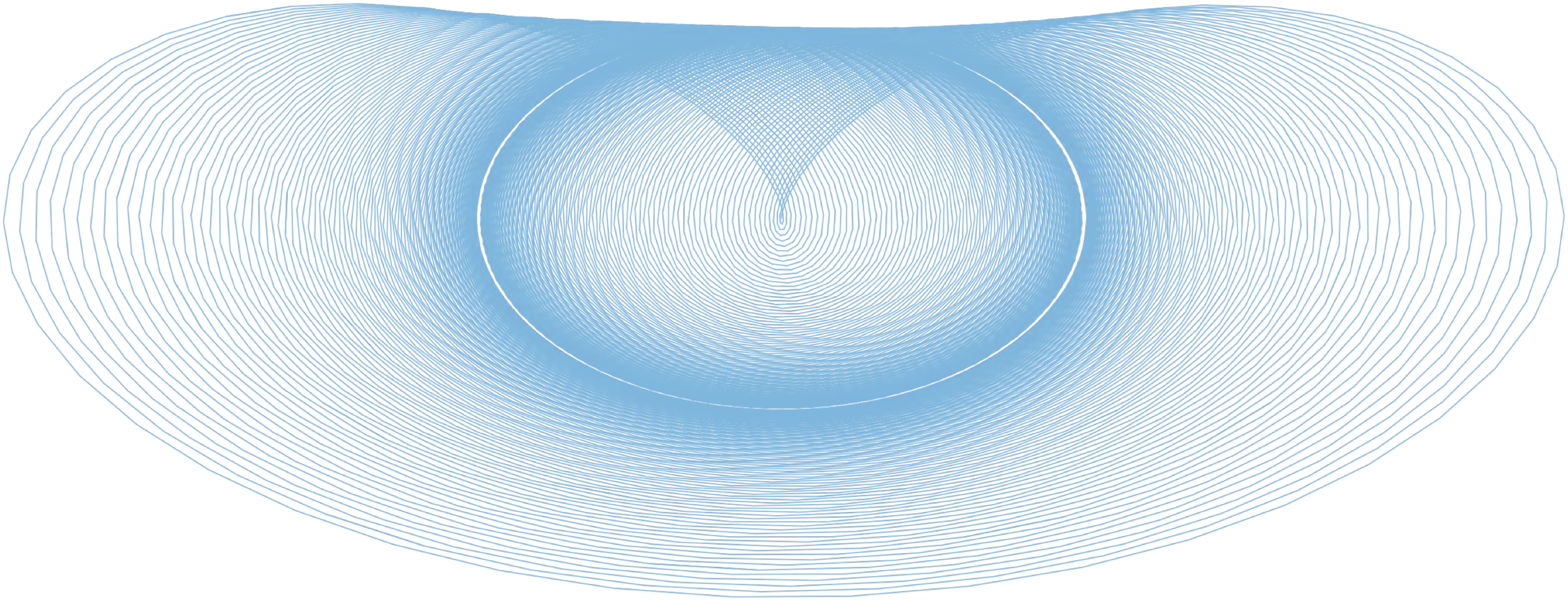
PHYS 1420 (F19)
Physics with Applications to Life Sciences



