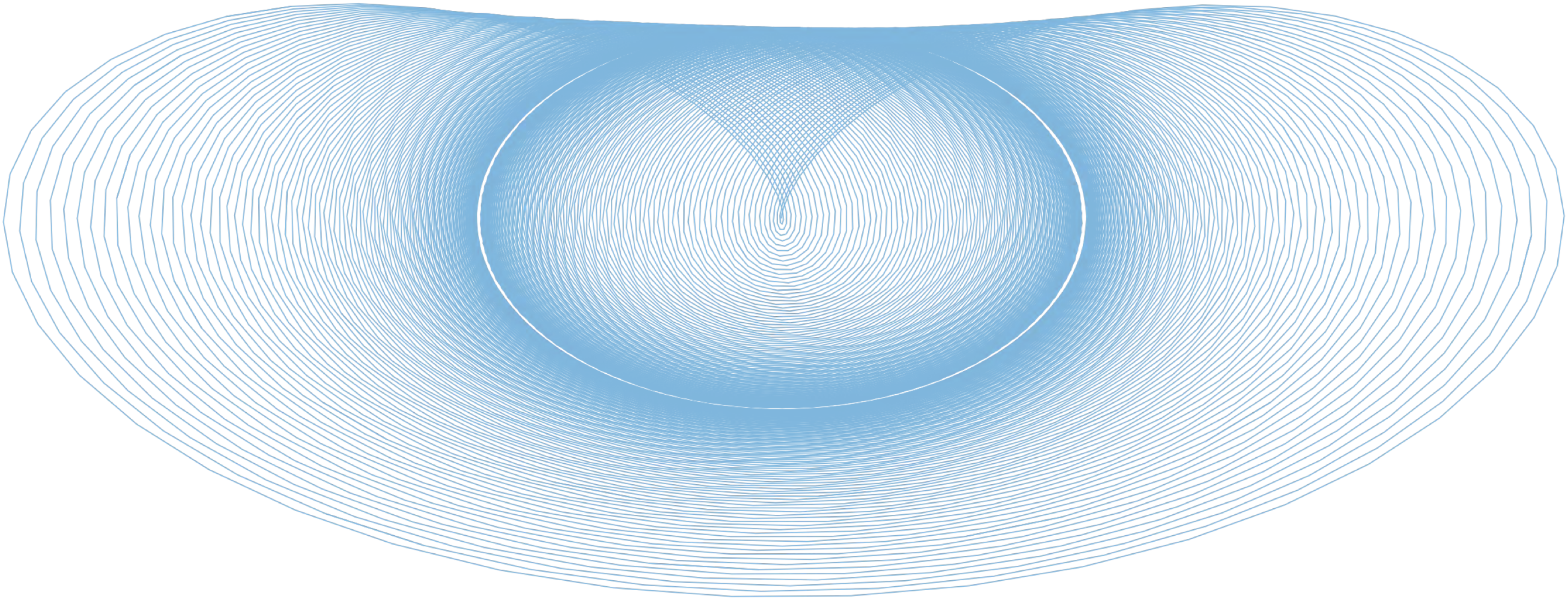
PHYS 1420 (F19)
Physics with Applications to Life Sciences



2019.10.11

Relevant reading:
Kesten & Tauck ch. 6.8

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Ref. (re images):
Wolfson (2007), Knight (2017)

Four sisters left a slice of pizza in the kitchen and went to watch a movie in the other room. Two hours later they went back into the kitchen – and the slice was gone!

Katy said "Holly ate it!"

Holly said "Amy ate it!"

Amy said "Holly's lying!"

Poppy said "Well, it wasn't me!"

If just ONE of the sisters' statements is true – who ate the pizza?

<u>Announcements & Key Concepts</u> (re Today)

→ Online HW #5: Posted and due next Monday (10/14)

→ No class next week (10/14-10/18): **READING WEEK**

→ Midterm exam coming up on Monday 10/21

Some relevant underlying concepts of the day…

- Conservation of energy
- Interdisciplinary connections
- Connection point to *momentum*…

Conservation of Energy

- Perhaps one of the most important concepts in all of science....

> *"Energy can neither be created nor destroyed; rather, it transforms from one form to another"*

Note: Such is consistent w/ our foundation at the heart of mechanics, that ***change*** is a key consideration

Physics: Context for, well, just about everything e.g., $E = mc^2$

Chemistry: Foundation of the "laws of thermodynamics" $\Delta U = q + w$

Biology: Much of evolution is geared around minimizing wasted energy

Engineering: Efficiency (of energy conversion) as a fundamental design principle

Interdisciplinary Connection (Biology)

