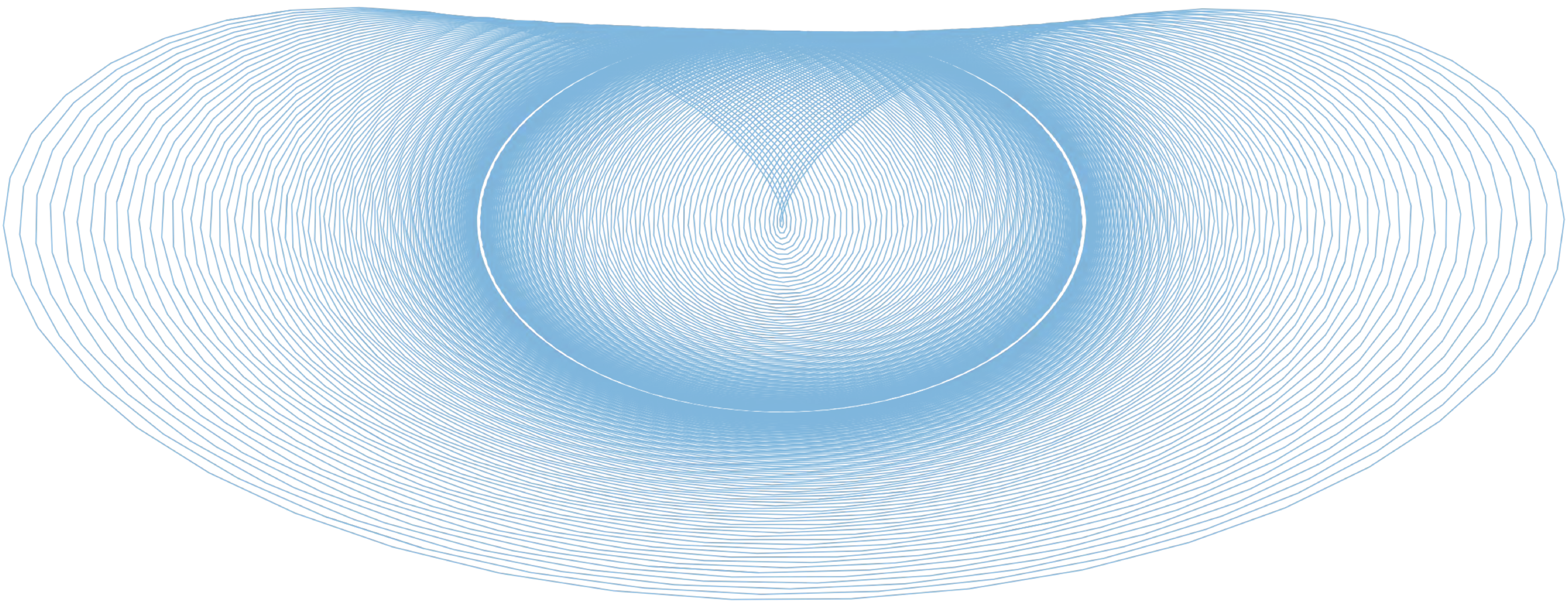
PHYS 1420 (F19)
Physics with Applications to Life Sciences



