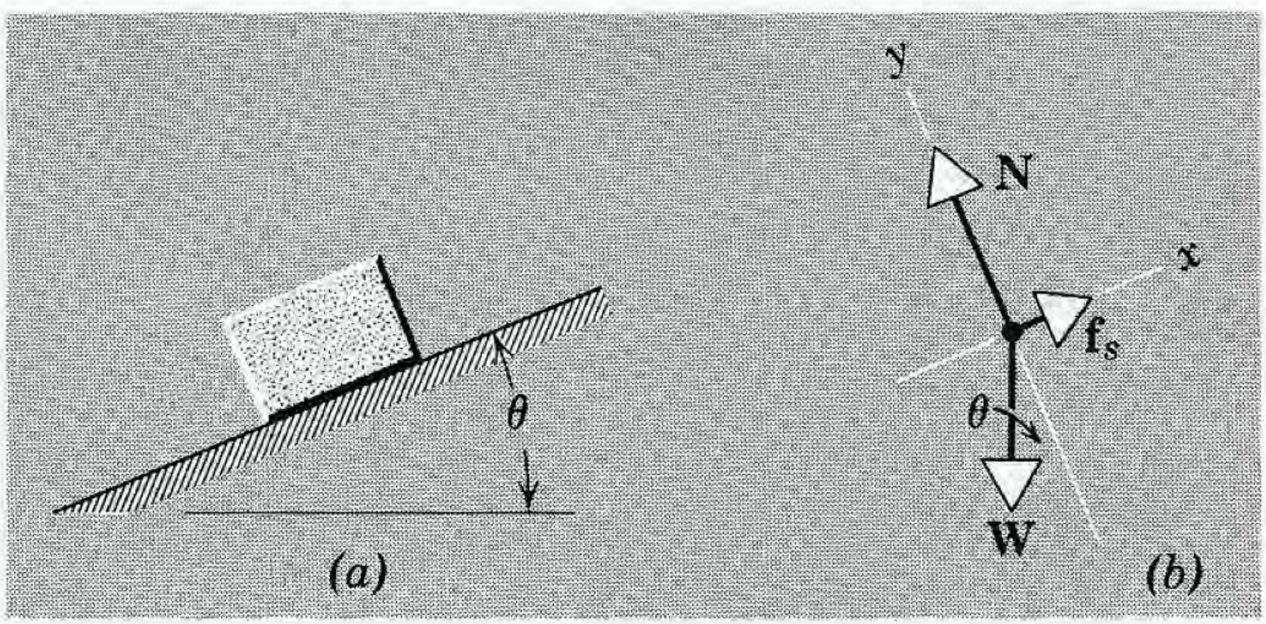
FIGURE 6.21 A car experiences both rolling friction and drag.



Knight (2013)

Ex.

A block is at rest on an inclined plane making an angle θ with the horizontal, as in Fig. 6–4*a*. As the angle of incline is raised, it is found that slipping just begins at an angle of inclination θ_s. What is the coefficient of static friction between block and incline?



Resnick & Halliday (1966)

Ex. (SOL)

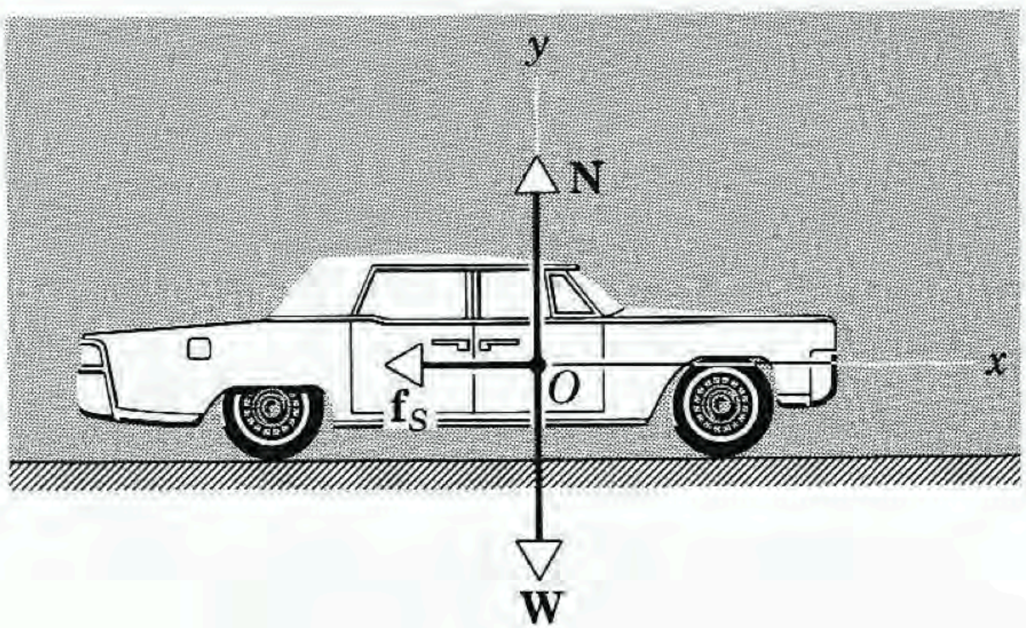
A block is at rest on an inclined plane making an angle θ with the horizontal, as in Fig. 6–4*a*. As the angle of incline is raised, it is found that slipping just begins at an angle of inclination θ_s. What is the coefficient of static friction between block and incline?

$$\mu_s = \tan \theta_s$$

Resnick & Halliday (1966)

Ex.

