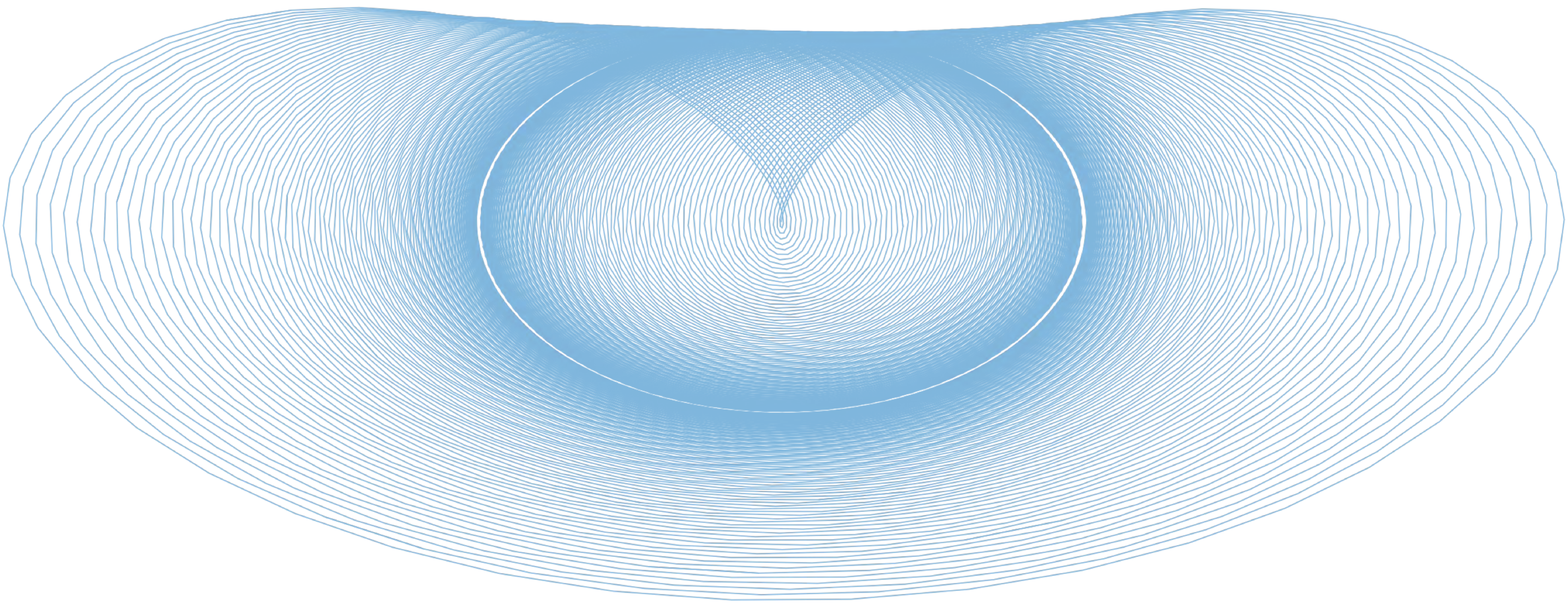
PHYS 1420 (F19)
Physics with Applications to Life Sciences



2019.10.07

Relevant reading:
Kesten & Tauck ch.6.3-6.4

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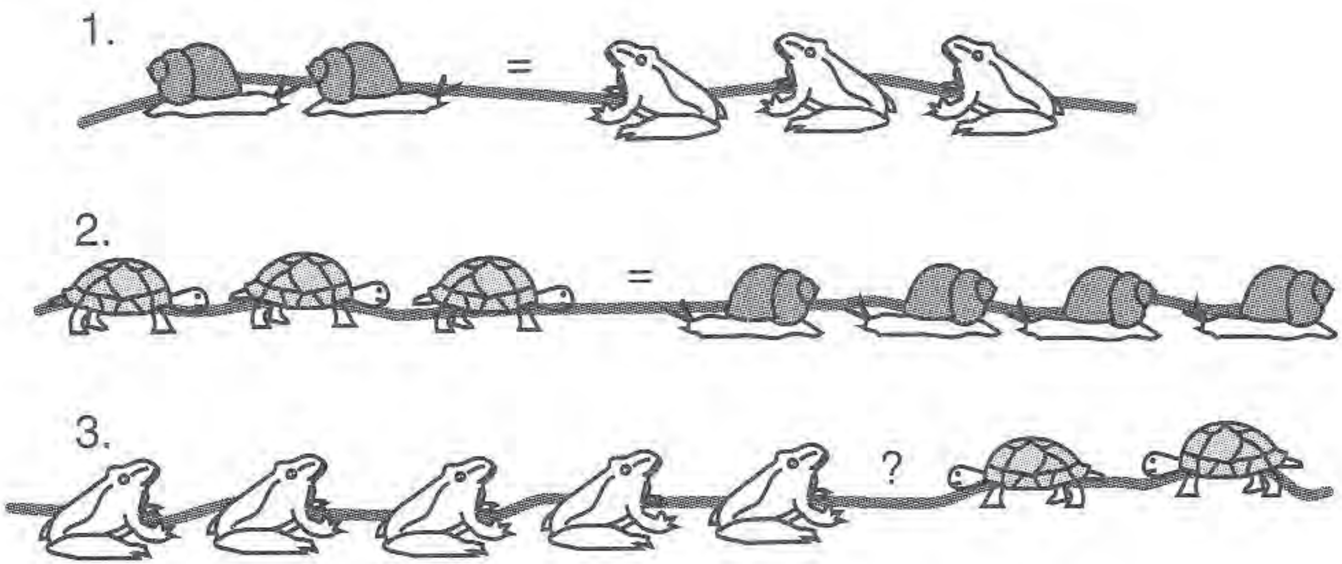
Ref. (re images):
Wolfson (2007), Knight (2017)

Animal Strength

If the first two tug of war contests shown here are ties, which group will win the third contest, or will it also be a draw?



Announcements & Key Concepts (re Today)

→ Online HW #5: Posted and due next Monday (10/14)

→ No class next week (10/14-10/18): **READING WEEK**

→ Midterm exam coming up on Monday 10/21

Some relevant underlying concepts of the day…

- Energy → Work
- Hashing out *work*….
- Integrals

Energy

- "Energy" is a fundamental concept in all of science

- Etymology is of Greek origin for "activity"

- Comes in many different flavors/contexts:

Potential

Elastic

Mechanical

Electrical

Thermal

Gravitational

Kinetic

Chemical

Nuclear

$$E = mc^2$$

→ Somehow, these are all different, but yet are all the same....

- At the most basic level, "something" has energy and can transfer/receive such from other "somethings" around it....

Force + Energy?

- How are these two connected?
- Intuitively.....

Niagara Falls

Robert Moses Niagara Power Plant

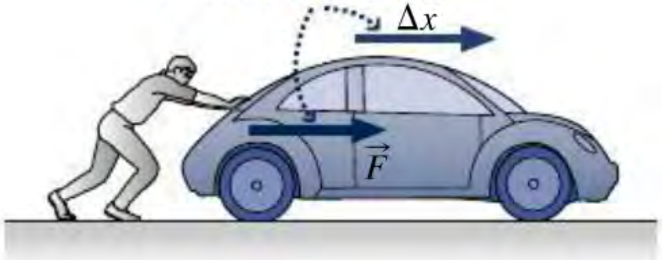
→ Work!

Work

- Work is the energy transferred between systems via an applied force

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$



→ A bit complicated once vectors are factored in (direction matters!). But basically....

Units

$(kg\ m/s^2) * (m) = kg\ (m/s)^2$

$= J$

For an object moving in one dimension, the work W done on the object by constant applied force $\vec{F}$ is

$$W = F_x \Delta x \qquad (6.1)$$

where F_x is the component of the force in the direction of the object's motion and Δx is the object's displacement.