2019.10.09

Relevant reading:
Kesten & Tauck ch. 6.5-6.7

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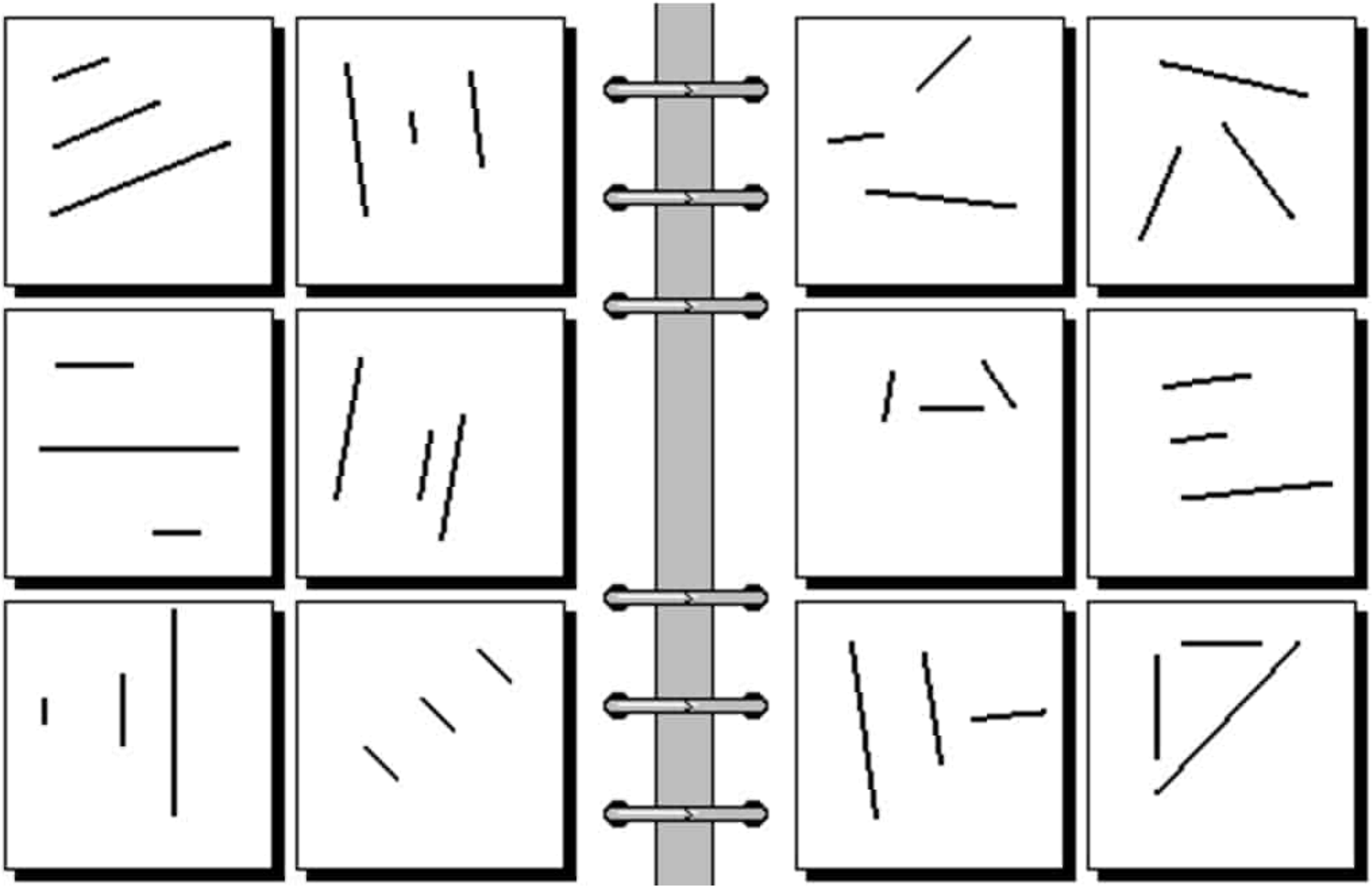
Ref. (re images):
Wolfson (2007), Knight (2017)



Announcements & Key Concepts (re Today)

→ Online HW #5: Posted and due next Monday (10/14)

→ No class next week (10/14-10/18): **READING WEEK**

→ Midterm exam coming up on Monday 10/21

Some relevant underlying concepts of the day…

- Work & Integrals → Spring as an example

- Interdisciplinary connections and (review) examples

- Different *types* of energy…

Work as an Integral

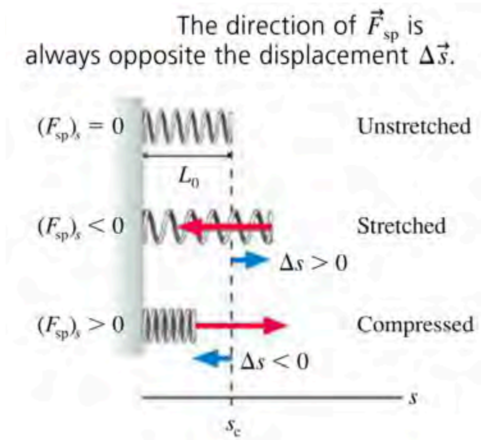
- So this whole integration thing.....