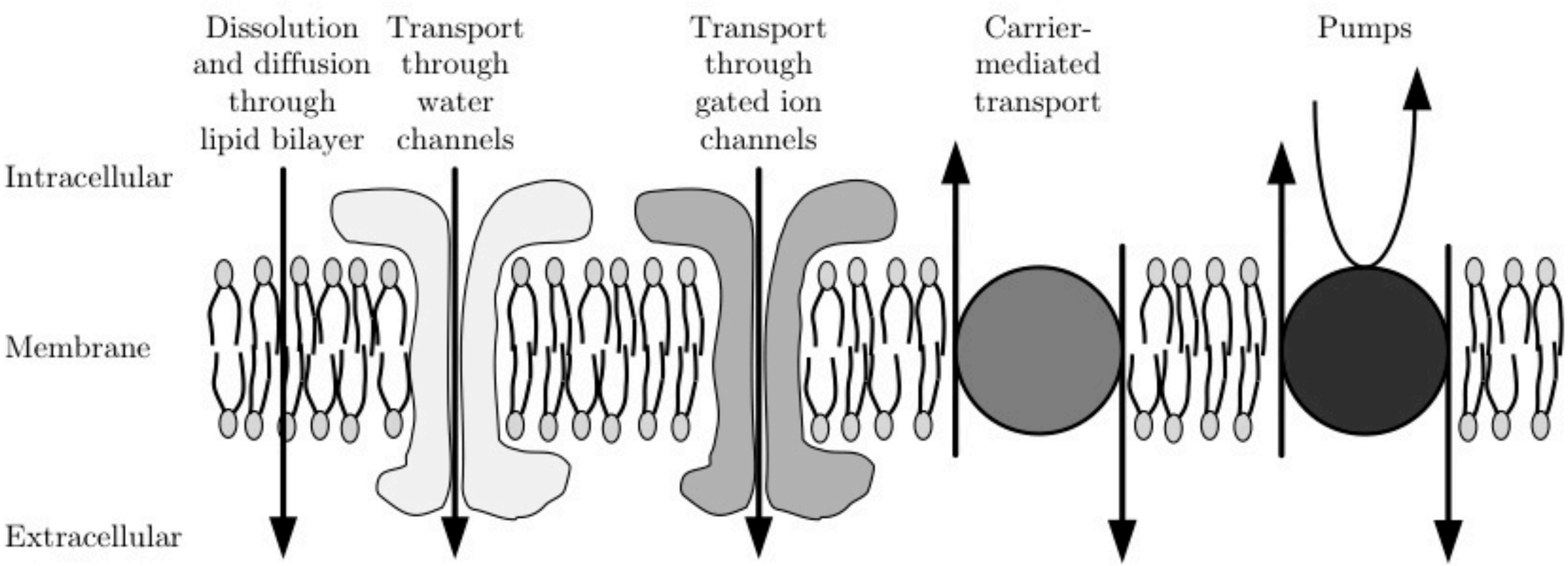


Figure 2.19

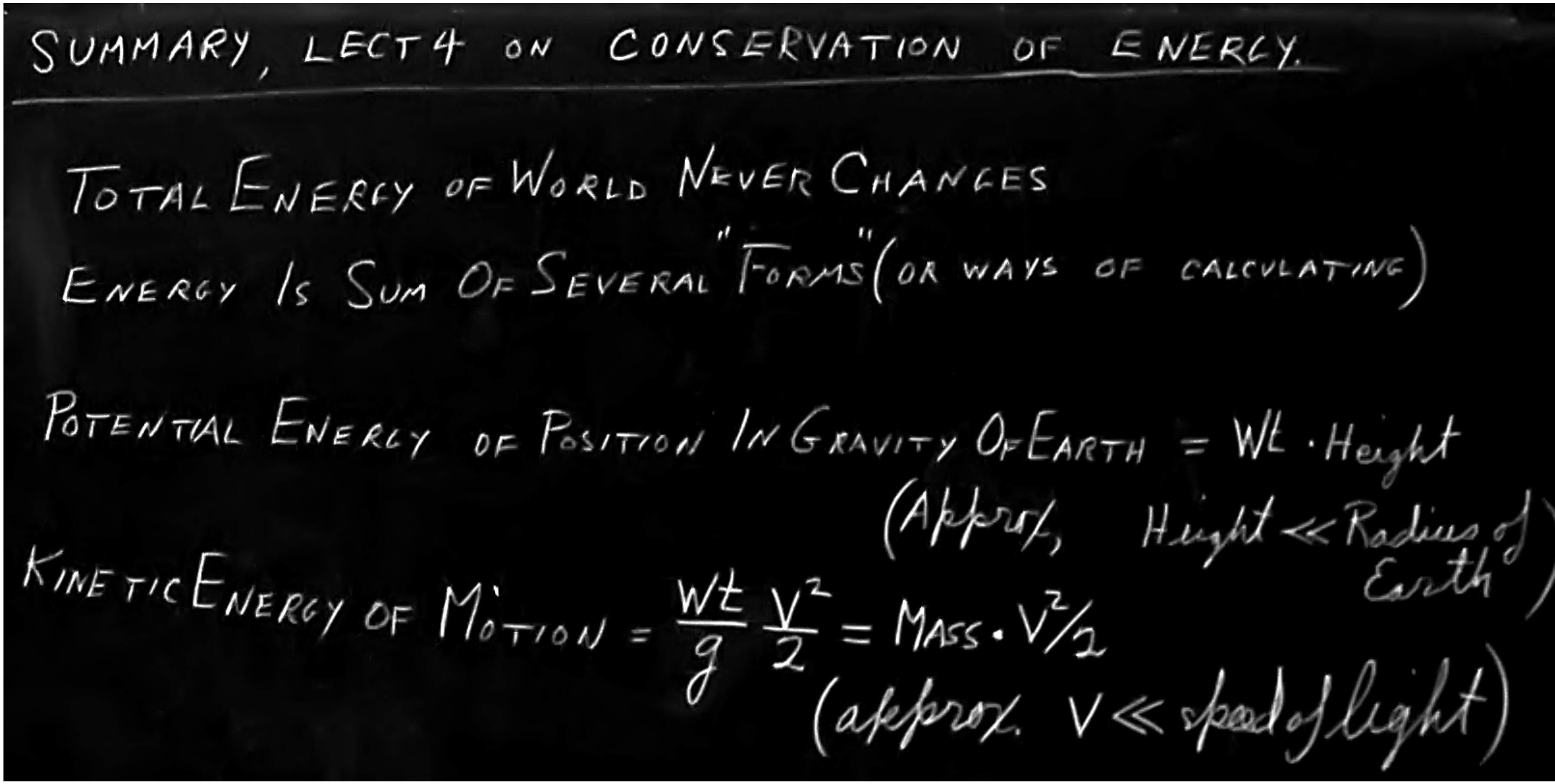
→ Mechanisms tied to membrane transport are well-approached through the lens of *energy*

Nernst-Plank Equation

$$J_n(x,t) = -z_n F D_n \frac{\partial c_n(x,t)}{\partial x} - u_n z_n^2 F^2 c_n(x,t) \frac{\partial \psi(x,t)}{\partial x}$$

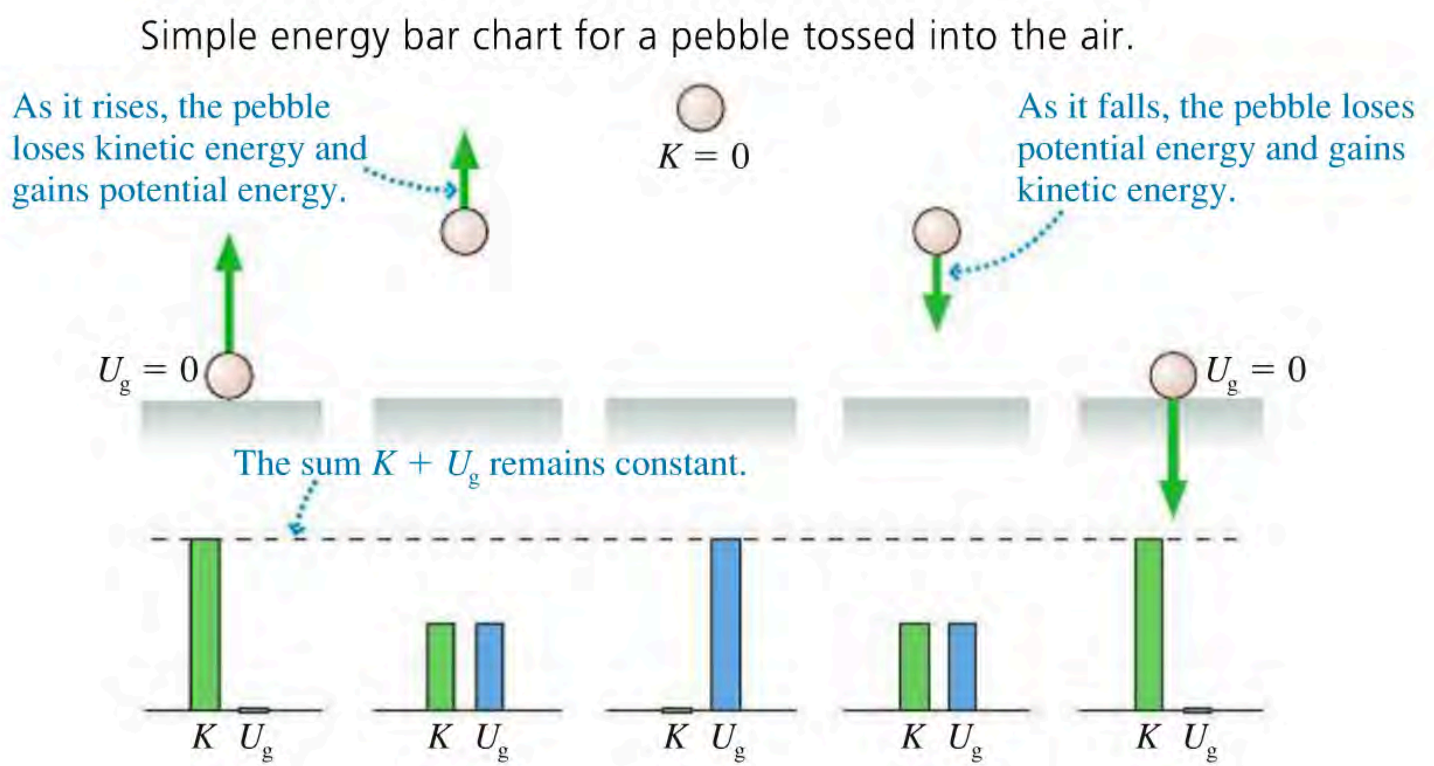
NOTE: This is beyond the scope of PHYS 1420!