2019.10.23

Relevant reading:
Kesten & Tauck ch. 7.1-7.3

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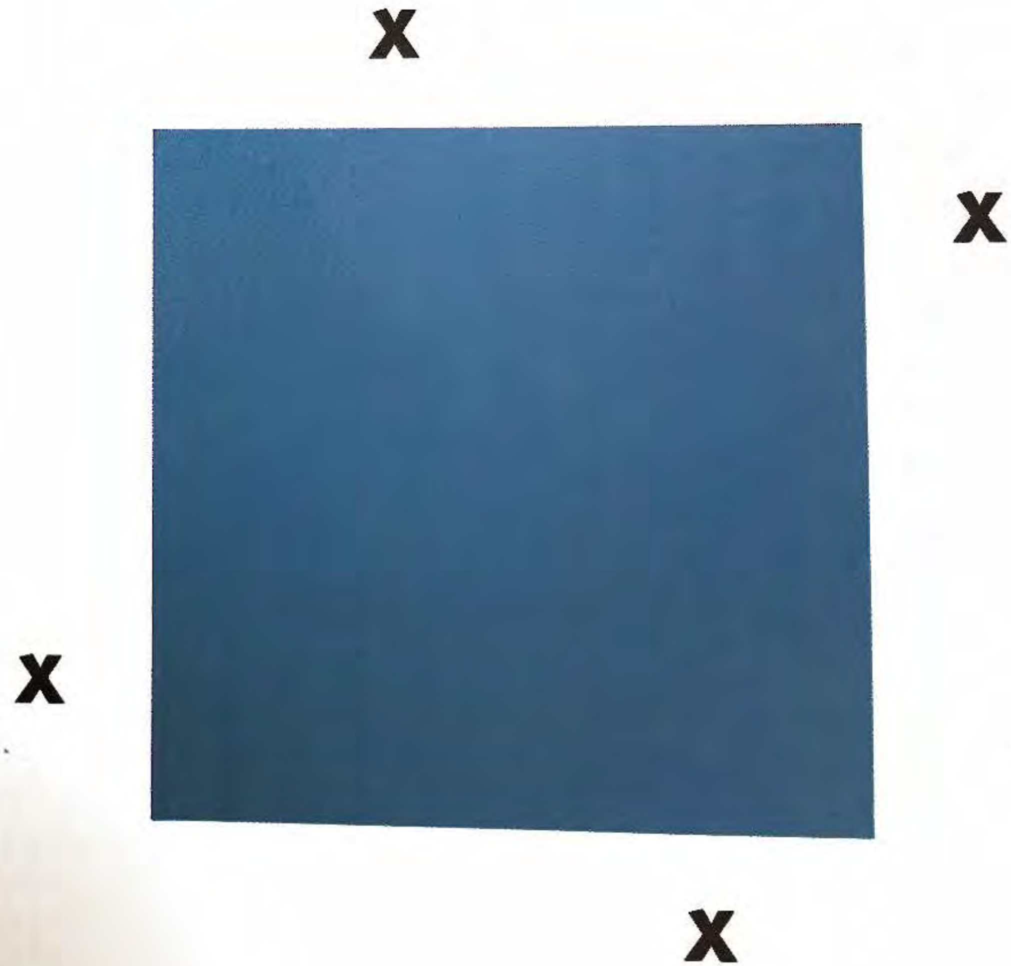
Ref. (re images):
Wolfson (2007), Knight (2017)

Can you draw a perfect square around the figure at right so that each side passes through one of the crosses and none of the sides touches the figure? There's just one hitch—the lines of the square you draw must not be parallel to those of the existing square.



Announcements & Key Concepts (re Today)

→ Online HW #6: Posted and due next Wednesday (10/30)

Note: There are two sections (one re *energy*, the other re *momentum*)

→ Midterm exams are being graded

→ No tutorial next Tuesday (10/29)

Some relevant underlying concepts of the day…

- Energy review → Notion of "conservation" and "reference"
- Momentum
- Conservation of momentum
- (Inelastic) Collisions

Review (re Energy)

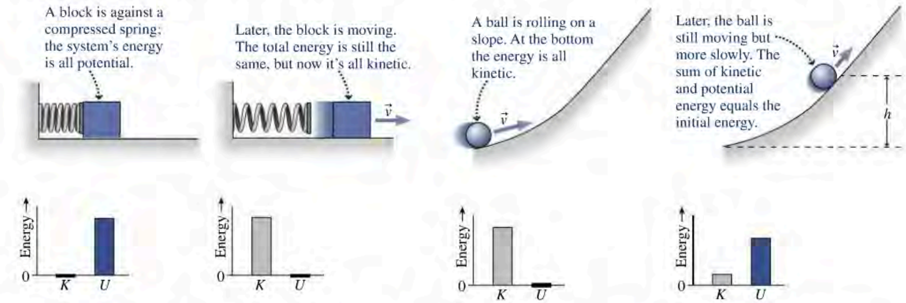
Note that there are plenty of interdisciplinary connections here!