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

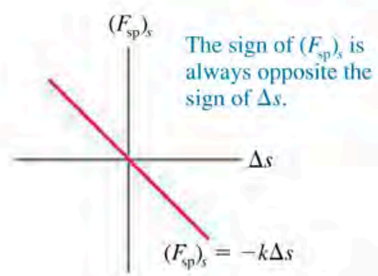
- When the force varies w/ position, the amount of work needed at a given point varies too

Consider stretching a spring:

$$F_s = -kx$$



Knight (2013)



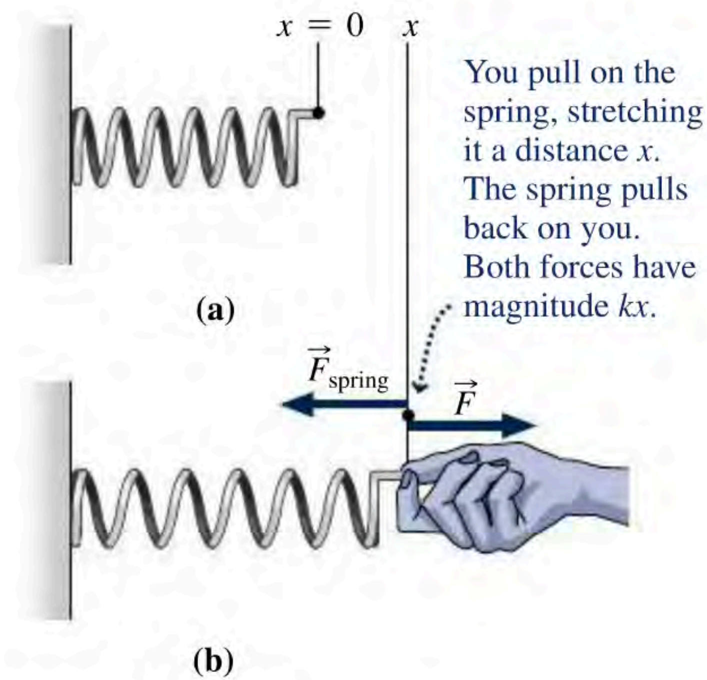


FIGURE 6.10 Stretching a spring.

$$W = \int_0^x F(x)\,dx = \int_0^x kx\,dx = \tfrac{1}{2}kx^2\Big|_0^x = \tfrac{1}{2}kx^2 - \tfrac{1}{2}k(0)^2 = \tfrac{1}{2}kx^2$$

Wolfson

Work as an Integral

- A bit more generally....