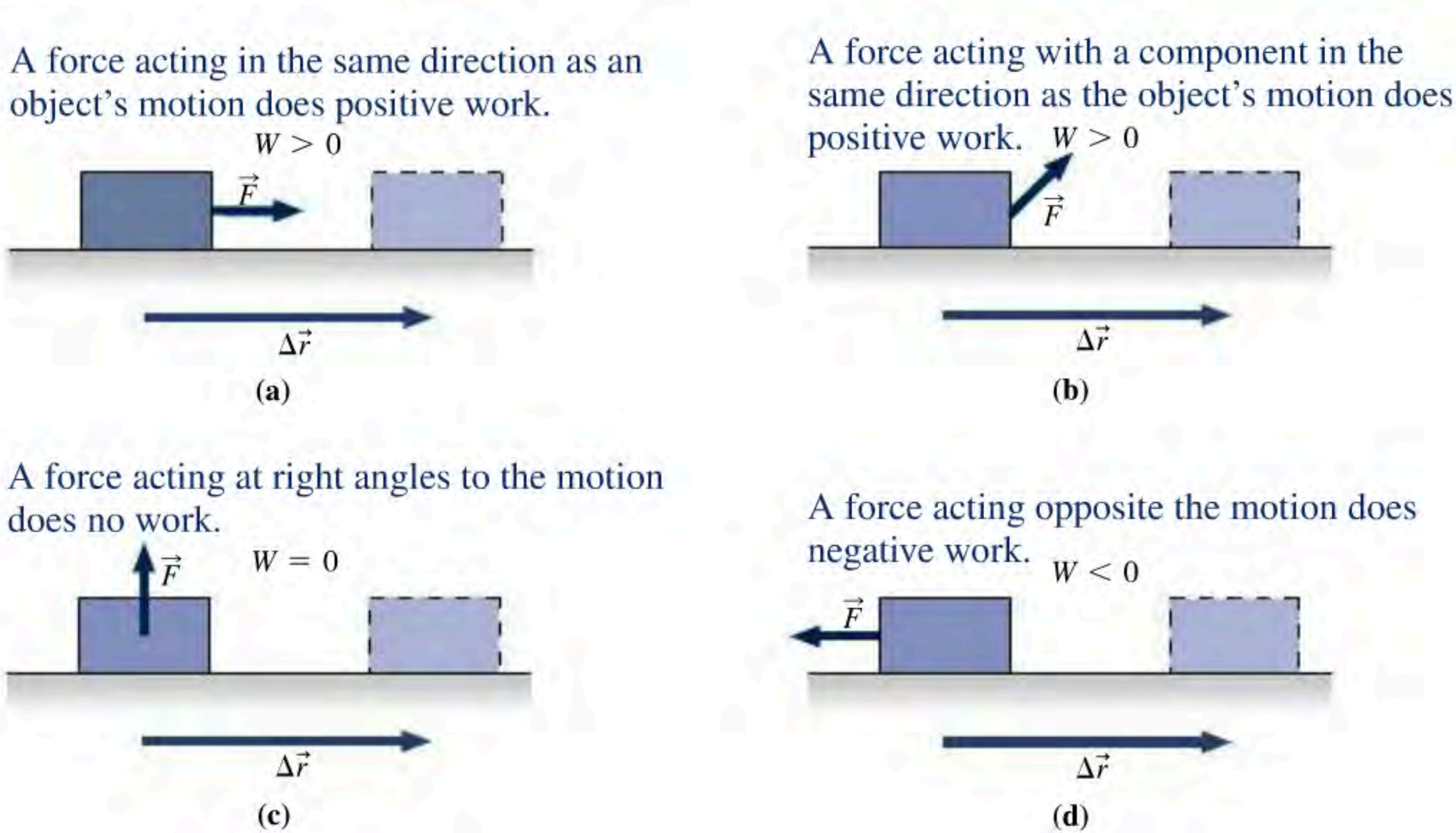
Wolfson

Work

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

$$W = F_x \Delta x$$

Note: The work (W) here is only that tied to force F. If there are other forces at play, the associated work needs to be calculated separately....



→ So work is energy. Note that unlike force, work/energy is a scalar

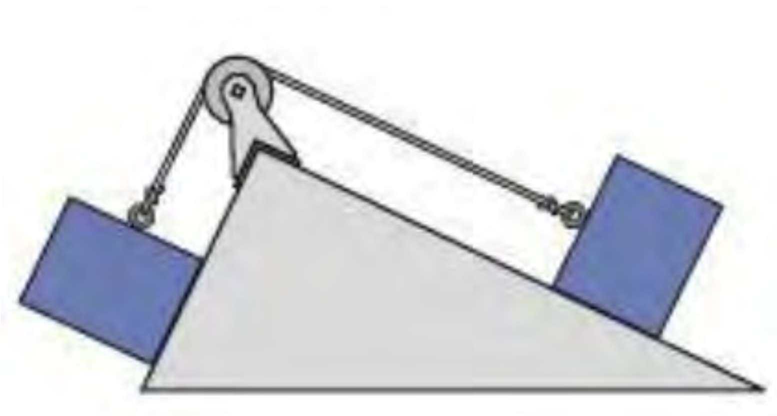
(this makes life much easier downstream!)

Work

- Direction matters! This does make sense intuitively....

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

$$W = F_x \Delta x$$



→ Think about what direction gravity works in and how changing the angle of the wedge would affect "work"

→ More fun when Earth does its work on the skier when on the steep part!

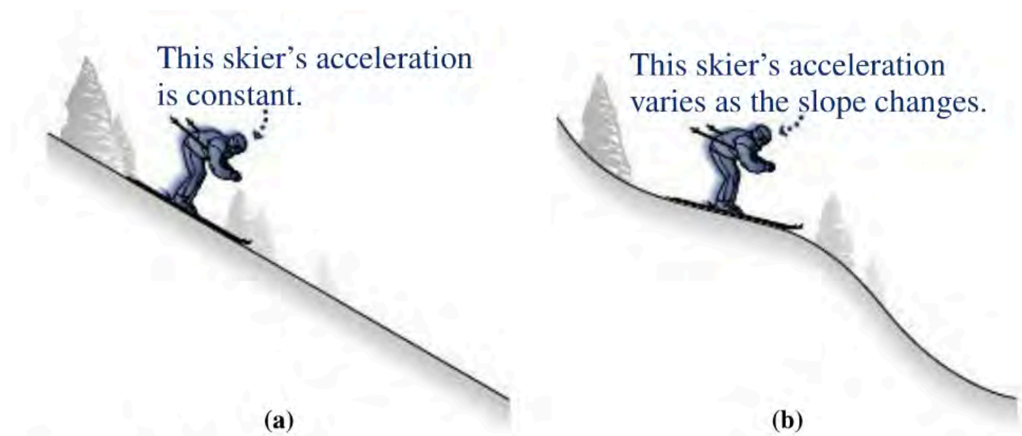


FIGURE 6.1 Two skiers.

Note: When forces are not constant per se, problems can be very hard via Newton's Laws. But they can be much more accessible via the lens of "energy" (as we'll see)

Wolfson

Ex.

How much work is done in lifting

(a) A 5-pound book 3 feet off the floor?

(b) A 1.5-kilogram book 2 meters off the floor?

Hughes-Hallet et al (2005)

Ex. (SOL)

How much work is done in lifting

(a) A 5-pound book 3 feet off the floor?

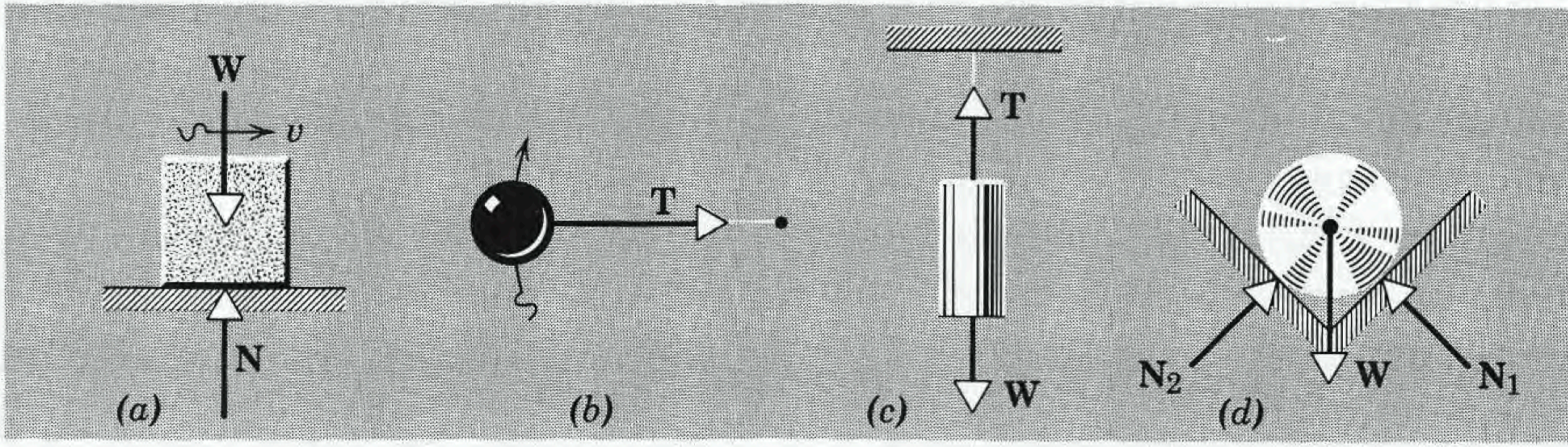
(b) A 1.5-kilogram book 2 meters off the floor?

(a) The force due to gravity is 5 lb, so $W = F \cdot d = (5 \text{ lb})(3 \text{ ft}) = 15$ foot-pounds.

(b) The force due to gravity is $mg = (1.5 \text{ kg})(g \text{ m/sec}^2)$, so

$$W = F \cdot d = [(1.5 \text{ kg})(9.8 \text{ m/sec}^2)] \cdot (2 \text{ m}) = 29.4 \text{ joules}.$$

Hughes-Hallet et al (2005)



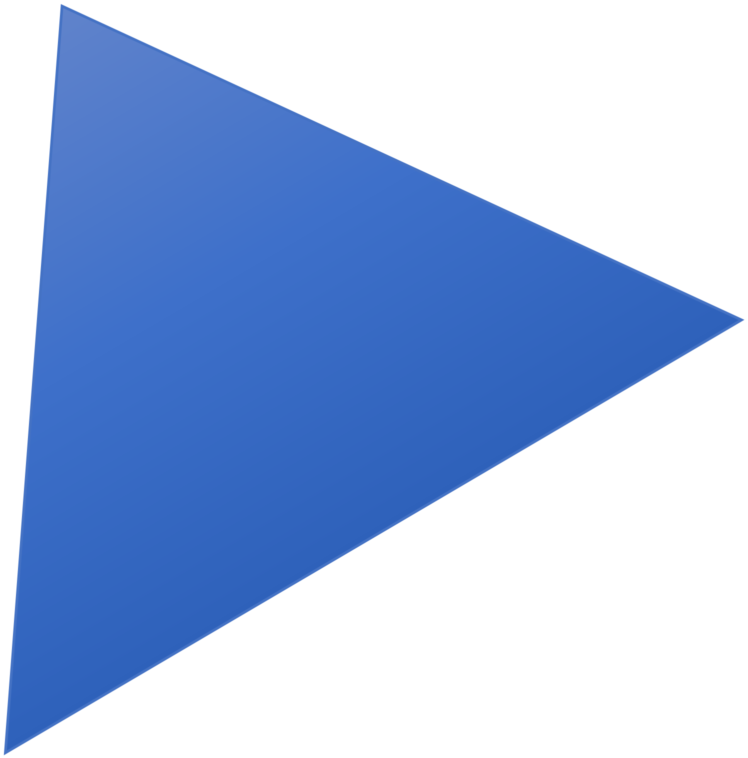
Work is not always done by a force that is applied to a body. (*a*) The block is moving to the right at constant speed v over a frictionless surface. Work is not done by either the weight **W** or the normal force **N**. (*b*) The ball moves in a circle under the influence of a centripetal force **T**. There is a centripetal acceleration **a** but no work is done by **T**. In both (*a*) and (*b*) the forces being considered (**W**, **N**, and **T**) are at right angles to the displacement so that $W = \mathbf{F} \cdot \mathbf{d} = Fd \cos \phi = Fd \cos 90° = 0$. (*c*) A cylinder hangs from a cord. No work is done either by **T**, the tension in the cord, or by **W** the weight of the cylinder. (*d*) A cylinder rests in a groove; no work is done by **W**, $\mathbf{N}_1$ or $\mathbf{N}_2$. In both (*c*) and (*d*) the work done by the individual forces is zero because the displacement is zero.