- integrals (re work)
- potential energy of chemical bonds
- potential energy of a folded protein
- notion of “lowest energy state”
- countless additional threads....

CHAPTER 7 SUMMARY

Big Ideas

The big idea here is conservation of energy. This chapter emphasizes the special case of systems subject only to conservative forces, in which case the total mechanical energy—the sum of kinetic and potential energy—cannot change. Energy may change from kinetic to potential, and vice versa, but the total remains constant. Applying conservation of mechanical energy requires the concept of potential energy—energy stored in a system as a result of work done against conservative forces.



If nonconservative forces act in a system, then mechanical energy isn't conserved; instead, mechanical energy gets converted to internal energy.

Key Concepts and Equations

The important new concept here is potential energy, defined as the negative of the work done by a conservative force. Only the change ΔU has physical significance. Expressions for potential energy include:

$$\Delta U_{AB} = -\int_A^B \vec{F} \cdot d\vec{r}$$

This one is the most general, but it's mathematically involved. The force can vary over an arbitrary path between points A and B.

$$\Delta U = -\int_{x_1}^{x_2} F(x)\,dx$$

This is a special case, when force and displacement are in the same direction and force may vary with position.

$$\Delta U = -F\,(x_2 - x_1)$$

This is the most specialized case, where the force is constant.

Given the concept of potential energy, the principle of conservation of mechanical energy follows from the work–kinetic energy theorem of Chapter 6. Here's the mathematical statement of mechanical energy conservation:

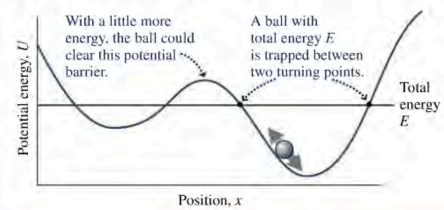
$$K + U = K_0 + U_0$$

K and U are the kinetic and potential energy at some point where we don't know one of these quantities.

The total mechanical energy is conserved, as indicated by the equal sign.

K_0 and U_0 are the kinetic and potential energy at some point where both are known. $K_0 + U_0$ is the *total mechanical energy*.

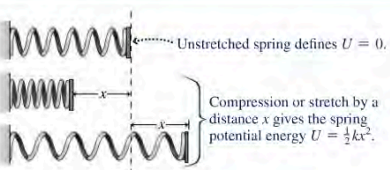
We can describe a wide range of systems—from molecules to roller coasters to planets—in terms of **potential-energy curves**. Knowing the total energy then lets us find **turning points** that determine the range of motion available to the system.

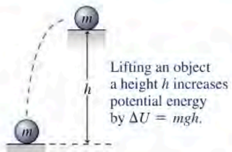


Applications

Two important cases of potential energy are the elastic potential energy of a spring, $U = \frac{1}{2}kx^2$, and the gravitational potential energy change, $\Delta U = mgh$, associated with lifting an object of mass m through a height h.

The former is limited to ideal springs for which $F = -kx$, the latter to the proximity of Earth's surface, where the variation of gravity with height is negligible.





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Ex.