Consider an automobile moving along a straight horizontal road with a speed v_0. If the coefficient of static friction between the tires and the road is μ_s, what is the shortest distance in which the automobile can be stopped?



Note: This problem is essentially identical in nature to Wolfson Ex.5.8. Also note here the (confusing?) distinction between "static" and "rolling" friction

Resnick & Halliday (1966)

Ex. (SOL)

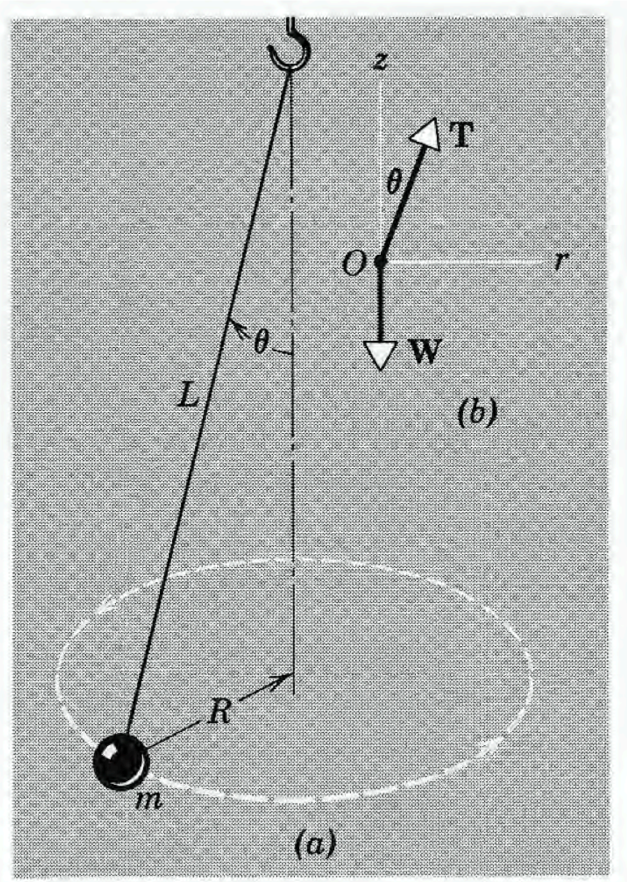
Consider an automobile moving along a straight horizontal road with a speed v_0. If the coefficient of static friction between the tires and the road is μ_s, what is the shortest distance in which the automobile can be stopped?

$$x = -v_0{}^2/2a = v_0{}^2/2g\mu_s$$

Resnick & Halliday (1966)

Ex.

The Conical Pendulum. Figure 6–7*a* represents a small body of mass m revolving in a horizontal circle with constant speed v at the end of a string of length L. As the body swings around, the string sweeps over the surface of a cone. This device is called a *conical pendulum.* Find the time required for one complete revolution of the body.



Resnick & Halliday (1966)

Ex. (SOL)

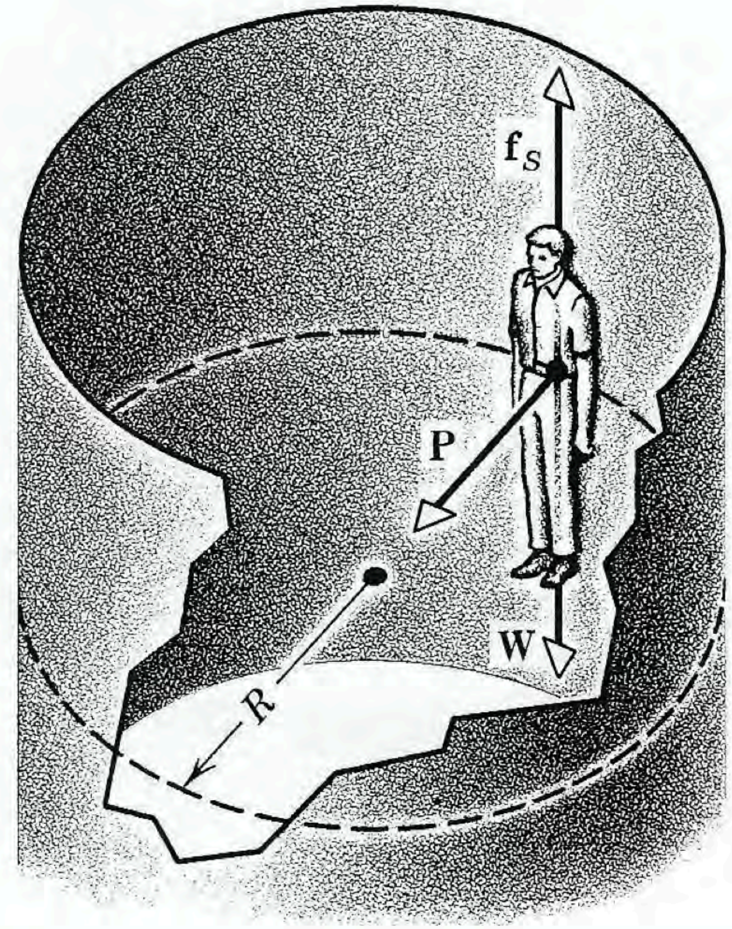
The Conical Pendulum. Figure 6–7a represents a small body of mass m revolving in a horizontal circle with constant speed v at the end of a string of length L. As the body swings around, the string sweeps over the surface of a cone. This device is called a *conical pendulum.* Find the time required for one complete revolution of the body.