Resnick & Halliday (1966)

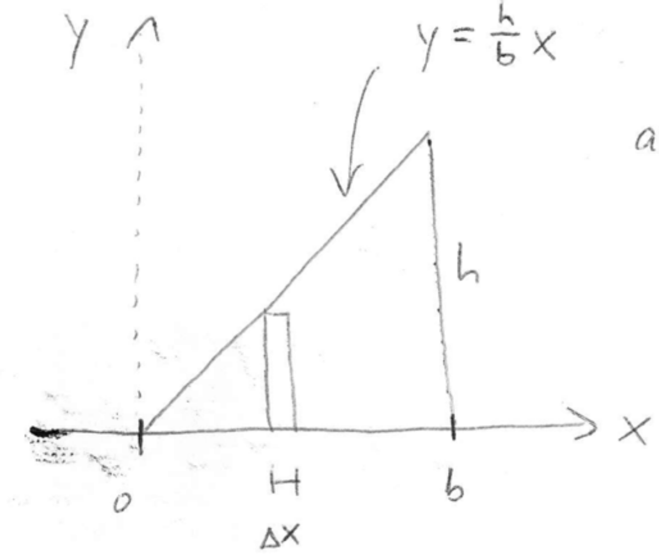
Integrals…

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

- So this whole integration thing.....
- Let's take a tangent: **What is the area of a triangle?**



Tangent: Integrals



$$\text{area}_R = \text{area of rectangle} = \Delta X \cdot \text{height}$$

$$\text{height}(x) = y = \frac{h}{b}x$$

$$\longrightarrow \text{area}_R = \frac{h}{b}x\,\Delta X$$

$$\text{area}_\Delta = \sum_{x=0}^{x=b} \text{area}_R$$

$$= \sum \frac{h}{b}x\,\Delta X$$

$$\text{let } \Delta X \to 0: \quad \text{area}_\Delta = \int_0^b \frac{h}{b}x\,dx = \frac{h}{b}\cdot\frac{1}{2}x^2\Big|_0^b$$

$$= \frac{h}{b}\cdot\frac{1}{2}b^2 = \frac{1}{2}bh$$

(area of triangle)

Example 1 Use horizontal slices to set up a definite integral to calculate the area of the isosceles triangle in Figure 8.1.

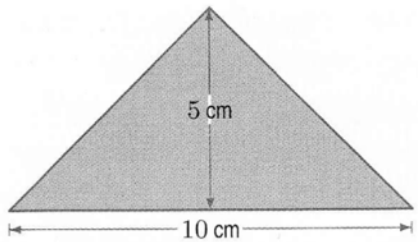


Figure 8.1: Isosceles triangle

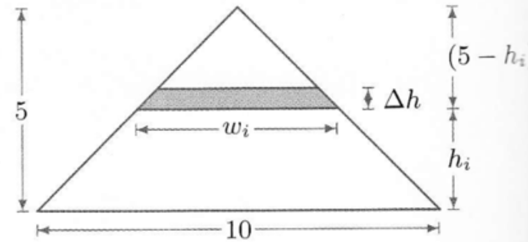


Figure 8.2: Horizontal slicing of isosceles triangle

Solution Notice that we can find the area of a triangle without using an integral; we will use this to check the result from integration:

$$\text{Area} = \frac{1}{2}\,\text{Base} \cdot \text{Height} = 25\ \text{cm}^2.$$

To calculate the area using horizontal slices, see Figure 8.2. A typical strip is approximately a rectangle of length w_i and width Δh, so

$$\text{Area of strip} \approx w_i \Delta h\ \text{cm}^2.$$

To get w_i in terms of h_i, the height above the base, use the similar triangles in Figure 8.2:

$$\frac{w_i}{10} = \frac{5-h_i}{5}$$
$$w_i = 2(5-h_i) = 10 - 2h_i.$$

Summing the areas of the strips gives the Riemann sum approximation

$$\text{Area of triangle} \approx \sum_{i=1}^{n} w_i \Delta h = \sum_{i=1}^{n} (10-2h_i)\Delta h\ \text{cm}^2.$$

Taking the limit as $n \to \infty$ and $\Delta h \to 0$ gives the integral:

$$\text{Area of triangle} = \lim_{n\to\infty} \sum_{i=1}^{n} (10-2h_i)\Delta h = \int_0^5 (10-2h)\,dh\ \text{cm}^2.$$

Evaluating the integral gives

$$\text{Area of triangle} = \int_0^5 (10-2h)\,dh = (10h - h^2)\Big|_0^5 = 25\ \text{cm}^2.$$

Hughes-Hallett et al (2005)

Tangent: Integrals to compute areas

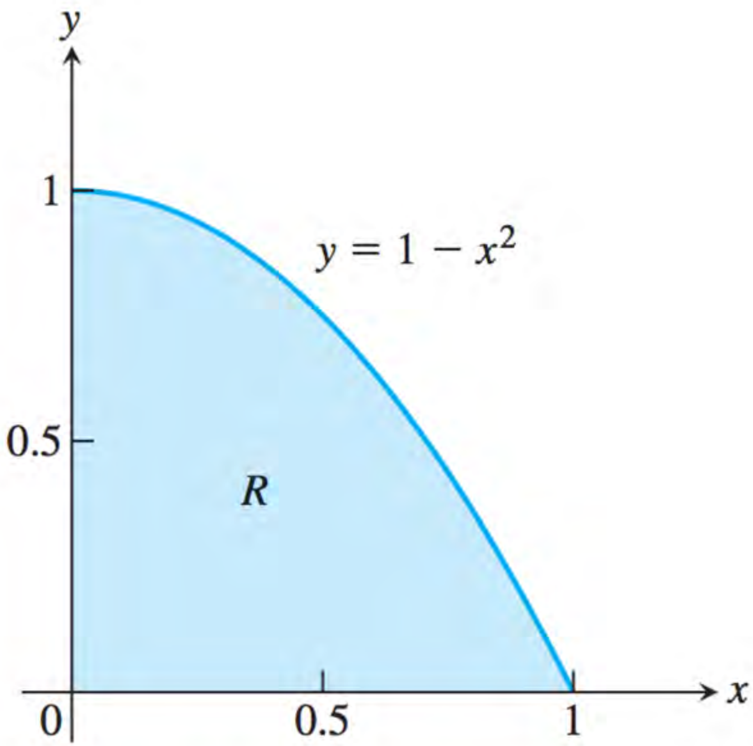


FIGURE 5.1 The area of the region R cannot be found by a simple formula.

→ The shape of the curve makes this kinda hard. But perhaps we can use an "easier" shape (and a lot of them) to get what we need….

Thomas (et al) (2014)

Tangent: Integrals to compute areas

"Riemann sums"