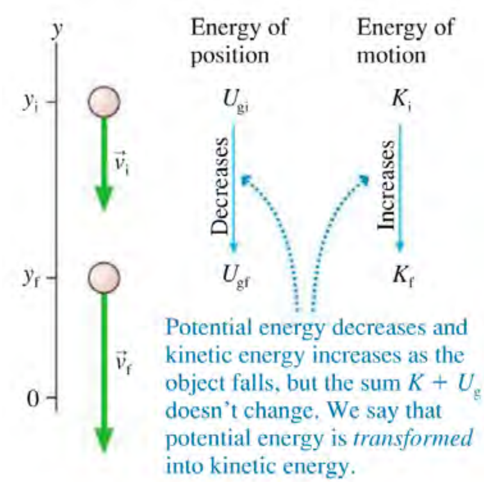
Weiss (Cellular Biophysics, 1996)



http://www.feynmanlectures.caltech.edu/I_04.html

Kinetic vs Potential Energy





→ Remember, ignoring air resistance and whatnot, the ball's total energy remains constant

Knight (2013)

Kinetic vs Potential Energy

Note: We'll derive these later...

$$K = \frac{1}{2}mv^2 \qquad \text{(kinetic energy)}$$

$$U_g = mgy \qquad \text{(gravitational potential energy)}$$

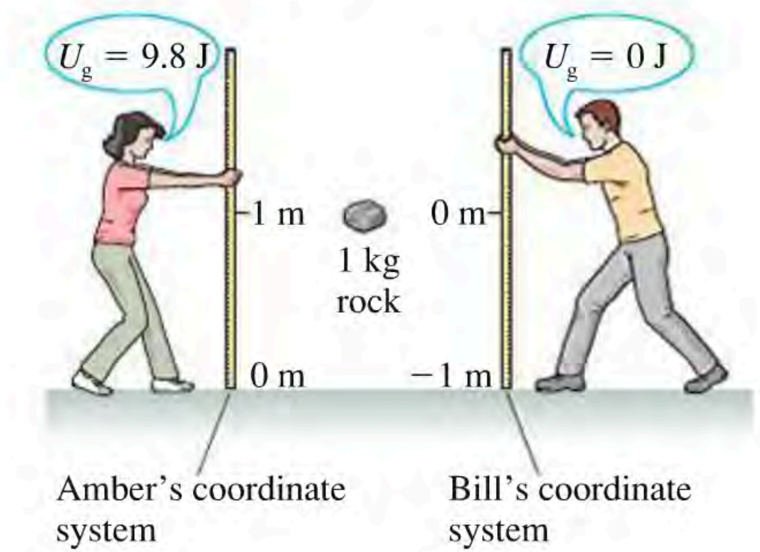
→ If energy remains constant, then:

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

Or more generally:

$$K_f + U_f = K_i + U_i$$

Amber and Bill use coordinate systems with different origins to determine the potential energy of a rock.



Wait a second, isn't there a degree of ambiguity here?

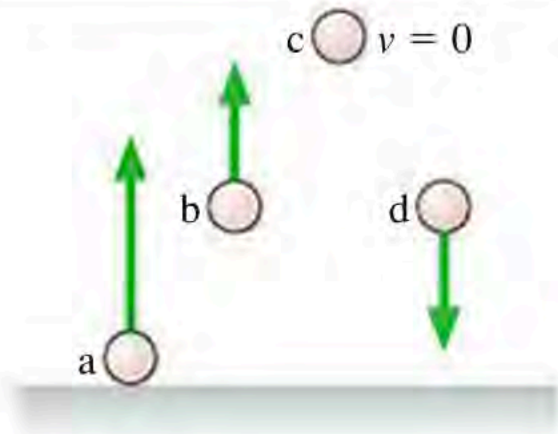
ANS: No. Because a constant offset doesn't affect what *change* in energy would arise if the rock moved

$$\Delta U = -mg(y_f - y_i)$$

Knight (2013)

Ex.

STOP TO THINK 10.1 Rank in order, from largest to smallest, the gravitational potential energies of balls a to d.



Knight (2013)

Ex. (SOL)

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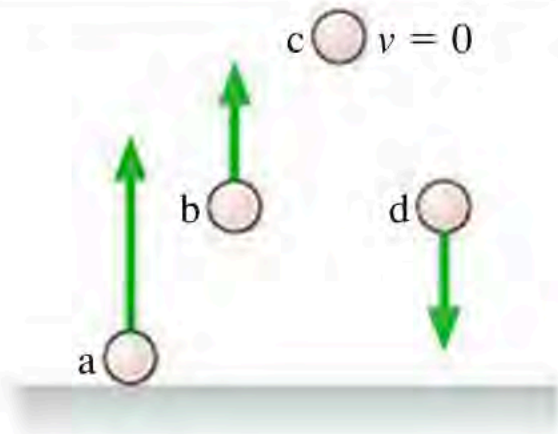
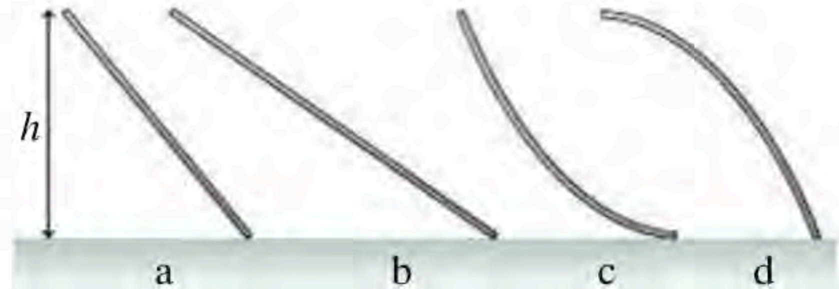


c > b = d > a

Knight (2013)

Ex.

STOP TO THINK 10.2 A small child slides down the four frictionless slides a–d. Each has the same height. Rank in order, from largest to smallest, her speeds v_a to v_d at the bottom.