→ Connection point to **Riemann sums**

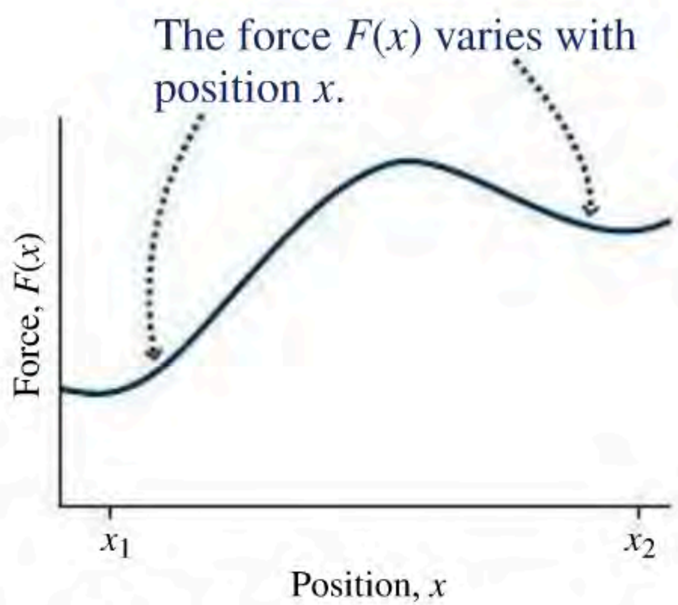
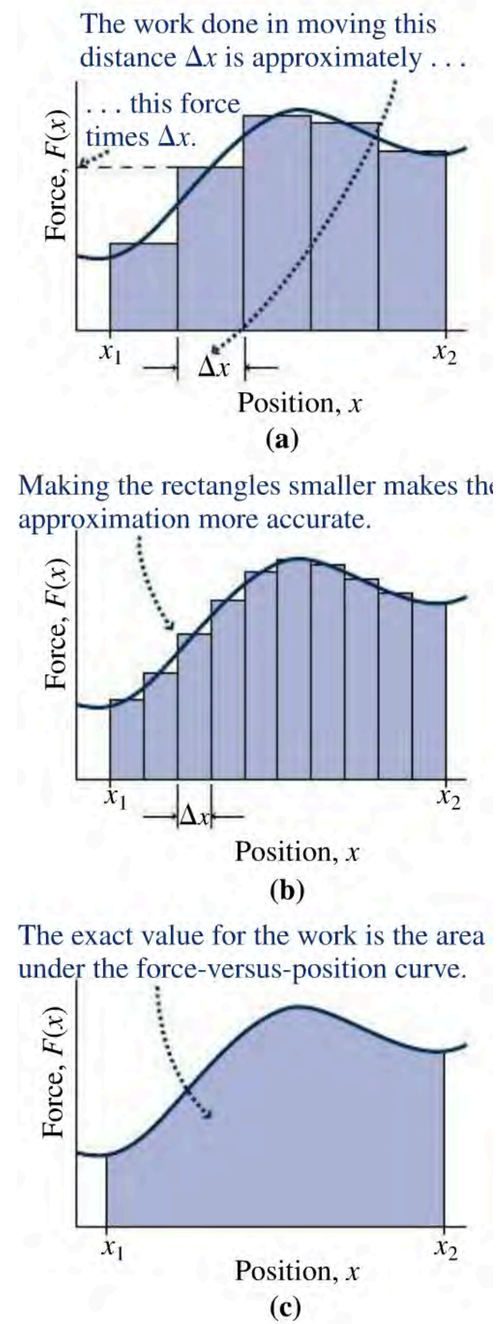


FIGURE 6.8 A varying force.



$$W = \int_{x_1}^{x_2} F(x)\,dx \qquad \left(\begin{array}{c}\text{work done by a varying}\\ \text{force in one dimension}\end{array}\right)$$