$$\tau = 2\pi \sqrt{(L \cos \theta)/g}.$$

Resnick & Halliday (1966)

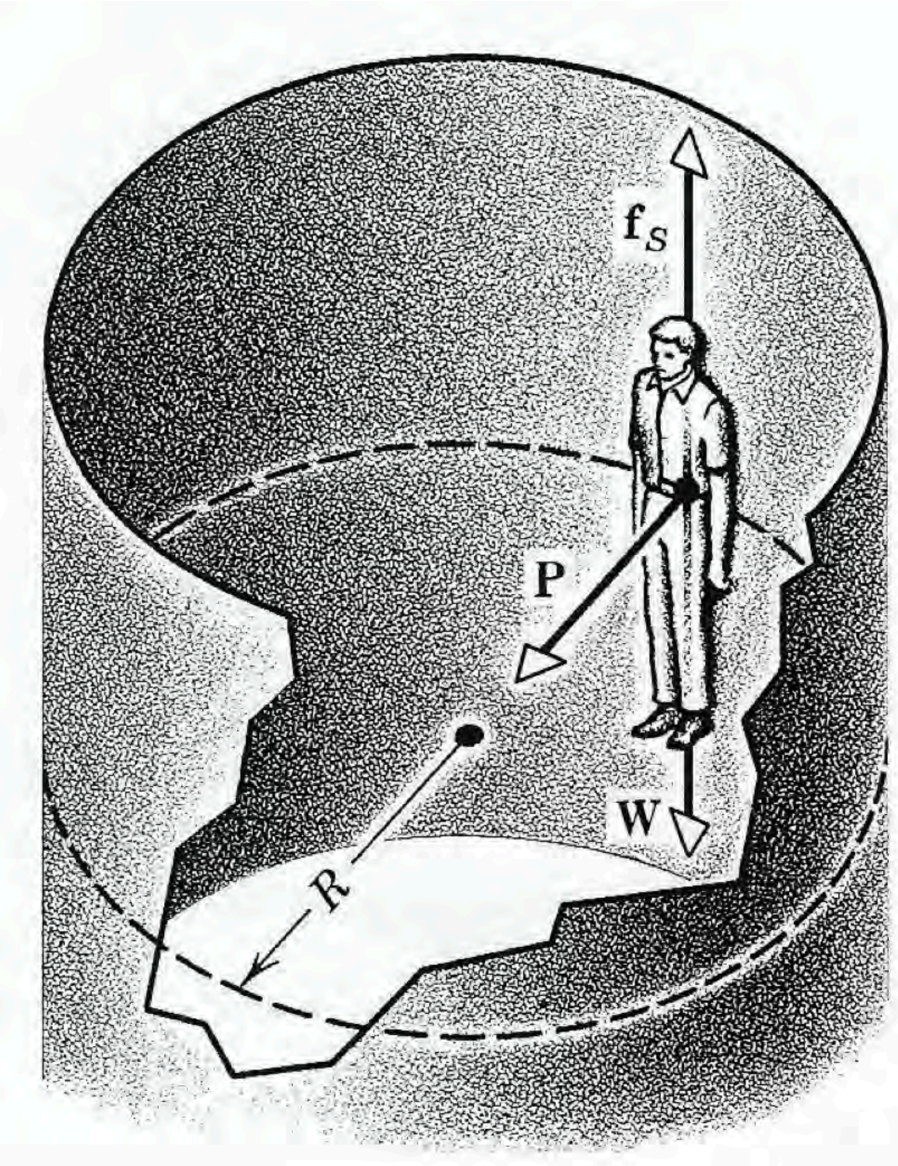
Ex.

The Rotor. In many amusement parks we find a device called the *rotor*. The rotor is a hollow cylindrical room which can be set rotating about the central vertical axis of the cylinder. A person enters the rotor, closes the door, and stands up against the wall. The rotor gradually increases its rotational speed from rest until, at a predetermined speed, the floor below the person is opened downward, revealing a deep pit. The passenger does not fall but remains "pinned up" against the wall of the rotor. Find the coefficient of friction necessary to prevent falling.



Resnick & Halliday (1966)

Ex. (SOL)



$$\mu_s = \frac{W}{P} = \frac{gR}{v^2}.$$

Resnick & Halliday (1966)