Knight (2013)

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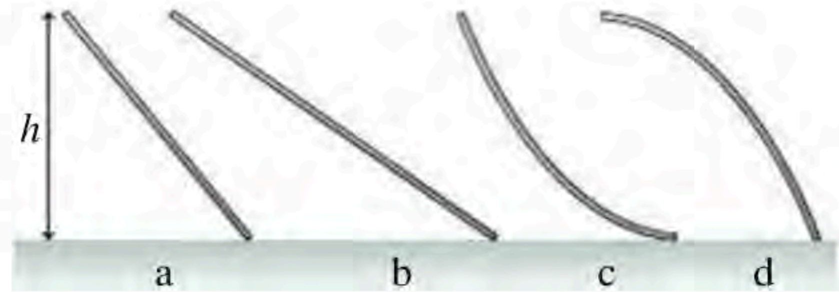


All the same

Knight (2013)

Ex.

STOP TO THINK 10.3 A box slides along the frictionless surface shown in the figure. It is released from rest at the position shown. Is the highest point the box reaches on the other side at level a, level b, or level c?

Knight (2013)

Ex. (SOL)

STOP TO THINK 10.3 A box slides along the frictionless surface shown in the figure. It is released from rest at the position shown. Is the highest point the box reaches on the other side at level a, level b, or level c?

b

http://wonders.physics.wisc.edu/bowling-ball-pendulum.htm

Knight (2013)

"Deriving" Kinetic Energy

- "Force is a vector, work and energy are scalars. Thus, it is often easier to solve problems using energy considerations instead of using Newton's laws (i.e. it is easier to work with scalars than vectors)."

- "The work-energy theorem states that the work done by all forces acting on a particle equals the change in the particle's kinetic energy."

- Here we will simply define work and kinetic energy, and derive the relationship between them via Newton's 2nd Law:

Consider a constant force F acting on an object.

$$a = \frac{v - v_0}{t} \qquad x = \frac{v + v_0}{2} \cdot t,$$

Here v_o is the velocity at $t=0$ and v is the velocity at t. Then:

→ What work is done by this force moving the object a distance x?

$$W = Fx = max$$
$$= m\left(\frac{v - v_0}{t}\right)\left(\frac{v + v_0}{2}\right)t = \tfrac{1}{2}mv^2 - \tfrac{1}{2}m{v_0}^2$$

Resnick & Halliday (1966)
http://theory.uwinnipeg.ca/physics/work/node3.html

"Deriving" Kinetic Energy

$$W = Fx = max$$
$$= m\left(\frac{v - v_0}{t}\right)\left(\frac{v + v_0}{2}\right)t = \tfrac{1}{2}mv^2 - \tfrac{1}{2}m{v_0}^2$$

Kinetic energy

$$K = \tfrac{1}{2}mv^2$$

→ The work done by the force acting on the object is equal to the change in kinetic energy of the object

So, what if the force is not constant?

$$W = \int \mathbf{F} \cdot d\mathbf{r} = \int_{x_0}^{x} F\,dx$$

$$a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{dv}{dx}v = v\frac{dv}{dx}.$$

We end up w/ the same answer!

$$W = \int_{x_0}^{x} F\,dx = \int_{x_0}^{x} mv\frac{dv}{dx}\,dx = \int_{v_0}^{v} mv\,dv = \tfrac{1}{2}mv^2 - \tfrac{1}{2}m{v_0}^2.$$

$$W \text{ (of the } \textit{resultant} \text{ force)} = K - K_0 = \Delta K.$$

Work-energy theorem

Resnick & Halliday (1966)

Conservation of Momentum

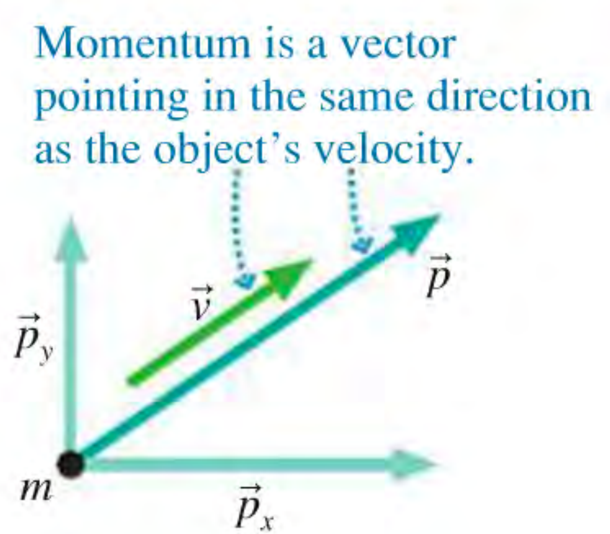
- Tied to conservation of energy is notion that *momentum is conserved*

- Unlike energy, momentum is a vector

$$\textbf{momentum} = \vec{p} \equiv m\vec{v}$$

Kinetic energy

$$K = \tfrac{1}{2}mv^2$$



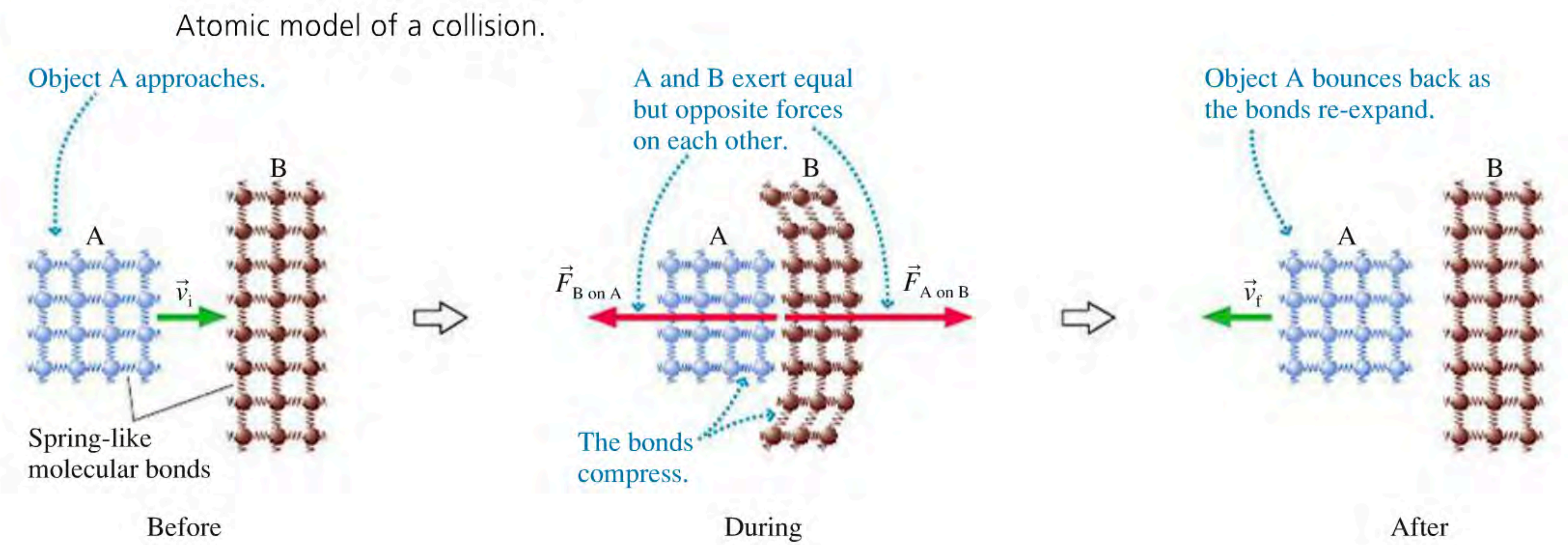
Law of conservation of momentum The total momentum $\vec{P}$ of an isolated system is a constant. Interactions within the system do not change the system's total momentum.