Wolfson

Computational Connection

INTexample1.m

```
% Numerical integration example - original source:
% http://ef.engr.utk.edu/ef230-2011-01/modules/matlab-integration/

clear;
% ----------------------
% User parameters
F = @(x)(sin(x)); % function to integrate
%F = @(x)(exp(-x.^2/2)); % function to integrate
xL= [0 pi]; % integration limits

N= 5;    % Method A - # of points for LEFT and RIGHT
pts= [3 4 5 10 25]; % Method B - # of points to consider integrating (via trapz function)
dur= 1;     % Method B - pause duration [s] for trapz loop
% ----------------------

% **************
% Show the curve
figure(1);
fplot(F,[xL(1),xL(2)]) % a quick way to plot a function
xlabel('x'); ylabel('F(x)');

% **************
% Method A
% Approximate the integral via brute force LEFT and RIGHT Riemann sums
sumL= 0; sumR=0;
delX= (xL(2)-xL(1))/N;    % step-size
x= linspace(xL(1),xL(2),N+1);  % add one since N is # of 'boxes' and is really N-1
for nn=1:N
    sumL= sumL + F(x(nn))*delX;
    sumR= sumR + F(x(nn+1))*delX;
end
disp(['left-hand rule yields =',num2str(sumL),' (for ',num2str(N),' steps)']);
disp(sprintf('right-hand rule yields = %g', sumR));

% **************
% Method B
% Approximate the integral via trapz for different numbers of points
for np=pts
    figure(2); clf % clear the current figure
    hold on % allow stuff to be added to this plot
    x = linspace(xL(1),xL(2),np); % generate x values
    y = F(x); % generate y values
    a2 = trapz(x,y); % use trapz to integrate
    % Generate and display the trapezoids used by trapz
    for ii=1:length(x)-1
        px=[x(ii) x(ii+1) x(ii+1) x(ii)];   py=[0 0 y(ii+1) y(ii)];
        fill(px,py,ii)
    end
    fplot(F,[xL(1),xL(2)]); xlabel('x'); ylabel('F(x)');
    disp(['area calculated by trapz.m for ',num2str(np),' points =',num2str(a2)]);
    title(['area calculated by trapz.m for ',num2str(np),' points =',num2str(a2)]);
    pause(dur);   % wait a bit
end

% **************
% Method C
a1 = quad(F,xL(1),xL(2)); % use quad to integrate
msg = ['area calculated by quad.m = ' num2str(a1,10)]; disp(msg);
```

$$A = \int_0^b f(x)\,\mathrm{d}x$$

Note: Three different approaches to doing the integral (via Riemann sums) in the code here

Interdisciplinary Connection (Mathematics)

Position, Velocity, and Acceleration

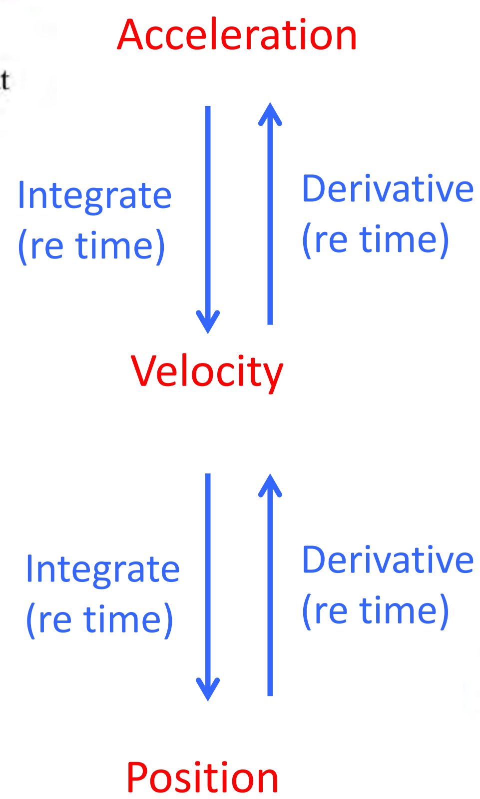
Derivative Form

If $s = s(t)$ is the position function of an object at time t, then

$$\text{Velocity} = v = \frac{ds}{dt} \qquad \text{Acceleration} = a = \frac{dv}{dt}$$

Integral Form

$$s(t) = \int v(t)dt \qquad v(t) = \int a(t)dt$$



→ Sometimes integration is called "anti-differentiation"

	Derivative Form
Position	$r(t)$
Velocity	$v(t) = \frac{dr}{dt}$
Acceleration	$a(t) = \frac{dv}{dt} = \frac{d^2r}{dt^2}$

→ So many "models" take the form of differential equations (e.g., Newton's 2^{nd}) and we use integration to solve such

Review (re integration)

Table 2.1 Equations of Motion for Constant Acceleration

Equation	Contains	Number
$v = v_0 + at$	v, a, t; no x	2.7
$x = x_0 + \frac{1}{2}(v_0 + v)t$	x, v, t; no a	2.9
$x = x_0 + v_0 t + \frac{1}{2}at^2$	x, a, t; no v	2.10
$v^2 = v_0^2 + 2a(x - x_0)$	x, v, a; no t	2.11

→ Using integration, you can easily derive the formulae up top!