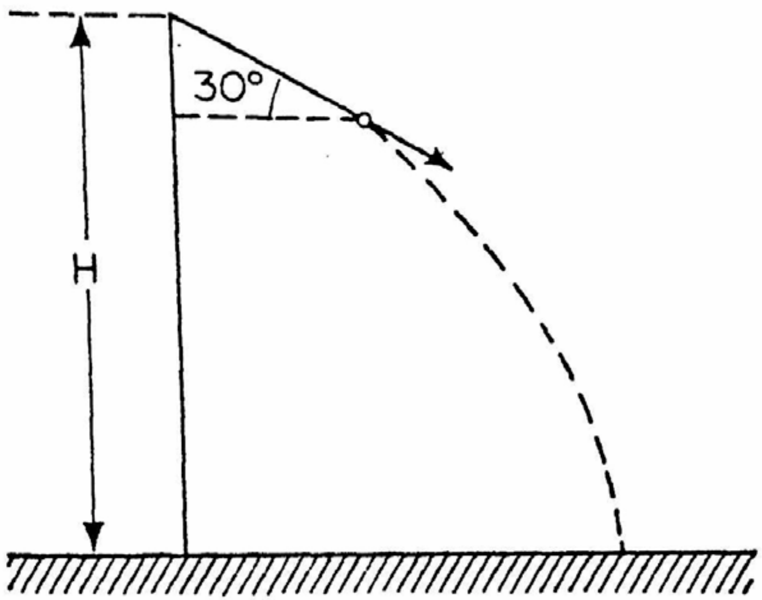
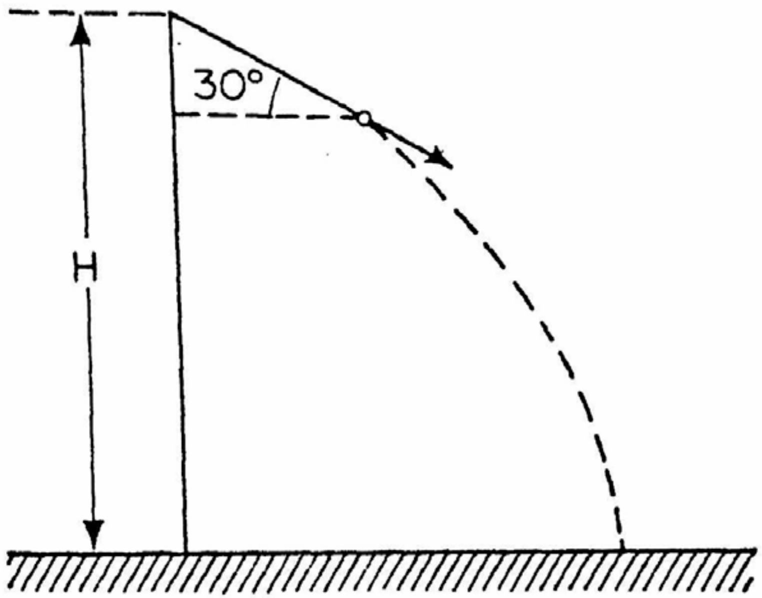


FIG. 43

Starting from a height H, a ball slips without friction, down a smooth plane inclined at an angle of 30° to the horizontal (Fig. 43). The length of the plane is $H/3$. The ball then falls on to a horizontal surface with an impact that may be taken as perfectly elastic. How high does the ball rise after striking the horizontal plane?

Shaskol'Skaya & El'Tsin (1963)

Ex. (SOL)



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$7/8\ H$

→ Key to approaching this problem is to consider through the lens of *energy*.... (PHYS and BPHS majors will see this in detail come 2nd year classical mechanics, PHYS 2010)

Shaskol'Skaya & El'Tsin (1963)

Energy: Positive? Negative?

Some useful connection points back to energy to highlight as we head towards *momentum*...

- Sometimes it seems that expressions for energy are *negative* (!?!)

The change ΔU_{AB} in potential energy associated with a conservative force is the negative of the work done by that force as it acts over any path from point A to point B:

$$\Delta U_{AB} = -\int_A^B \vec{F} \cdot d\vec{r} \qquad \text{(potential energy)} \qquad (7.2)$$

$$\Delta K + \Delta U = 0$$

$$K + U = \text{constant} = K_0 + U_0$$

- Clearly the *change* in things (i.e., Δ) matters....

- Energy is never negative (well, at least normal convention dictates such). But convention-wise, a "negative energy" (or better put, a "negative change in energy") can be useful

- The lynchpin here is that things are ***relative*** to something...

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Ex.

Some useful connection points back to energy to highlight as we head towards *momentum*...

A bug lands on top of the frictionless, spherical head of a bald man. It begins to slide down his head (Fig. 7.24). Show that the bug leaves the head when it has dropped a vertical distance one-third of the head's radius.