The total momentum of the rocket + gases system is conserved, so the rocket accelerates forward as the gases are expelled backward.

Note: We'll deal more w/ momentum soon (e.g. rocket-themed tangents)

Knight (2013)

Aside: Molecular Underpinnings of Momentum (e.g., collisions)



Knight (2013)

Pulling it together....

"Warmth" represents (in part) a means to do work...

... though we are also tempered by the First Law of Thermodynamics

$$\Delta U = q + w$$

Additional Problems

(some w/ solutions, some w/o)

Ex.

Hooke's Law says that the force, F, required to compress the spring in Figure 8.59 by a distance x, in meters, is given by $F = kx$, for some constant k. Find the work done in compressing the spring by 0.1 m if $k = 8$ nt/m.

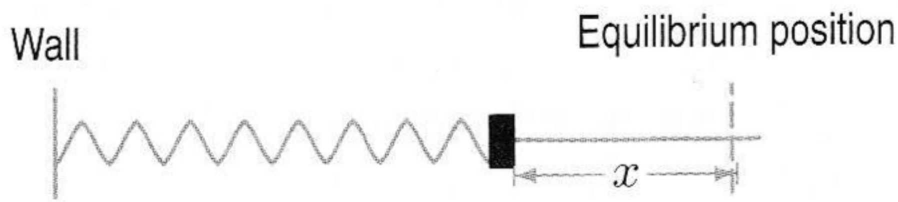


Figure 8.59: Compression of spring: Force is kx

Hughes-Hallet et al (2005)

Ex. (SOL)

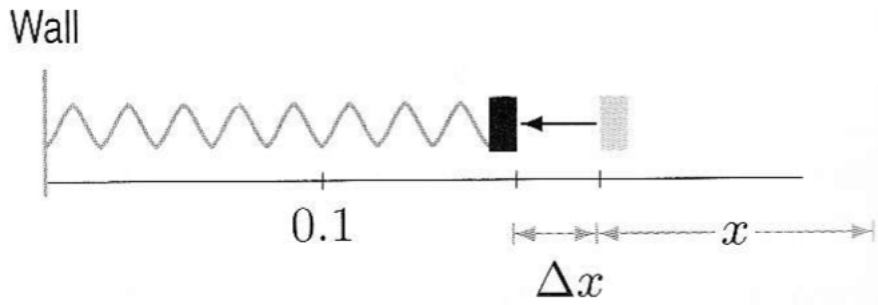


Figure 8.60: Work done in compressing spring a small distance Δx is $kx\Delta x$

Since k is in newtons/meter and x is in meters, we have $F = 8x$ newtons. Since the force varies with x, we divide the distance moved into small increments, Δx, as in Figure 8.60. Then

$$\text{Work done in moving through an increment} \approx F\Delta x = 8x\Delta x \text{ joules.}$$

So, summing over all increments gives the Riemann sum approximation

$$\text{Total work done} \approx \sum 8x\Delta x.$$

Taking the limit as $\Delta x \to 0$ gives

$$\text{Total work done} = \int_0^{0.1} 8x\,dx = 4x^2\Big|_0^{0.1} = 0.04 \text{ joules.}$$

Hughes-Hallet et al (2005)

Ex.

Calculate the work done in pumping oil from the cone-shaped tank in Figure 8.62 to the rim. The oil has density 800 kg/m^3 and its vertical depth is 10 m.

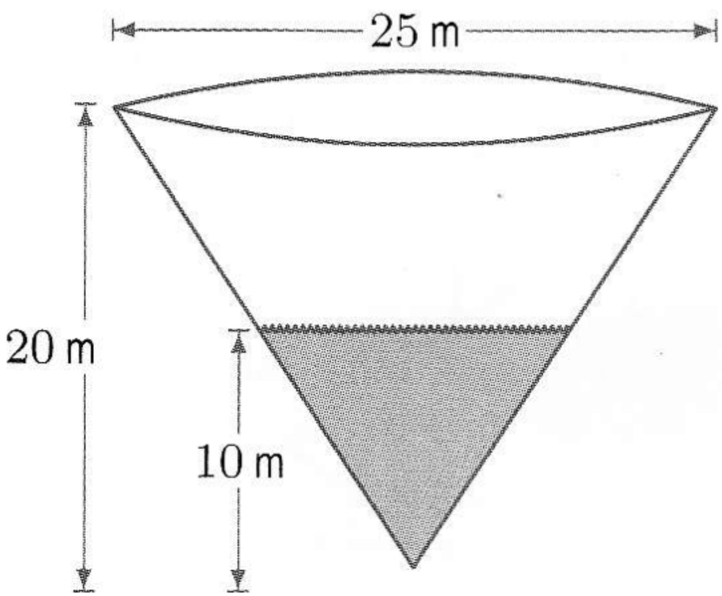


Figure 8.62: Cone-shaped tank containing oil

Hughes-Hallet et al (2005)

Ex. (SOL)

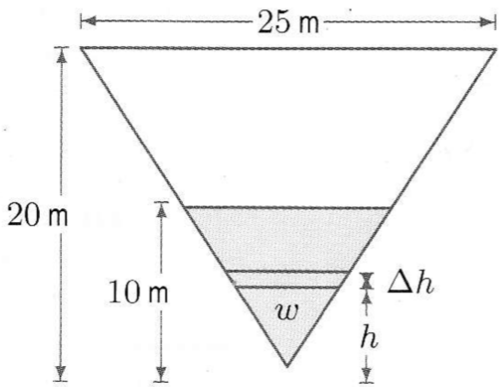


Figure 8.63: Slicing the oil horizontally to compute work

We slice the oil horizontally because each part of such a slice moves the same vertical distance. Each slice is approximately a circular disk with radius $w/2$ m, so

$$\text{Volume of slice} \approx \pi\left(\frac{w}{2}\right)^2 \Delta h = \frac{\pi}{4}w^2\Delta h \text{ m}^3.$$

$$\text{Force of gravity on slice} = \text{Density} \cdot g \cdot \text{Volume} = 800g\frac{\pi}{4}w^2\Delta h = 200\pi g w^2 \Delta h \text{ nt}.$$

Since each part of the slice has to move a vertical distance of $(20-h)$ m, we have

$$\begin{aligned}\text{Work done on slice} \approx \text{Force} \cdot \text{Distance} &= 200\pi g w^2 \Delta h \text{ nt} \cdot (20-h) \text{ m} \\ &= 200\pi g w^2 (20-h)\Delta h \text{ joules.}\end{aligned}$$

To find w in terms of h, we use the similar triangles in Figure 8.63:

$$\frac{w}{h} = \frac{25}{20} \quad \text{so} \quad w = \frac{5}{4}h = 1.25h.$$

Thus,

$$\text{Work done on strip} \approx 200\pi g(1.25h)^2(20-h)\Delta h = 312.5\pi g h^2(20-h)\Delta h \text{ joules.}$$

Summing and taking the limit as $\Delta h \to 0$ gives an integral with upper limit $h = 10$, the depth of the oil.

$$\text{Total work} = \lim_{\Delta h \to 0}\sum 312.5\pi g h^2(20-h)\Delta h = \int_0^{10} 312.5\pi g h^2(20-h)\, dh \text{ joules.}$$