Position, Velocity, and Acceleration

Derivative Form

If $s = s(t)$ is the position function of an object at time t, then

Velocity $= v = \frac{ds}{dt}$ Acceleration $= a = \frac{dv}{dt}$

Integral Form

$s(t) = \int v(t)dt$ $v(t) = \int a(t)dt$

Wolfson

<u>Interdisciplinary Connection</u> (Mathematics)

→ Connection point back to friction/drag

Falling body: Terminal velocity

Assume air resistance is proportional to velocity, the Newton's 2nd Law leads to:

$$m\frac{dv}{dt} = mg - kv$$

$$\frac{dv}{dt} = -\frac{k}{m}\left(v - \frac{mg}{k}\right)$$

Free-body diagram

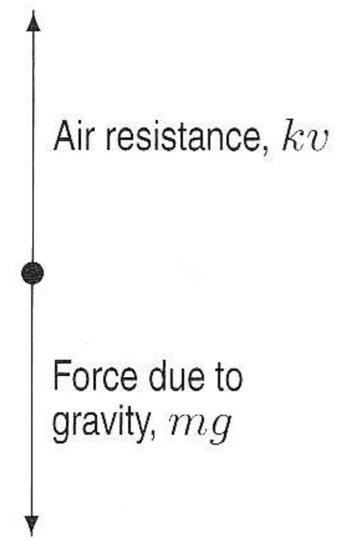


Figure 11.44: Forces acting on a falling object

Solution

$$v = \frac{mg}{k}\left(1 - e^{-kt/m}\right)$$

<u>Note</u>: This approach is a bit more powerful than the steady-state one in that we know the explicit time-dependence!

Hughes-Hallett et al. (2005)

Review: Terminal Velocity

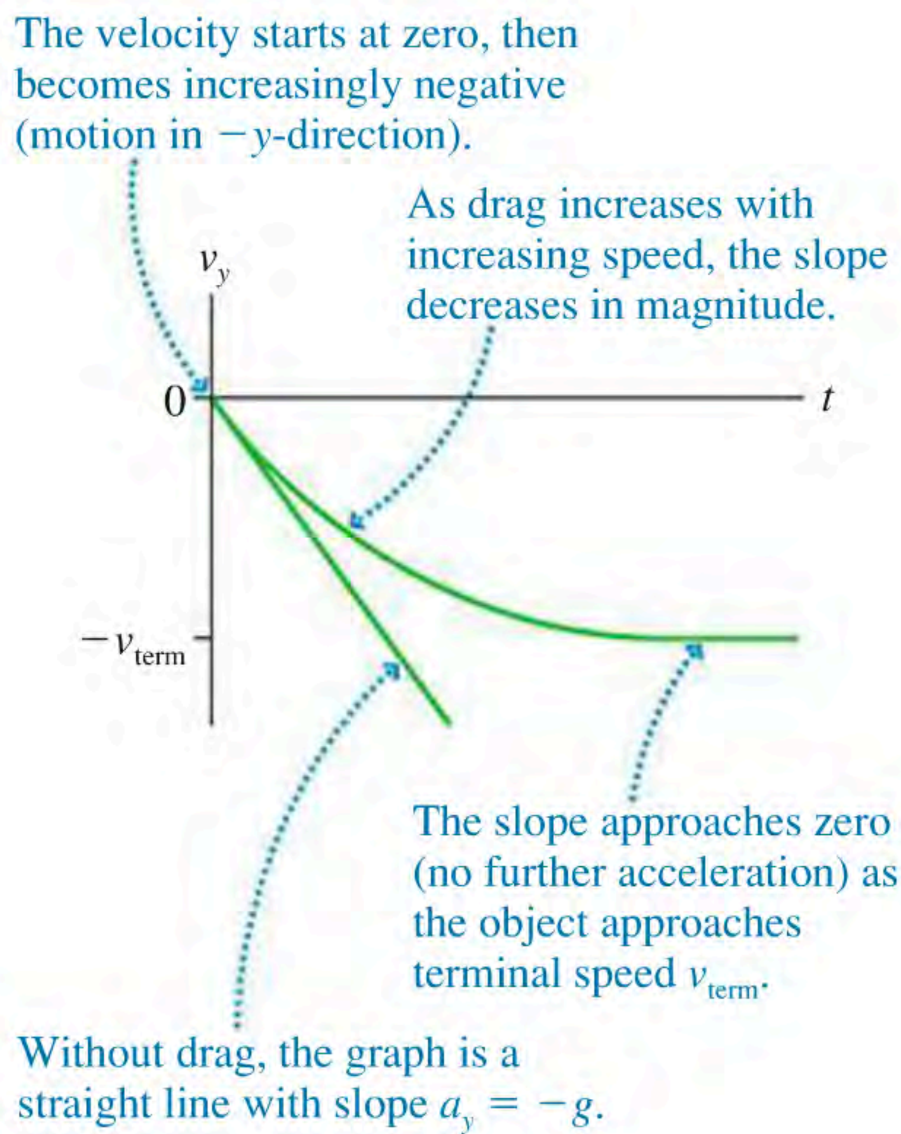
$$v_{\text{term}} = \sqrt{\frac{2mg}{C\rho A}}$$