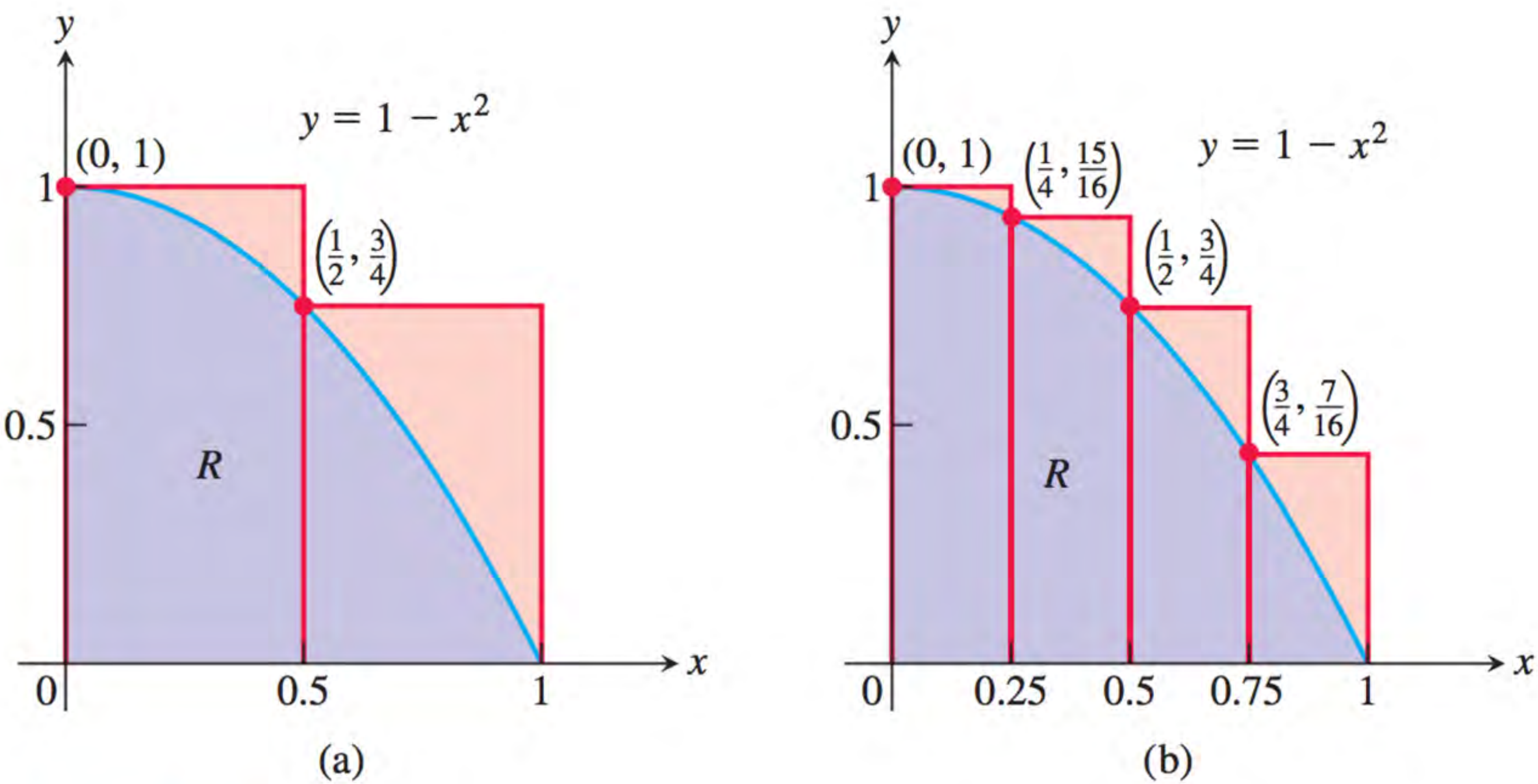


FIGURE 5.2 (a) We get an upper estimate of the area of *R* by using two rectangles containing *R*. (b) Four rectangles give a better upper estimate. Both estimates overshoot the true value for the area by the amount shaded in light red.

Thomas (et al) (2014)

Tangent: Integrals to compute areas

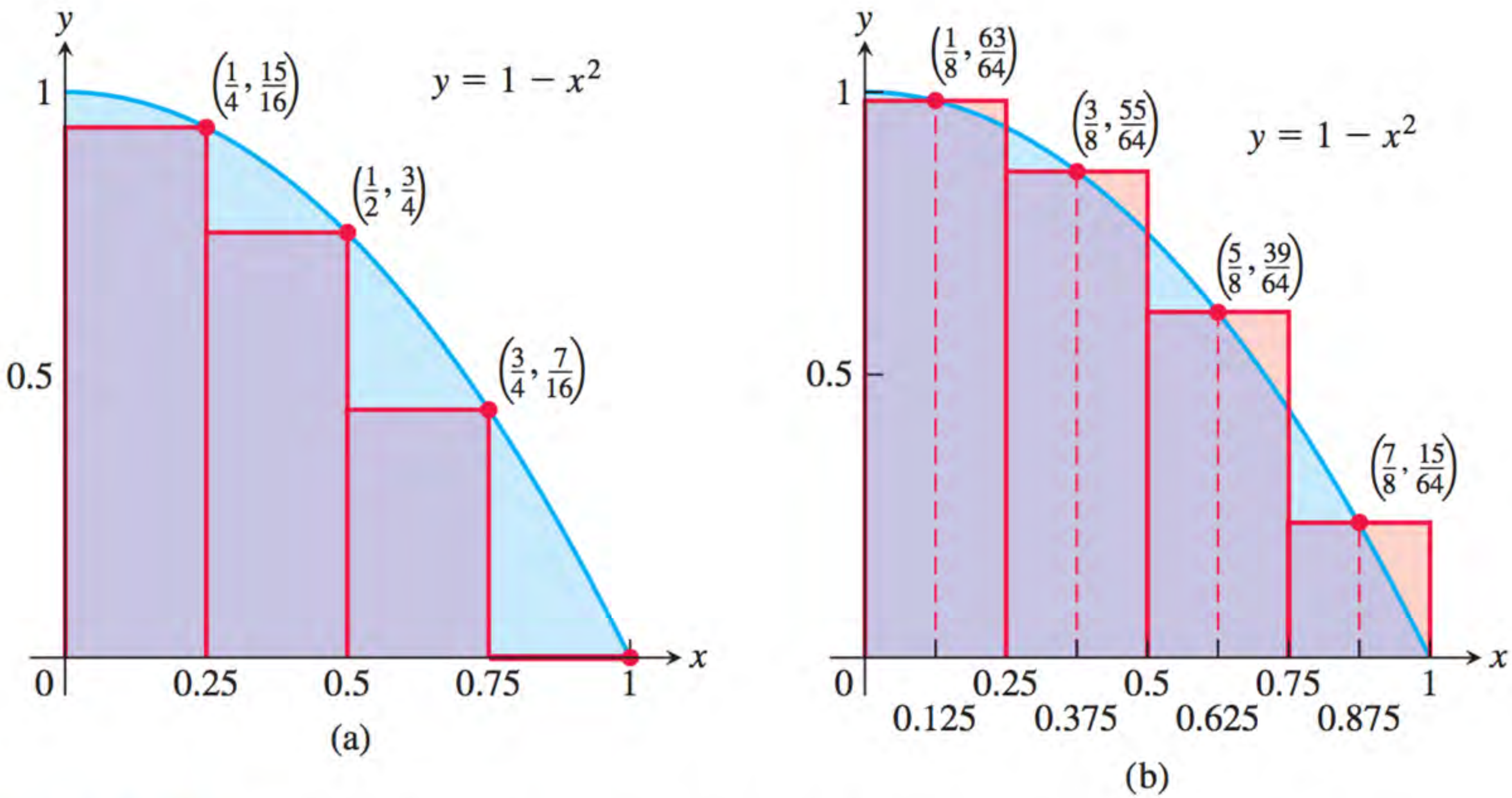


FIGURE 5.3 (a) Rectangles contained in R give an estimate for the area that undershoots the true value by the amount shaded in light blue. (b) The midpoint rule uses rectangles whose height is the value of $y = f(x)$ at the midpoints of their bases. The estimate appears closer to the true value of the area because the light red overshoot areas roughly balance the light blue undershoot areas.

Thomas (et al) (2014)

Tangent: Integrals to compute areas

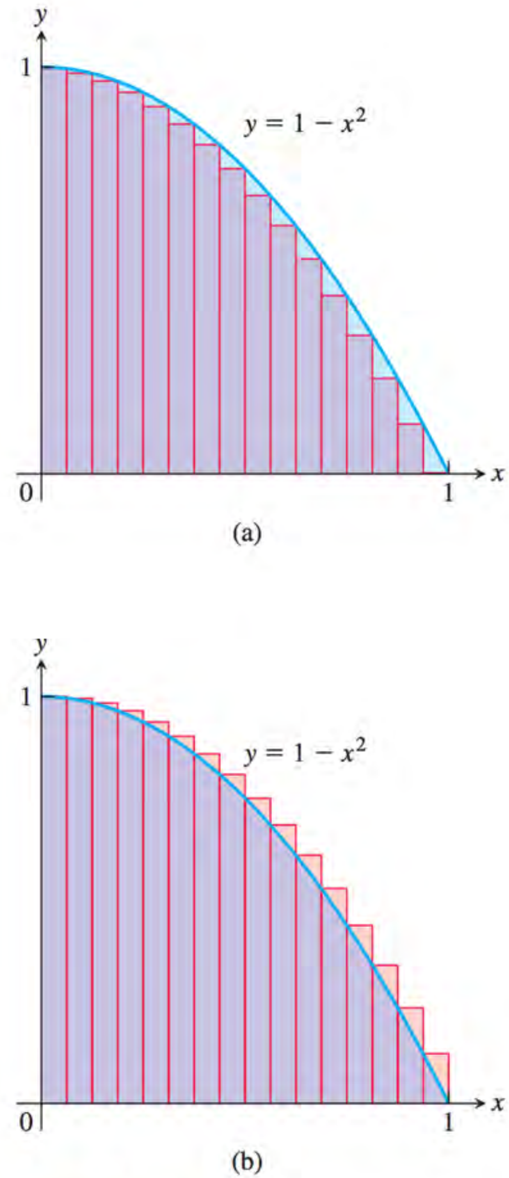


FIGURE 5.4 (a) A lower sum using 16 rectangles of equal width $\Delta x = 1/16$. (b) An upper sum using 16 rectangles.