FIGURE 7.24 Problem 60

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Ex. (SOL)

INTERPRET This problem involves Newton's second law and conservation of total mechanical energy. The force of gravity that acts on the bug is a conservative force, and we shall take the geometric center of the man's spherical head to be the zero of gravitational potential energy.

DEVELOP Draw a diagram of the situation (see figure below). Apply conservation of mechanical energy to express the bug's speed as a function of its vertical position d on the man's head. This gives

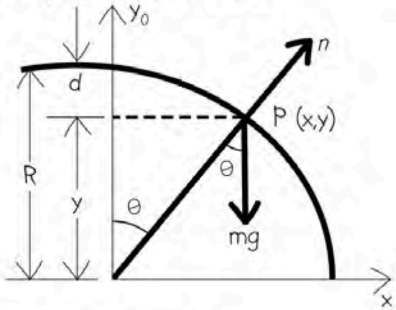
$$U_0 + \overbrace{K_0}^{=0} = U + K$$

$$mgR = mg(R-d) + \frac{1}{2}mv^2$$

The bug will no longer be in contact with the man's head when the normal force goes to zero. From the radial component of Newton's second law, the normal force can be found.

$$n + mg\cos\theta = m\frac{v^2}{R}$$

Combining these equations and letting $n = 0$, we can solve for d.



EVALUATE Using the fact that $\cos\theta = (R-d)/R$, we have

$$mg\frac{R-d}{R} = m\frac{v^2}{R}$$

$$v^2 = g(R-d)$$

Inserting this result into the result from conservation of energy gives the distance d to be

$$mgR = mg(R-d) + \frac{1}{2}mg(R-d)$$

$$d = \frac{R}{3}$$

ASSESS If the bug is perfectly positioned on the top of the man's head and it doesn't move, it will of course stay there. However, this is an unstable equilibrium, so any slight perturbation will send it sliding down the man's head.

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Ex. Energy & A Point of reference

Some useful connection points back to energy to highlight as we head towards *momentum*...

$$K = \frac{1}{2}mv^2$$

So what is v for the bug?

What if the man was walking up the stairs?

What about the fact that Earth is moving!?!

→ Clearly there has to be some sort of meaningful reference...

Aside

Zur Elektrodynamik bewegter Körper; von A. Einstein.

- Another one of Einstein's *Annus Mirabilis* papers from 1905!

Daß die Elektrodynamik Maxwells — wie dieselbe gegenwärtig aufgefaßt zu werden pflegt — in ihrer Anwendung auf bewegte Körper zu Asymmetrien führt, welche den Phänomenen nicht anzuhaften scheinen, ist bekannt. Man denke z. B. an die elektrodynamische Wechselwirkung zwischen einem Magneten und einem Leiter. Das beobachtbare Phänomen hängt hier nur ab von der Relativbewegung von Leiter und Magnet, während nach der üblichen Auffassung die beiden Fälle, daß der eine oder der andere dieser Körper der bewegte sei, streng voneinander zu trennen sind. Bewegt sich nämlich der Magnet und ruht der Leiter, so entsteht in der Umgebung des Magneten ein elektrisches Feld von gewissem Energiewerte, welches an den Orten, wo sich Teile des Leiters befinden, einen Strom erzeugt. Ruht aber der Magnet und bewegt sich der Leiter, so entsteht in der Umgebung des Magneten kein elektrisches Feld, dagegen im Leiter eine elektromotorische Kraft, welcher an sich keine Energie entspricht, die aber — Gleichheit der Relativbewegung bei den beiden ins Auge gefaßten Fällen vorausgesetzt — zu elektrischen Strömen von derselben Größe und demselben Verlaufe Veranlassung gibt, wie im ersten Falle die elektrischen Kräfte.