- This equation applies to *"steady-state"* (i.e., doesn't tell you how things are changing w/ time)

$$D = F_{\text{G}}$$

$$v = \frac{mg}{k}\left(1 - e^{-kt/m}\right)$$

The velocity-versus-time graph of a falling object with and without drag.



Knight (2013)

Interdisciplinary Connection (Mathematics)

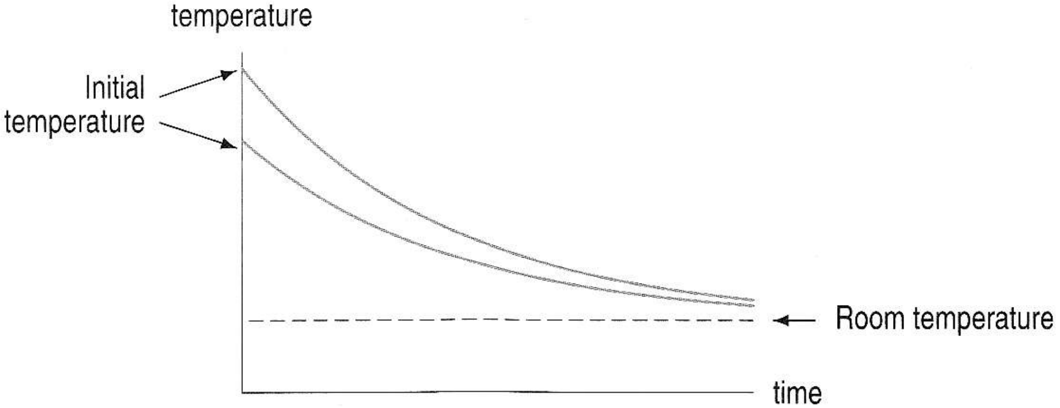
- Connecting back to *warmth*....

Newton's law of heating/cooling

$$\frac{dT}{dt} = \alpha(T_o - T)$$

Solution

$$T(t) = T_0 + Ce^{-\alpha t}$$



Reminder (general solution):