Thomas (et al) (2014)

Aside: Computational approaches (via Matlab)

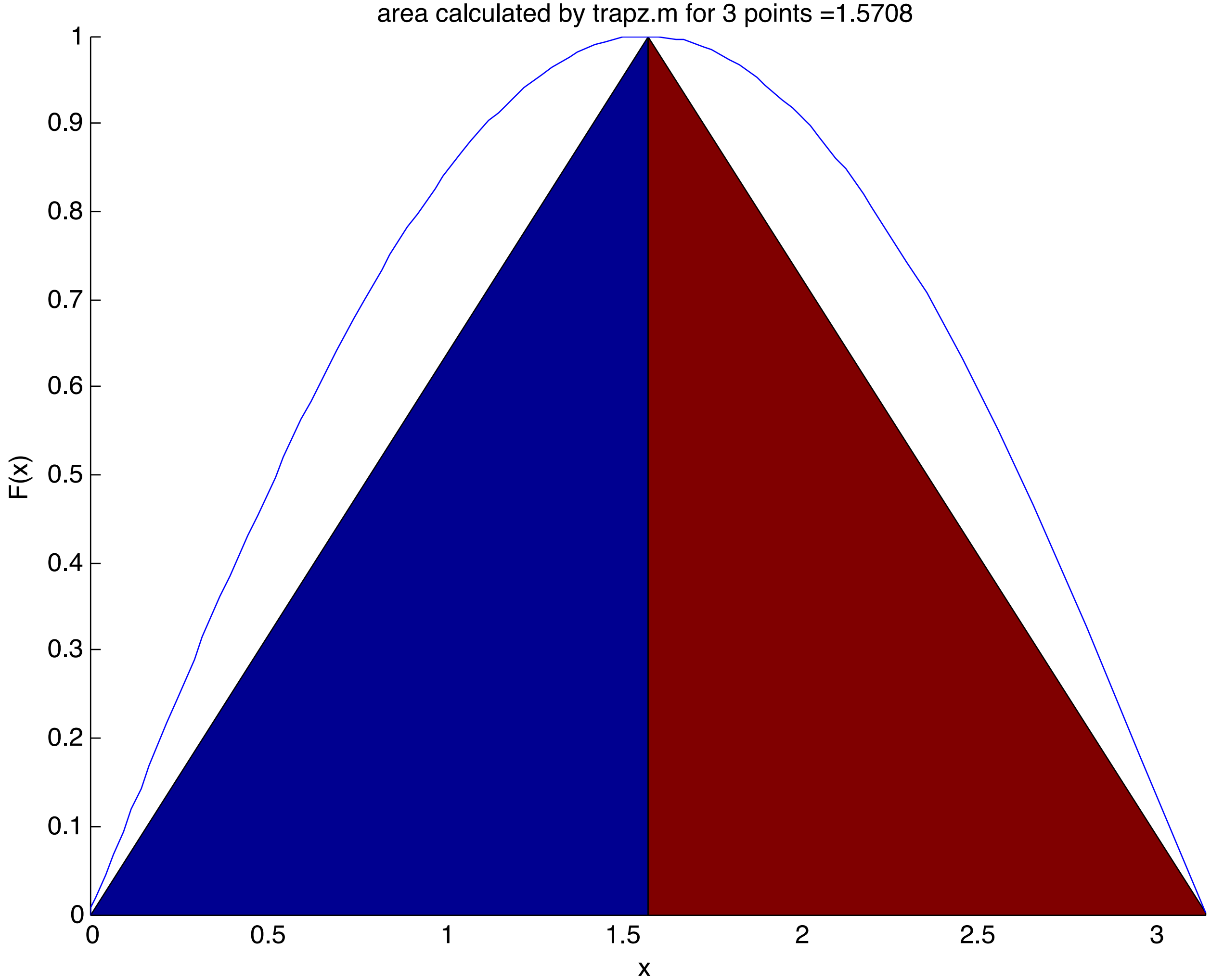
INTexample1.m

```
% Numerical integration example - original source:
% http://ef.engr.utk.edu/ef230-2011-01/modules/matlab-integration/

clear;
% ----------------------
% User parameters
F = @(x)(sin(x)); % function to integrate
%F = @(x)(exp(-x.^2/2)); % function to integrate
xL= [0 pi]; % integration limits

N= 5;    % Method A - # of points for LEFT and RIGHT
pts= [3 4 5 10 25]; % Method B - # of points to consider integrating (via trapz function)
dur= 1;     % Method B - pause duration [s] for trapz loop
% ----------------------

% **************
% Show the curve
figure(1);
fplot(F,[xL(1),xL(2)]) % a quick way to plot a function
xlabel('x'); ylabel('F(x)');

% **************
% Method A
% Approximate the integral via brute force LEFT and RIGHT Riemann sums
sumL= 0; sumR=0;
delX= (xL(2)-xL(1))/N;    % step-size
x= linspace(xL(1),xL(2),N+1);  % add one since N is # of 'boxes' and is really N-1
for nn=1:N
    sumL= sumL + F(x(nn))*delX;
    sumR= sumR + F(x(nn+1))*delX;
end
disp(['left-hand rule yields =',num2str(sumL),' (for ',num2str(N),' steps)']);
disp(sprintf('right-hand rule yields = %g', sumR));

% **************
% Method B
% Approximate the integral via trapz for different numbers of points
for np=pts
    figure(2); clf % clear the current figure
    hold on % allow stuff to be added to this plot
    x = linspace(xL(1),xL(2),np); % generate x values
    y = F(x); % generate y values
    a2 = trapz(x,y); % use trapz to integrate
    % Generate and display the trapezoids used by trapz
    for ii=1:length(x)-1
        px=[x(ii) x(ii+1) x(ii+1) x(ii)];   py=[0 0 y(ii+1) y(ii)];
        fill(px,py,ii)
    end
    fplot(F,[xL(1),xL(2)]); xlabel('x'); ylabel('F(x)');
    disp(['area calculated by trapz.m for ',num2str(np),' points =',num2str(a2)]);
    title(['area calculated by trapz.m for ',num2str(np),' points =',num2str(a2)]);
    pause(dur);   % wait a bit
end

% **************
% Method C
a1 = quad(F,xL(1),xL(2)); % use quad to integrate
msg = ['area calculated by quad.m = ' num2str(a1,10)]; disp(msg);
```

$$A = \int_0^b f(x)\,\mathrm{d}x$$

Three different approaches to doing the integral in the code here

```
Trapezoid method (Method B)
np= 3
```

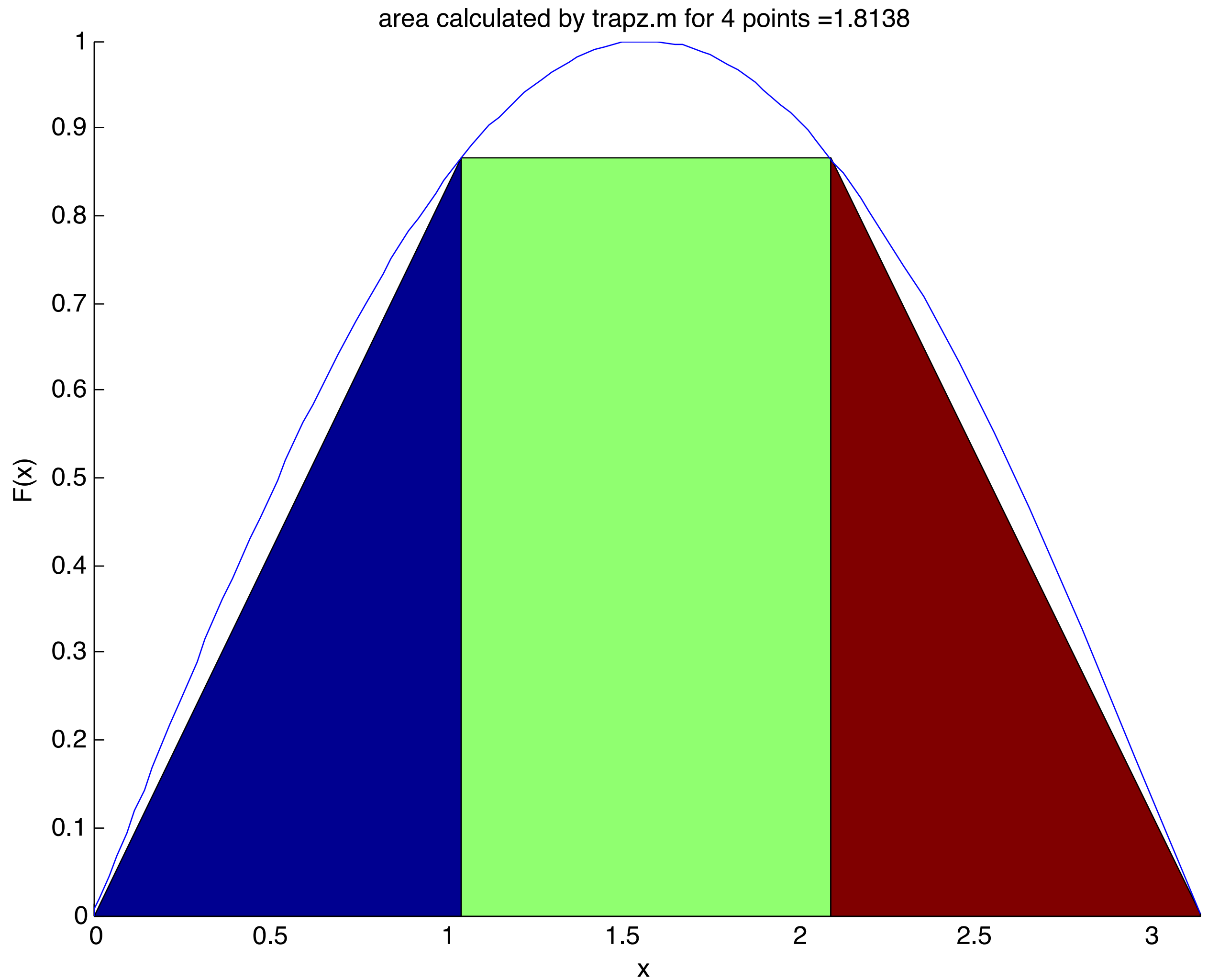


→ Are these rectangles? Why not?

→ Three points means how many ‘rectangles’?