Aside

ON THE ELECTRODYNAMICS OF MOVING BODIES

By A. Einstein

June 30, 1905

It is known that Maxwell's electrodynamics—as usually understood at the present time—when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. Take, for example, the reciprocal electrodynamic action of a magnet and a conductor. The observable phenomenon here depends only on the relative motion of the conductor and the magnet, whereas the customary view draws a sharp distinction between the two cases in which either the one or the other of these bodies is in motion. For if the magnet is in motion and the conductor at rest, there arises in the neighbourhood of the magnet an electric field with a certain definite energy, producing a current at the places where parts of the conductor are situated. But if the magnet is stationary and the conductor in motion, no electric field arises in the neighbourhood of the magnet. In the conductor, however, we find an electromotive force, to which in itself there is no corresponding energy, but which gives rise—assuming equality of relative motion in the two cases discussed—to electric currents of the same path and intensity as those produced by the electric forces in the former case.

→ In short, the speed of light (c) sets the ultimate benchmark

TO DEMONSTRATE KINETIC ENERGY AND INERTIA, Tom Carter breaks a cinder block over his brave colleague, Dave Fazzini. Most of the hammer's kinetic energy is absorbed in an inelastic collision with the block, the inertia of which saves the sandwiched professor from the nails in the upper board. The demonstration is for a class in conceptual physics that Fazzini teaches at the College of DuPage in Illinois.

OVERVIEW

Why Some Things Don't Change

Part I of this textbook was about *change*. One particular type of change—motion—is governed by Newton's second law. Although Newton's second law is a very powerful statement, it isn't the whole story. Part II will now focus on things that *stay the same* as other things around them change.

Consider, for example, an explosive chemical reaction taking place inside a closed, sealed box. No matter how violent the explosion, the total mass of the products—the final mass M_f —is the same as the initial mass M_i of the reactants. In other words, matter cannot be created or destroyed, only rearranged. This is an important and powerful statement about nature.

A quantity that *stays the same* throughout an interaction is said to be *conserved*. Our knowledge about mass can be stated as a *conservation law*:

Law of conservation of mass The total mass in a closed system is constant. Mathematically, $M_f = M_i$.*

The qualification "in a closed system" is important. Mass certainly won't be conserved if you open the box halfway through and remove some of the matter. Other conservation laws we'll discover also have qualifications stating the circumstances under which they apply.

In a nutshell: In most cases, on one hand we are fundamentally concerned with *change*. But on the other, we deal with what does *not change*

→ "conservation" laws

*Surprisingly, Einstein's 1905 theory of relativity showed that there are circumstances in which mass is *not* conserved but can be converted to energy in accordance with his famous formula $E = mc^2$. Nonetheless, conservation of mass is an exceedingly good approximation in nearly all applications of science and engineering.

Knight

- Momentum is a property of an object, and is equal to the product of the objects mass and velocity.

$$\vec{p} = m\vec{v}$$

- Momentum is a vector.
- Momentum points in the same direction as the velocity vector.
- Momentum has units of $kg\frac{m}{s}$ or equivalently Ns.
- We call this type of momentum Linear Momentum to distinguish it from Angular momentum which we will see later on in the course.

- Since momentum depends on velocity, the change in momentum usually involves a change in velocity (rather than mass).
- This change in velocity requires an acceleration.
- This acceleration requires a force.
- The longer the time that force is acting on an object, the greater the change in velocity, and hence the greater the change in momentum. In general:

$$\Delta\vec{p} = \int_{t_i}^{t_f} \vec{F} dt = m\overrightarrow{v_f} - m\overrightarrow{v_i}$$

if the force is constant, then

$$\Delta\vec{p} = \vec{F}\Delta\text{t}$$

Momentum & Newtons' 2nd Law

I. Newton

The *momentum* of a single particle is a vector **p** defined as the product of its mass m and its velocity **v**. That is,

$$\mathbf{p} = m\mathbf{v}.$$

Note:
Momentum is a vector!

Newton, in his famous *Principia*, expressed the second law of motion in terms of momentum (which he called "quantity of motion"). Expressed in modern terminology Newton's second law reads: *The rate of change of momentum of a body is proportional to the resultant force acting on the body and is in the direction of that force.* In symbolic form this becomes

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}.$$

If our system is a single particle of (constant) mass m, this formulation of the second law is equivalent to the form $\mathbf{F} = m\mathbf{a}$, which we have used up to now. That is, if m is a constant, then

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d}{dt}(m\mathbf{v}) = m\frac{d\mathbf{v}}{dt} = m\mathbf{a}.$$

So for "classical" cases, the following two are equivalent (when mass is const.):

$\mathbf{F} = m\mathbf{a}$ and $\mathbf{F} = d\mathbf{p}/dt$

→ So what about "relativity"?