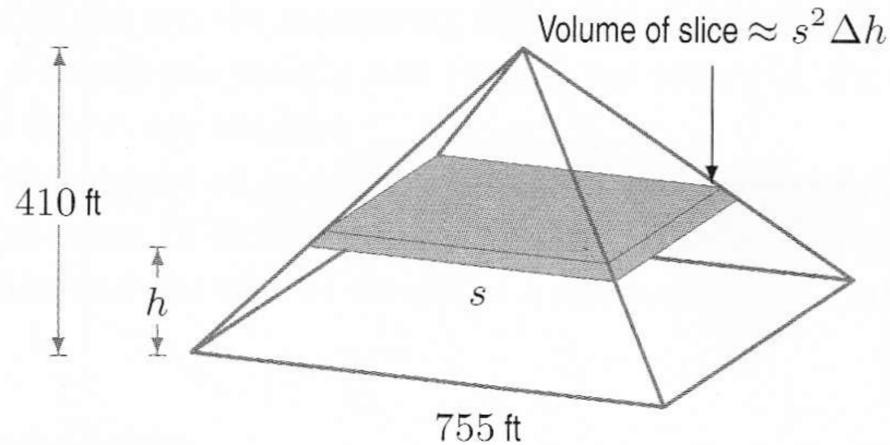
Evaluating the integral using $g = 9.8$ m/sec^2 gives

$$\text{Total work} = 312.5\pi g\left(20\frac{h^3}{3} - \frac{h^4}{4}\right)\Bigg|_0^{10} = 1{,}302{,}083\pi g \approx 4.0 \cdot 10^7 \text{ joules.}$$

Hughes-Hallet et al (2005)

Ex.

It is reported that the Great Pyramid of Egypt was built in 20 years. If the stone making up the pyramid has density 200 pounds per cubic foot, find the total amount of work done in building the pyramid. The pyramid is 410 feet high and has a square base 755 feet by 755 feet. Estimate how many workers were needed to build the pyramid.



Hughes-Hallet et al (2005)

Ex. (SOL)

$$\text{Total work} = \int_0^{410} 200 \left(\frac{755}{410}\right)^2 (410-h)^2 h\, dh \approx 1.6 \cdot 10^{12} \text{ foot-pounds.}$$

We have calculated the total work done in building the pyramid; now we want to estimate the total number of workers needed. Let's assume every laborer worked 10 hours a day, 300 days a year, for 20 years. Assume that a typical worker lifted ten 50 pound blocks a distance of 4 feet every hour, thus performing 2000 foot-pounds of work per hour (this is a very rough estimate). Then each laborer performed $(10)(300)(20)(2000) = 1.2 \cdot 10^8$ foot-pounds of work over a twenty year period. Thus, the number of workers needed was about $(1.6 \cdot 10^{12})/(1.2 \cdot 10^8)$, or about 13,000.

Hughes-Hallet et al (2005)

Ex.

- How does one measure the speed of a bullet?

A 10 g bullet is fired into a 1200 g wood block hanging from a 150-cm-long string. The bullet embeds itself into the block, and the block then swings out to an angle of 40°. What was the speed of the bullet? (This is called a *ballistic pendulum.*)

Ex. (SOL)

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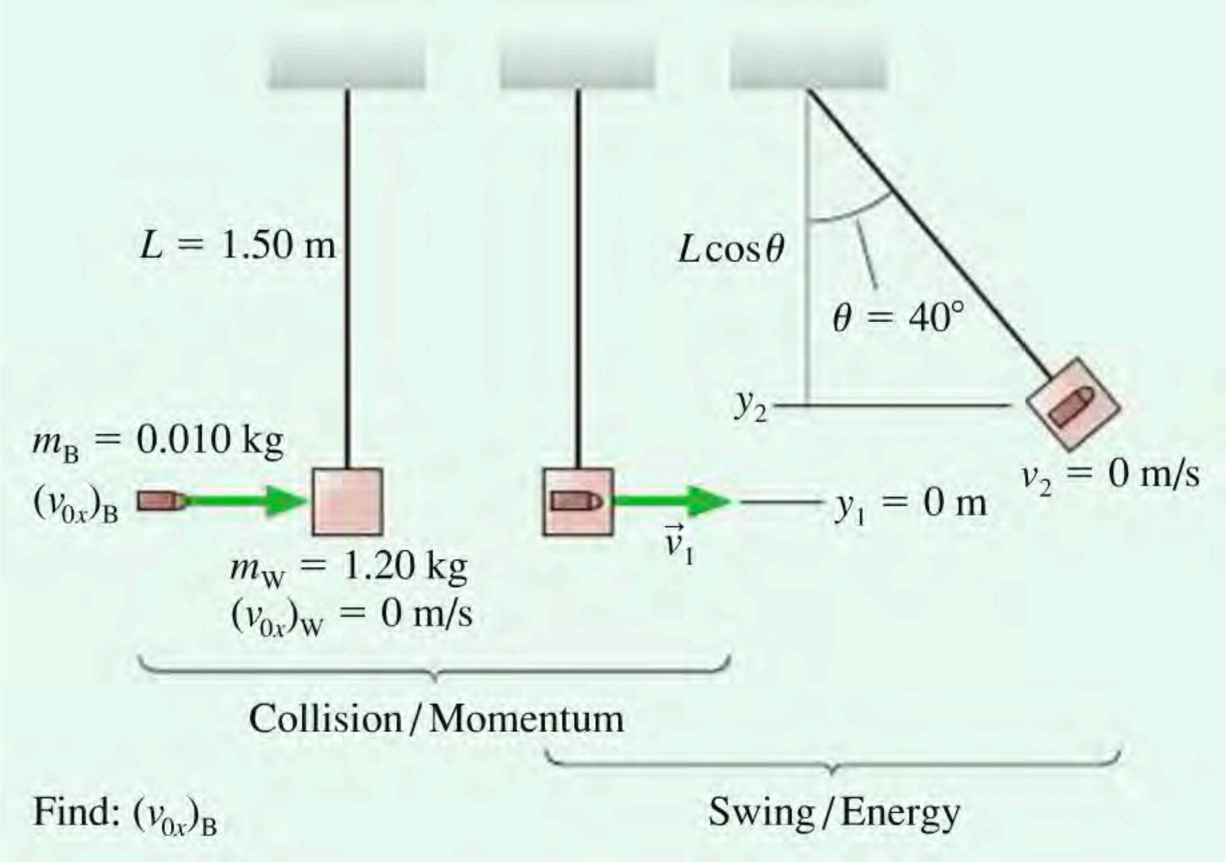

320 m/s

→ Relatively easy problem through the lens of conservation of momentum and energy

Knight (2013)

Force and Pressure

We can use the definite integral to compute the force exerted by a liquid on a surface, for example, the force of water on a dam. The idea is to get the force from the *pressure*. The pressure in a liquid is the force per unit area exerted by the liquid. Two things you need to know about pressure are:

- At any point, pressure is exerted equally in all directions—up, down, sideways.
- Pressure increases with depth. (That is one of the reasons why deep sea divers have to take much greater precautions than scuba divers.)