Trapezoid method (Method B)

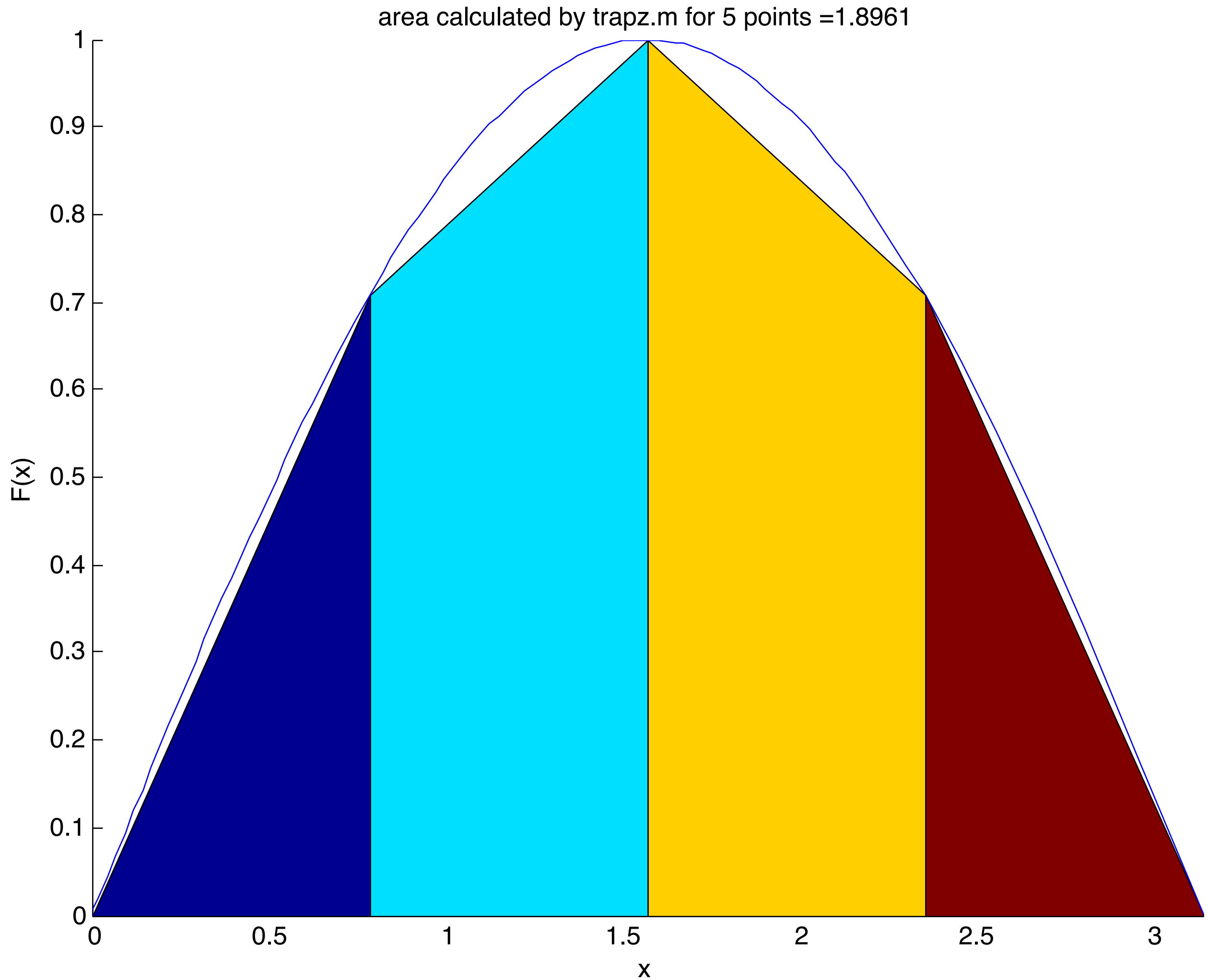
np= 4



→ What is the associated ‘error’?

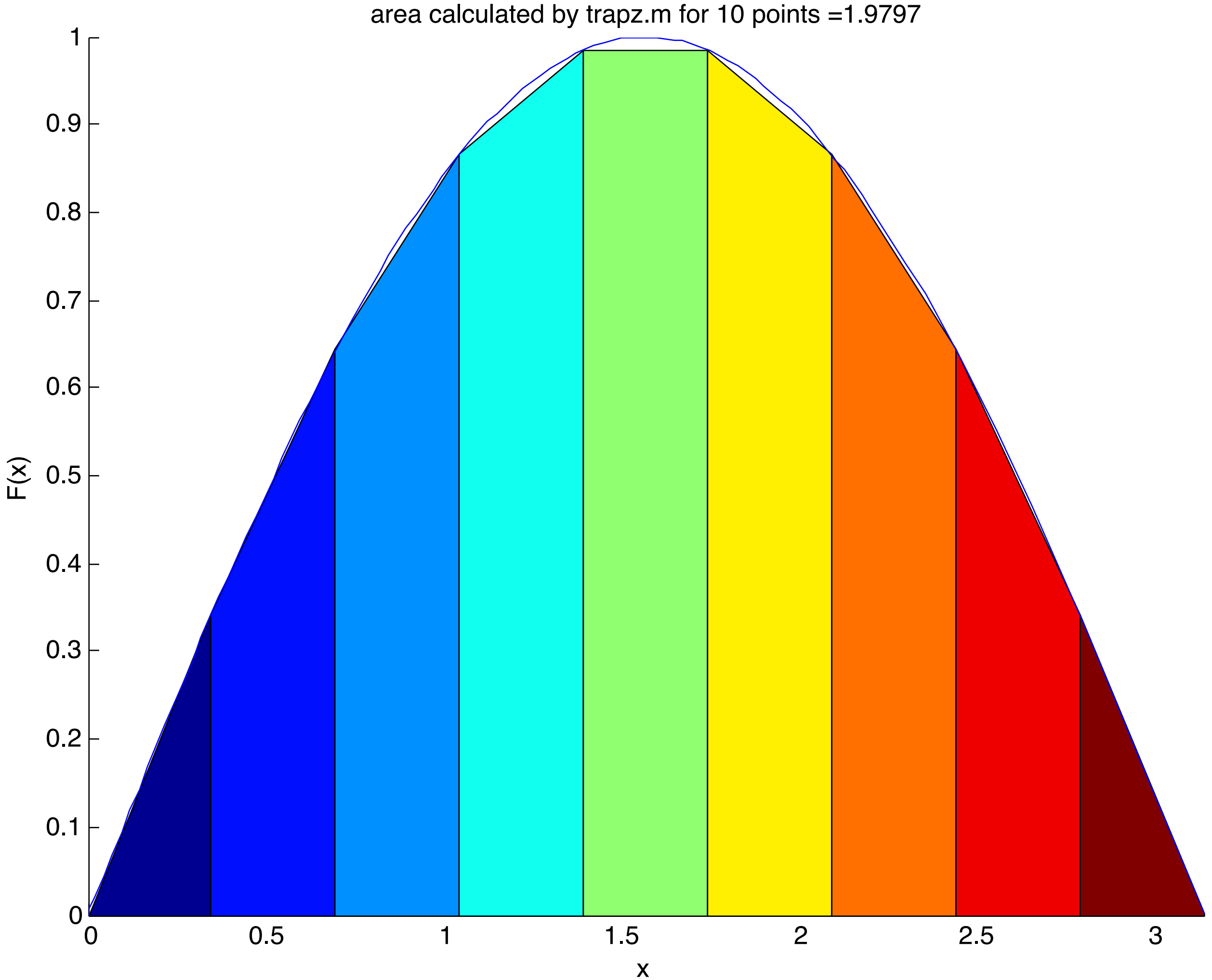
Trapezoid method (Method B)

np= 5



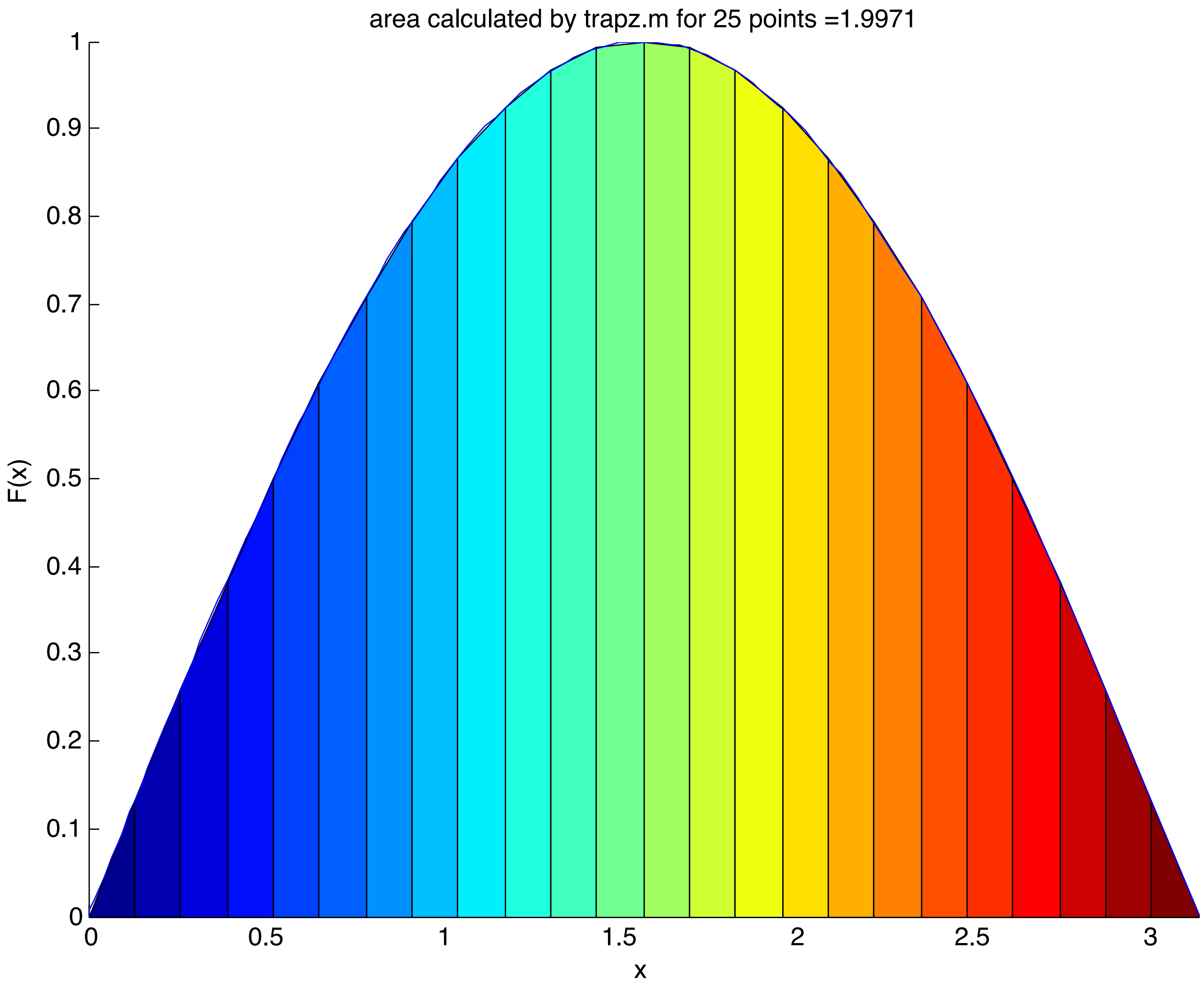
Trapezoid method (Method B)

np= 10



Trapezoid method (Method B)

np= 25



Tangent: Integrals

Basic idea of adding up smaller bits readily scales up from 2-D to 3-D (i.e., “areas” → “slices”)