Resnick & Halliday (1966)

<u>Aside</u>: Special Relativity

$$\mathbf{F} = m\mathbf{a}$$ Not valid

- For things moving "very" fast:

$$\mathbf{F} = d\mathbf{p}/dt$$ Valid

$$\mathbf{p} = \frac{m_0\mathbf{v}}{\sqrt{1 - v^2/c^2}}.$$

Zur Elektrodynamik bewegter Körper;
von A. Einstein.

→ In short, the speed of light (c) sets the ultimate benchmark

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}},$$

Mass changes w/ speed!

In atomic and nuclear systems particles may acquire enormous speeds, comparable to the speed of light. This concept can be put to a direct test in such systems because the increase in mass over the rest mass for such particles is large enough to measure accurately.

Wolfson

(Linear) Momentum of a "System" of Particles

Suppose that instead of a single particle we have a system of n particles, with masses m_1, m_2, etc. *We shall continue to assume*, as we did in Section 9–2, *that no mass enters or leaves the system*, so that the mass M $(= \Sigma\, m_i)$ of the system remains constant with time. The particles may interact with each other and external forces may act on them as well. Each particle will have a velocity and a momentum.

$$\begin{aligned}\mathbf{P} &= \mathbf{p}_1 + \mathbf{p}_2 + \cdots + \mathbf{p}_n \\ &= m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + \cdots + m_n\mathbf{v}_n.\end{aligned}$$

Recall:

$$M\frac{d\mathbf{r}_{\text{cm}}}{dt} = m_1\frac{d\mathbf{r}_1}{dt} + m_2\frac{d\mathbf{r}_2}{dt} + \cdots + m_n\frac{d\mathbf{r}_n}{dt}$$

$$\mathbf{P} = M\mathbf{v}_{\text{cm}}$$

The total momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its center of mass.

→ Generalizing this leads back to the (most general form of) Newton's 2nd: $\mathbf{F} = d\mathbf{p}/dt$

Resnick & Halliday (1966)