Differential equation

$$\tau \frac{dn(t)}{dt} + n(t) = n_\infty$$

Solution

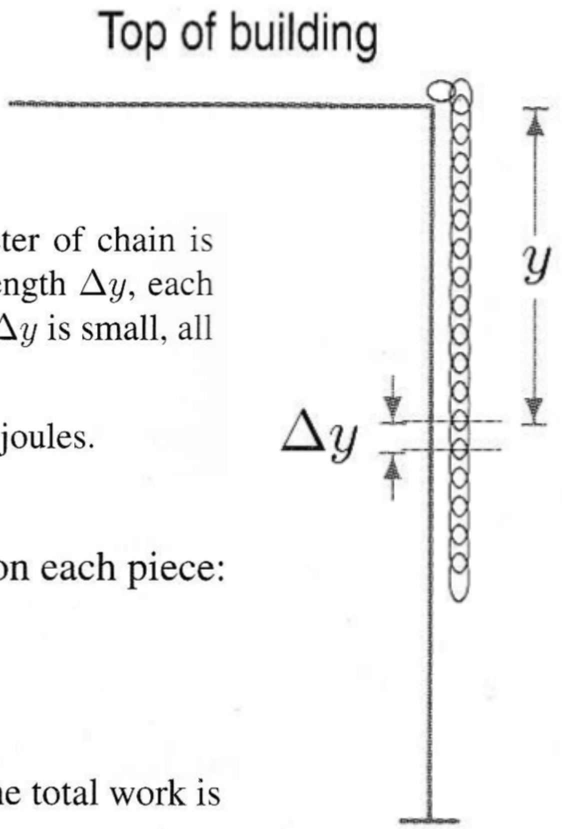
$$n(t) = n_\infty + \left(n_0 - n_\infty\right) e^{-t/\tau}$$

Work as an Integral (intuitive example)

Note: Be careful re the specified units!

A 28-meter uniform chain with a mass 2 kilograms per meter is dangling from the roof of a building. How much work is needed to pull the chain up onto the top of the building?

→ Very useful starting point is to draw a diagram and set up the relevant variables!



Since 1 meter of the chain has mass density 2 kg, the gravitational force per meter of chain is $(2\,\text{kg})(9.8\,\text{m/sec}^2) = 19.6$ newtons. Let's divide the chain into small sections of length Δy, each requiring a force of $19.6\,\Delta y$ newtons to move it against gravity. See Figure 8.61. If Δy is small, all of this piece is hauled up approximately the same distance, namely y, so

$$\text{Work done on the small piece} \approx (19.6\,\Delta y\,\text{newtons})(y\,\text{meters}) = 19.6y\,\Delta y\,\text{joules}.$$

The work done on the entire chain is given by the total of the work done on each piece:

$$\text{Work done} \approx \sum 19.6y\,\Delta y\,\text{joules}.$$

As $\Delta y \to 0$, we obtain a definite integral. Since y varies from 0 to 28 meters, the total work is

$$\text{Work done} = \int_0^{28} (19.6y)\,dy = 9.8y^2\Big|_0^{28} = 7683.2\,\text{joules}.$$

Hughes-Hallet et al (2005)

Exercise

A 28-meter uniform chain with a mass 2 kilograms per meter is dangling from the roof of a building. How much work is needed to pull the chain up onto the top of the building?

A chain of length x and mass m is hanging over the edge of a tall building and does not touch the ground. How much work is required to lift the chain to the top of the building?

→ Rework the problem, finding a more general solution that eschews quantitative values

→ More problems along these lines are at the “back” of the slides

<u>Different Types of Energy</u>

- At the most basic level, there are two ways to characterize energy:

  - Kinetic vs Potential energy
  - "Good" vs "Bad" energy (i.e., free energy vs entropy)

    → Such is embodied here.... $\Delta U = q + w$

> Thermal energy is the sum of the microscopic kinetic and potential energies of all the atoms and bonds that make up the object. An object has more thermal energy when hot than when cold.

Knight (2013)

→ So "warmth" fits in here somehow...

- Think of kinetic energy as tied to *motion* while potential energy is *stored*

Conservation of Energy

- Perhaps one of the most important concepts in all of science....

> *"Energy can neither be created nor destroyed; rather, it transforms from one form to another"*

Note: Such is consistent w/ our foundation at the heart of mechanics, that ***change*** is a key consideration

Physics: Context for, well, just about everything   e.g., $E = mc^2$

Chemistry: Foundation of the "laws of thermodynamics"

$$\Delta U = q + w$$

Biology: Much of evolution is geared around minimizing wasted energy

Engineering: Efficiency (of energy conversion) as a fundamental design principle