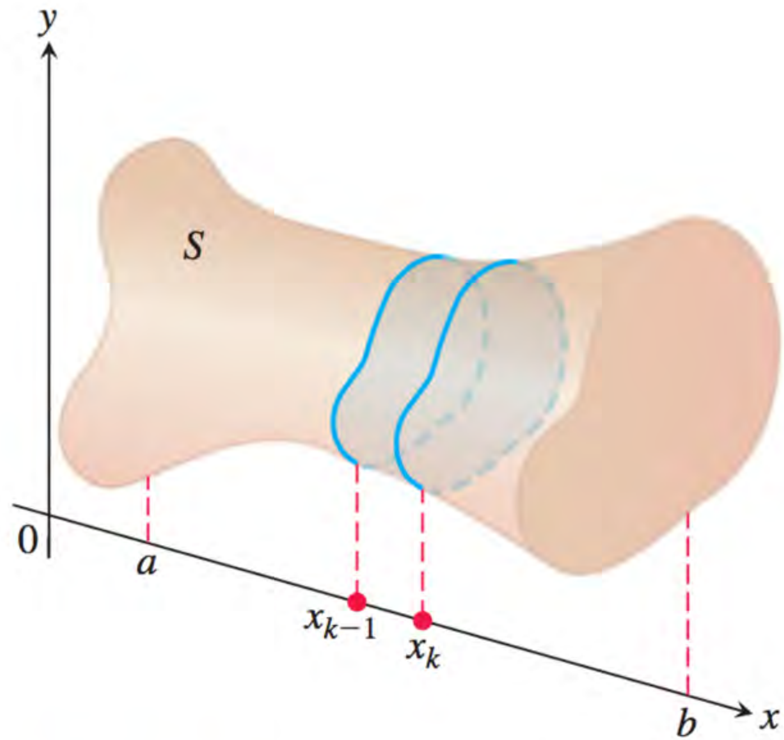


FIGURE 6.3 A typical thin slab in the solid S.

DEFINITION The **volume** of a solid of integrable cross-sectional area $A(x)$ from $x = a$ to $x = b$ is the integral of A from a to b,

$$V = \int_a^b A(x)\, dx.$$

Thomas (et al) (2014)

Example 2 Use horizontal slicing to find the volume of the cone in Figure 8.5.

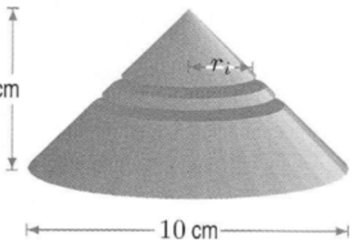


Figure 8.5: Cone

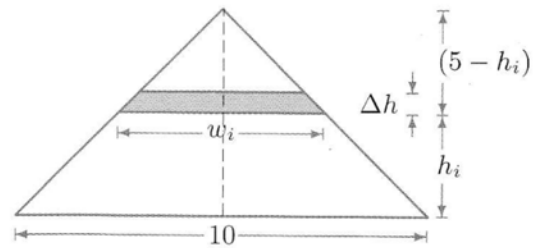


Figure 8.6: Vertical cross-section of cone in Figure 8.5

Solution Each slice is a circular disk of thickness Δh. See Figure 8.5. The disk at height h_i above the base has radius $r_i = \frac{1}{2}w_i$. From Figure 8.6 and the previous example, we have

$$w_i = 10 - 2h_i \quad \text{so} \quad r_i = 5 - h_i.$$

Each slice is approximately a cylinder of radius r_i and thickness Δh, so

$$\text{Volume of slice } \approx \pi r_i^2 \Delta h = \pi(5 - h_i)^2 \Delta h \text{ cm}^3.$$

Summing over all slices, we have

$$\text{Volume of cone } \approx \sum_{i=1}^{n} \pi(5 - h_i)^2 \Delta h \text{ cm}^3.$$

Taking the limit as $n \to \infty$, so $\Delta h \to 0$, gives

$$\text{Volume of cone } = \lim_{n\to\infty} \sum_{i=1}^{n} \pi(5 - h_i)^2 \Delta h = \int_0^5 \pi(5 - h)^2 \, dh \text{ cm}^3.$$

The integral can be evaluated using the substitution $u = 5 - h$ or by multiplying out $(5 - h)^2$. Using the substitution, we have

$$\text{Volume of cone } = \int_0^5 \pi(5 - h)^2 dh = -\frac{\pi}{3}(5 - h)^3 \Big|_0^5 = \frac{125}{3}\pi \text{ cm}^3.$$

Hughes-Hallett et al (2005)

Tangent: Integrals

Note: Multiple approaches all lead to the same answer, but some are *easier* than others…

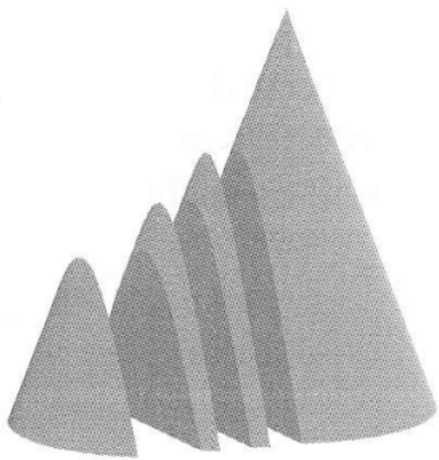


Figure 8.3: Cone cut into vertical slices

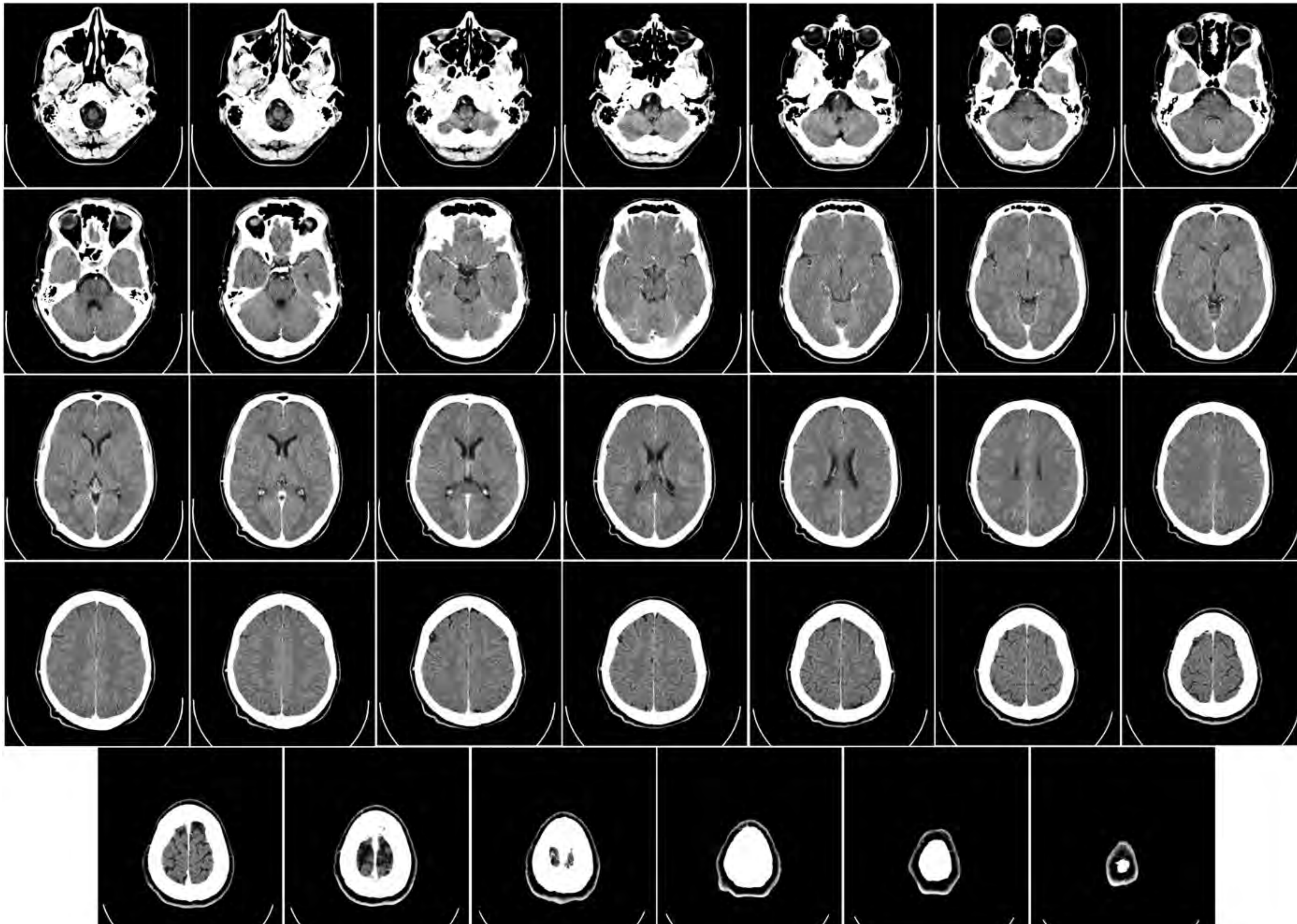
Figure 8.4: Cone cut into horizontal slices

Let's see how we might slice a cone standing with the vertex uppermost. We could divide the cone vertically into arch-shaped slices; see Figure 8.3. We could also divide the cone horizontally, giving coin-shaped slices; see Figure 8.4.