At a depth of h meters, the pressure, p, exerted by the liquid, measured in newtons per square meter, is given by computing the total weight of a column of liquid h meters high with a base of 1 square meter. The volume of such a column of liquid is just h cubic meters. If the liquid has density δ (mass per unit volume), then its weight per unit volume is δg, where g is the acceleration due to gravity. The weight of the column of liquid is $\delta g h$, so

$$\text{Pressure} = \text{Mass density} \cdot g \cdot \text{Depth} \quad \text{or} \quad p = \delta g h.$$

Provided the pressure is constant over a given area, we also have the following relation:

$$\text{Force} = \text{Pressure} \cdot \text{Area}.$$

The units and data we will generally use are given in the following table:

	Density of water	Force	Area	Pressure
SI units	1000 kg/m^3 (mass)	newton (nt)	meter^2	pascal (nt/m^2)
British units	62.4 lb/ft^3 (weight)	pound (lb)	foot^2	lb/ft^2

Conversions
1 lb = 4.45 nt
1ft^2 = 0.093 m^2
1 lb/ft^2 = 47.9 pa

Hughes-Hallet et al (2005)

In International units, the mass density of water is 1000 kg/m^3, so the pressure at a depth of h meters is $\delta g h = 1000 \cdot 9.8h = 9800h$ nt/m^2. See Figure 8.65.

In British units, the density of the liquid is usually given as a weight per unit volume, rather than a mass per unit volume. In that case, we do not need to multiply by g because it has already been done. For example, water weighs 62.4 lb/ft^3, so the pressure at depth h feet is $62.4h$ lb/ft^2. See Figure 8.66.

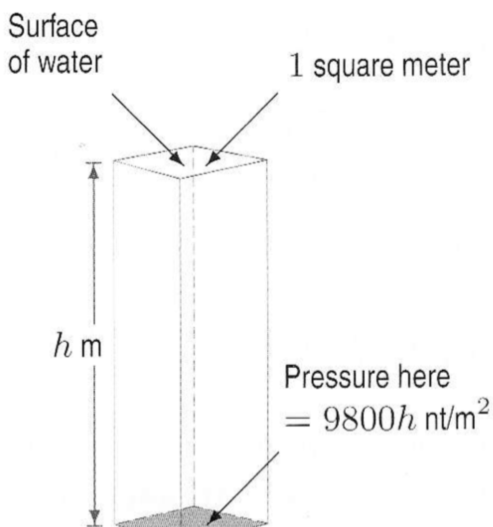

Figure 8.65: Pressure exerted by column of water (International units)

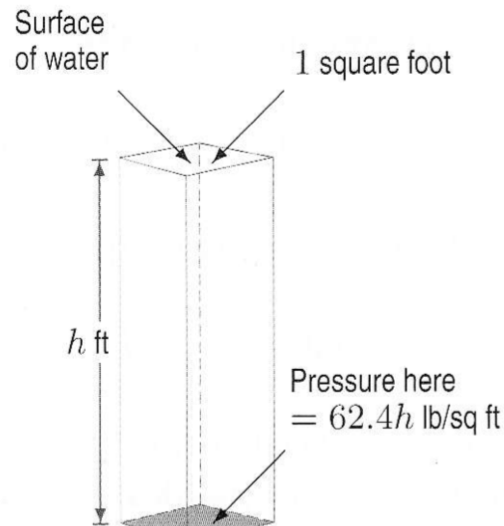

Figure 8.66: Pressure exerted by a column of water (British units)

If the pressure is constant over a surface, we calculate the force on the surface by multiplying the pressure by the area of the surface. If the pressure is not constant, we divide the surface into small pieces *in such a way that the pressure is nearly constant on each one* to obtain a definite integral for the force on the surface. Since the pressure varies with depth, we divide the surface into horizontal strips, each of which is at an approximately constant depth.

Hughes-Hallet et al (2005)

Ex.

Figure 8.68 shows a dam approximately the size of Hoover Dam, which stores water for California, Nevada, and Arizona. Calculate: (a) The water pressure at the base of the dam. (b) The total force of the water on the dam.

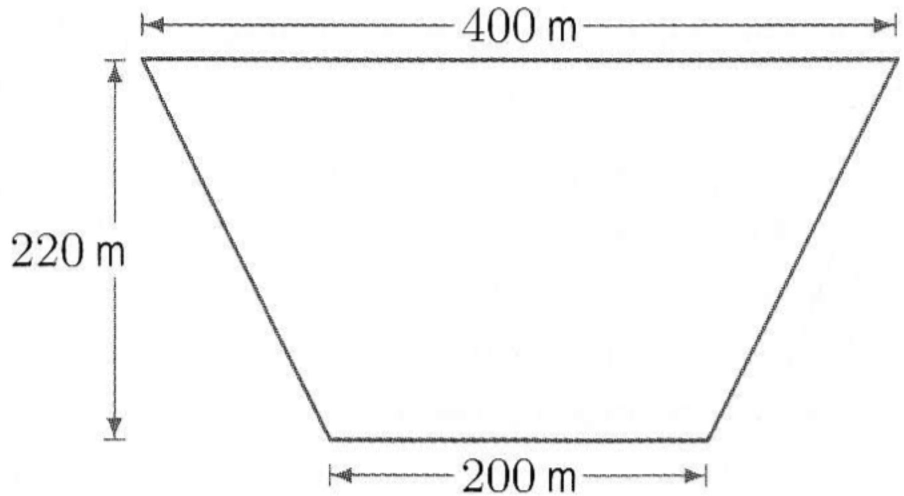

Figure 8.68: Trapezoid-shaped dam

Hughes-Hallet et al (2005)

Ex. (SOL)

(a) Since the density of water is $\delta = 1000$ kg/m^3, at the base of the dam,

$$\text{Water pressure } = \delta g h = 1000 \cdot 9.8 \cdot 220 = 2.156 \cdot 10^6 \text{ nt/m}^2.$$

(b) To calculate the force on the dam, we divide the dam into horizontal strips because the pressure along each strip is approximately constant. See Figure 8.69. Since each strip is approximately rectangular,

$$\text{Area of strip } \approx w\Delta h \text{ m}^2.$$

The pressure at a depth of h meters is $\delta g h = 9800h$ nt/m^2. Thus,

$$\text{Force on strip } \approx \text{ Pressure } \cdot \text{ Area } = 9800hw\Delta h \text{ nt.}$$

To find w in terms of h, we use the fact that w decreases linearly from $w = 400$ when $h = 0$ to $w = 200$ when $h = 220$. Thus w is a linear function of h, with slope $(200 - 400)/220 = -10/11$, so

$$w = 400 - \frac{10}{11}h.$$

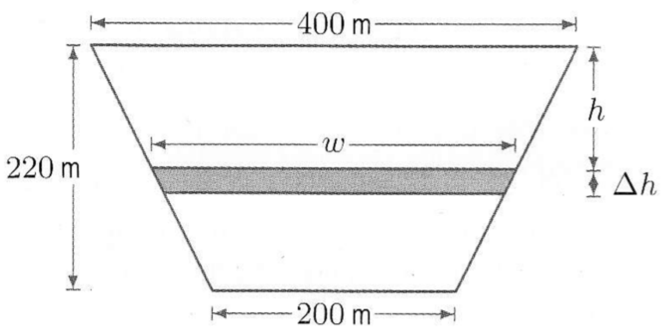

Figure 8.69: Dividing dam into horizontal strips

Thus

$$\text{Force on strip } \approx 9800h\left(400 - \frac{10}{11}h\right)\Delta h \text{ nt.}$$

Summing over all strips and taking the limit as $\Delta h \to 0$ gives

$$\begin{aligned}\text{Total force on dam } &= \lim_{\Delta h \to 0} \sum 9800h\left(400 - \frac{10}{11}h\right)\Delta h \\ &= \int_0^{220} 9800h\left(400 - \frac{10}{11}h\right) dh \text{ newtons.}\end{aligned}$$

Evaluating the integral gives

$$\text{Total force } = 9800\left(200h^2 - \frac{10}{33}h^3\right)\Bigg|_0^{220} = 6.32 \cdot 10^{10} \text{ newtons.}$$

In fact, Hoover Dam is not flat, as the problem assumed, but arched, to better withstand the pressure.