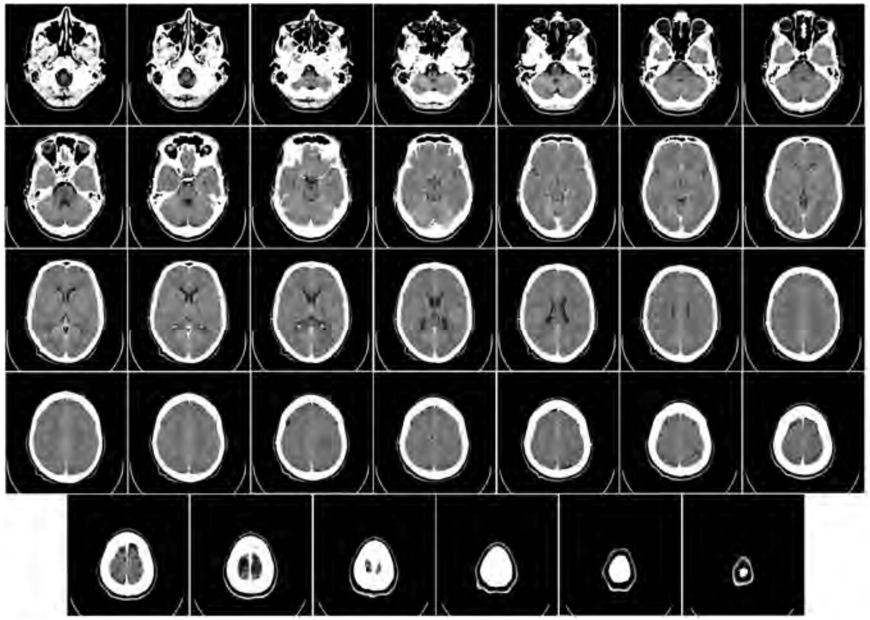
To calculate the volume of the cone, we choose the circular slices because it is easier to estimate the volumes of the coin-shaped slices.

Hughes-Hallett et al (2005)

Aside: Imaging



Wikipedia



Observation: Stacks of 2-D images are ‘sliced’ from a 3-D object

Idea: (Re-)Build up 3-D object from series of 2-D images*

* MRI, CT imaging, two photon imaging, confocal microscopy, etc.... all allow for ‘3-D imaging’, but work under very different principles

Wikipedia

<u>Aside</u>: Tomography

- Uses x-rays (i.e., ionizing radiation)
- Different tissue types have different attenuation coefficients
- Detector signal depends upon effective *attenuation coefficient* of what is in path between it and source
- Source and detector rotate around object, thereby tracing out a series of *projected images*

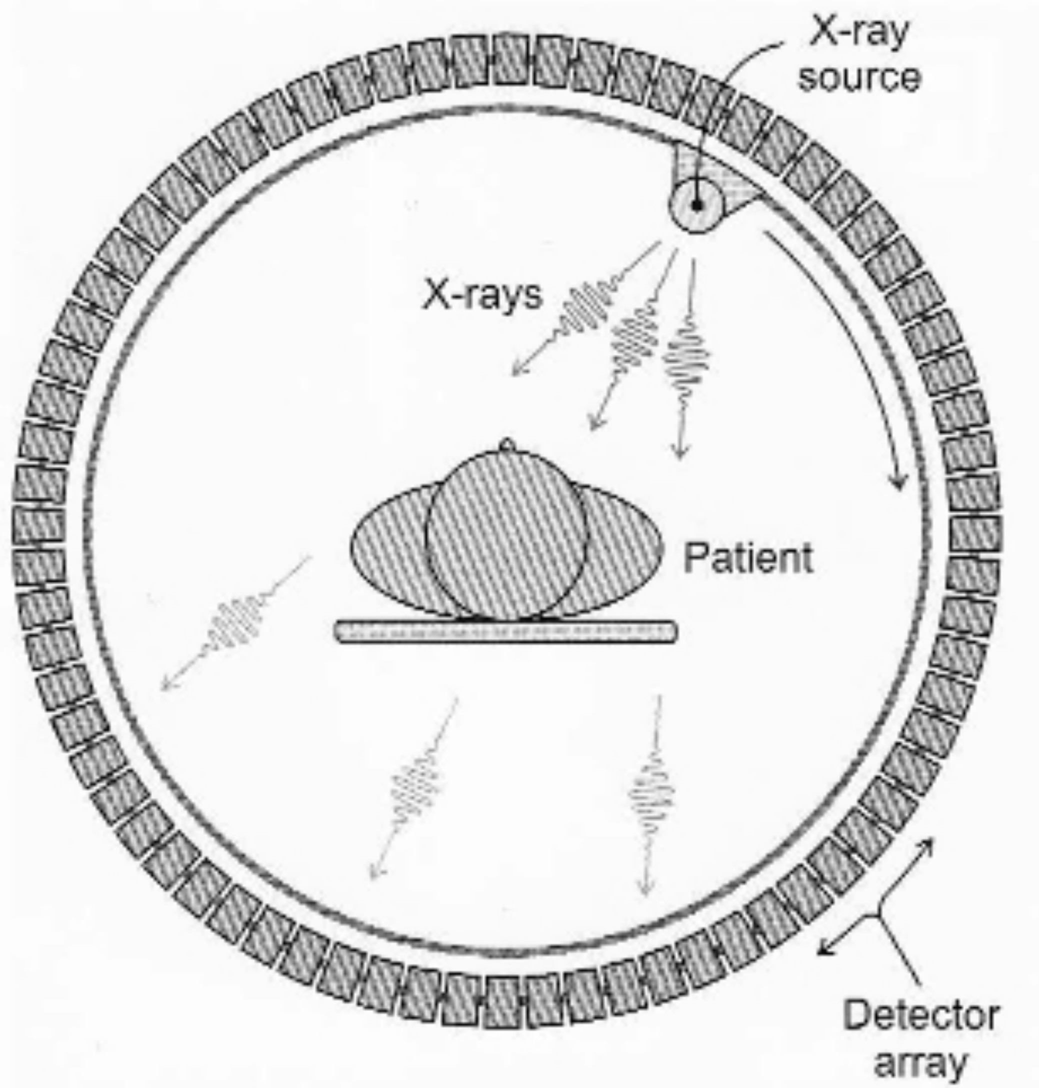


Figure 38.2 Fourth-generation CT scanners employ a circular array of detectors and a rotating source.

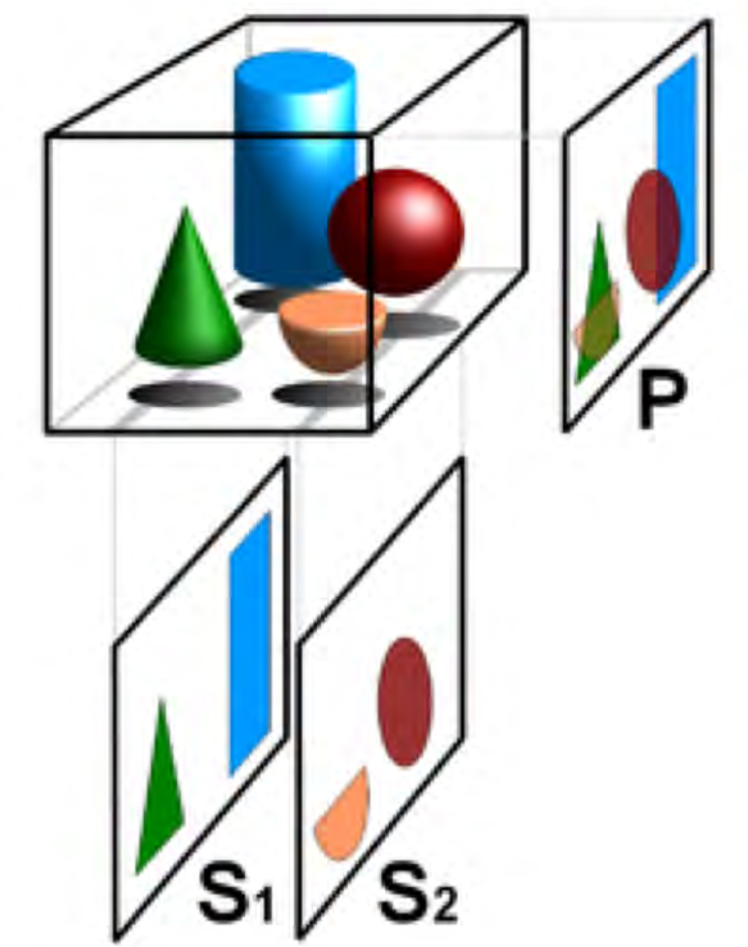
Franklin et al.

Aside: Tomography

Derived from the Greek *tomē* ("cut") or *tomos* ("part" or "section") and *graphein* ("to write")

→ Think of 2-D 'projections' as the sum of 'slices' of a 3-D object

→ From the 2-D projections, goal is to reconstruct 3-D object



Note: We can only (directly) measure P, not S_1 or S_2
[we can only know the 'slices' from the reconstruction]

Wikipedia