Hughes-Hallet et al (2005)

Ex.

A block of mass 10.0 kg is to be raised from the bottom to the top of an incline 5.00 meters long and 3.00 meters off the ground at the top. Assuming frictionless surfaces, how much work must be done by a force parallel to the incline pushing the block up at *constant speed* at a place where $g = 9.80$ meters/sec^2.

Resnick & Halliday (1966)

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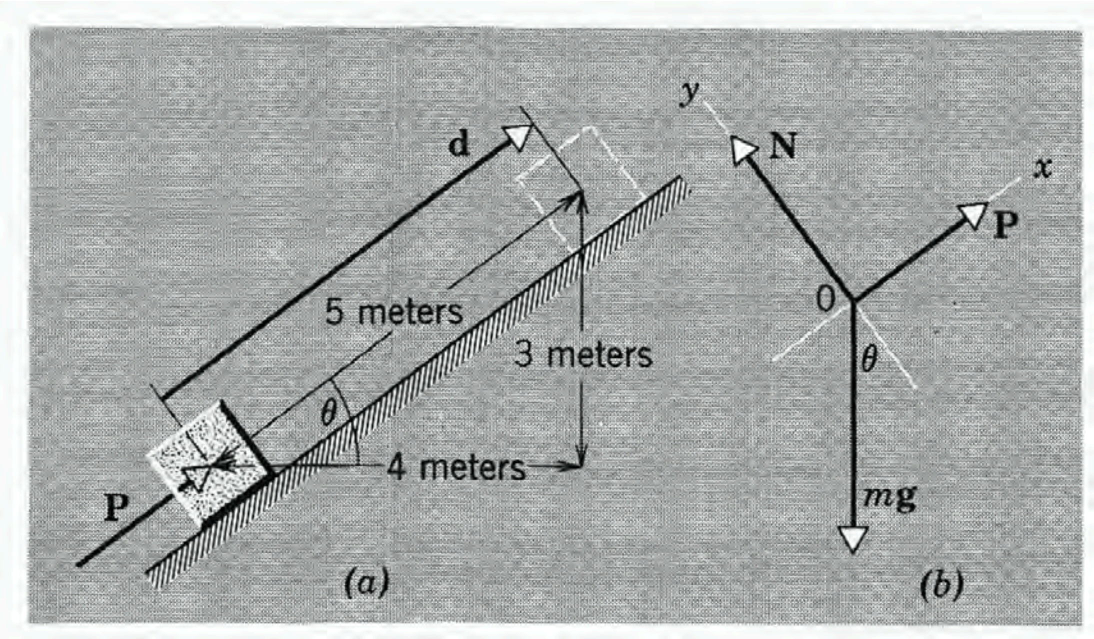

294 joules

Resnick & Halliday (1966)

Ex.

A boy pulls a 10-lb sled 30 ft along a horizontal surface at a *constant speed.* What work does he do on the sled if the coefficient of kinetic friction is 0.20 and his pull makes an angle of 45° with the horizontal?

Resnick & Halliday (1966)

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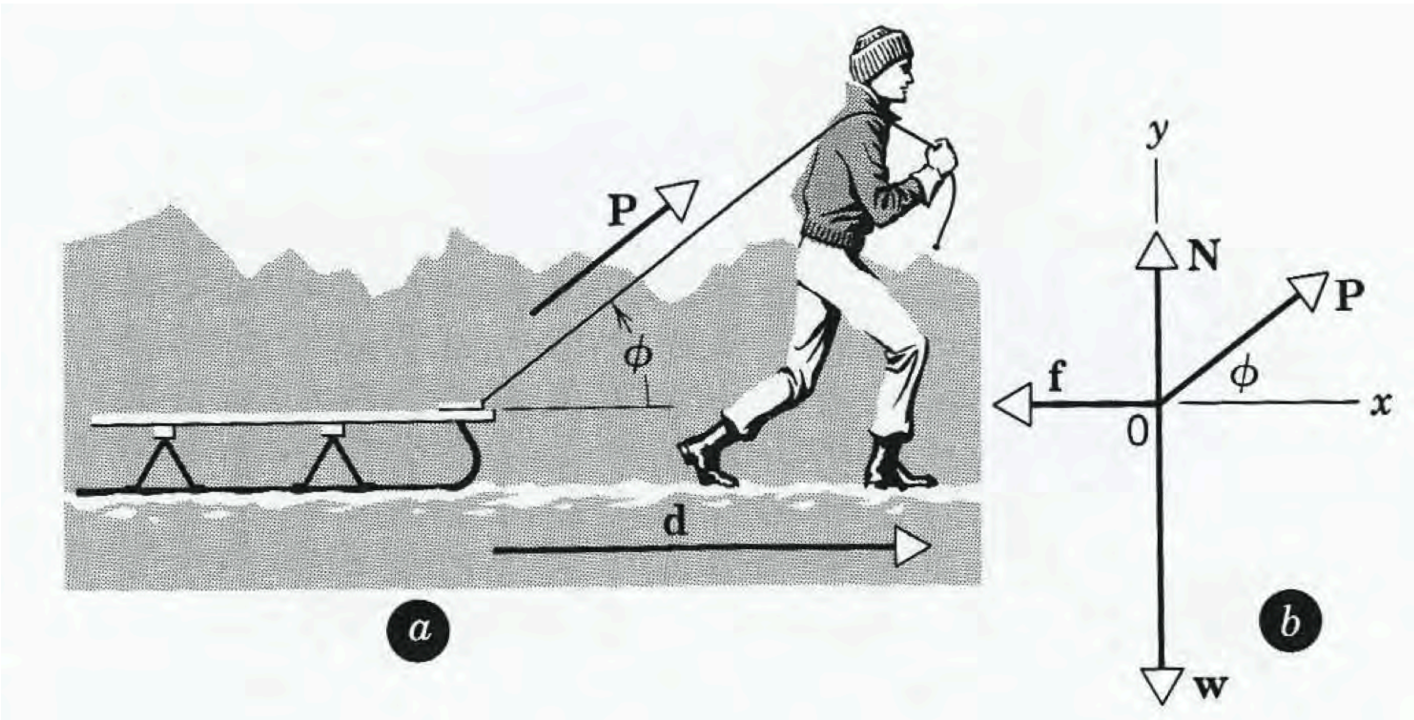

51 ft-lb

Resnick & Halliday (1966)