(Linear) Momentum of a "System" of Particles

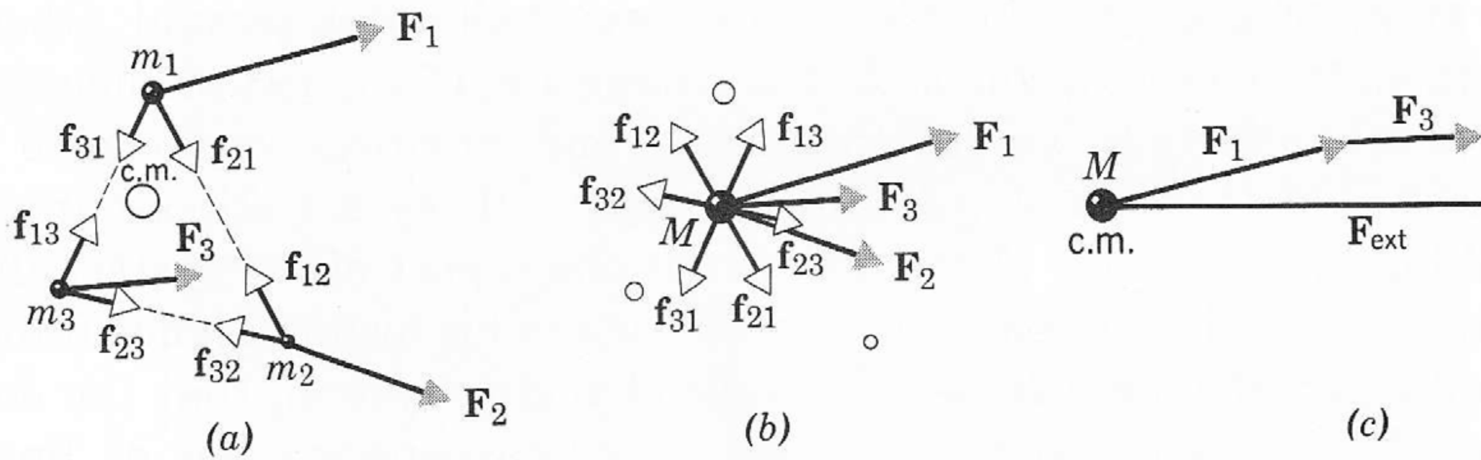

Fig. 9–7 Relationship between the forces acting on a system of three masses m_1, m_2, and m_3. (*a*) All the forces acting on each mass are shown here, as well as the location of the center of mass. On m_1 act forces $\mathbf{f}_{21}$ and $\mathbf{f}_{31}$ exerted by m_2 and m_3 respectively, as well as $\mathbf{F}_1$, a force from some external agent. Similar sets of forces act on m_2 and m_3. However, according to Newton's third law, internal forces $\mathbf{f}_{31}$ and $\mathbf{f}_{13}$ must be equal and opposite and must both lie along the line of centers of m_1 and m_3. Similar statements hold for the other two pairs of action-reaction forces. (*b*) If we are interested only in the motion of the system as a whole, we may consider all the forces to act on a mass $M = m_1 + m_2 + m_3$, located at the center of mass. Owing to the equality of the action-reaction pairs of internal forces as just stated, they cancel each other identically, leaving only the three external forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$. These are added graphically in (*c*) to yield a net force $\mathbf{F}_{ext}$ acting on the center of mass of the system.

Resnick & Halliday (1966)

System?

- Note that many of our “definitions” thus far included the notion of a “system”

This states that *the center of mass of a system of particles moves as though all the mass of the system were concentrated at the center of mass and all the external forces were applied at that point.*

Suppose that the sum of the external forces acting on a system is zero. Then, from Eq. 9–17,

$$\frac{d\mathbf{P}}{dt} = 0 \qquad \text{or} \qquad \mathbf{P} = \text{constant}.$$

Conservation of linear momentum: When the net external force on a system is zero, the total momentum $\vec{P}$ of the system—the vector sum of the individual momenta $m\vec{v}$ of its constituent particles—remains constant.

→ But what is a *system*!?!

Conservation of (Linear) Momentum

The total momentum of the rocket + gases system is conserved, so the rocket accelerates forward as the gases are expelled backward.

→ A very general/fundamental principle is at hand here...

Knight

Conservation of (Linear) Momentum

- This "law" is stated in different way, but all nail down the same notion

Knight

Law of conservation of momentum The total momentum $\vec{P}$ of an isolated system is a constant. Interactions within the system do not change the system's total momentum.

Resnick & Halliday

Suppose that the sum of the external forces acting on a system is zero. Then, from Eq. 9–17,

$$\frac{d\mathbf{P}}{dt} = 0 \qquad \text{or} \qquad \mathbf{P} = \text{constant}.$$

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$$\vec{P} = \text{constant} \qquad \text{(conservation of linear momentum)}$$

Conservation of linear momentum: When the net external force on a system is zero, the total momentum $\vec{P}$ of the system—the vector sum of the individual momenta $m\vec{v}$ of its constituent particles—remains constant.

When the net force acting on an object (or a system of objects) is zero, the total linear momentum is conserved.

- This is another fundamental law of physics (just like the law of conservation of energy)
- If you have a group of objects (like billiard balls on a pool table), the initial momentum imparted to the cue ball by the shooter is distributed to one or more other balls.

$$\sum_{all\ objects} \overrightarrow{p_i} = \sum_{all\ objects} \overrightarrow{p_f}$$

$$\sum_{all\ objects} \vec{p_i} = \sum_{all\ objects} \vec{p_f}$$

- note that this is a vector equation. We can break up this equation into its components

$$\sum_{all\ objects} p_{x_i} = \sum_{all\ objects} p_{x_f}$$

$$\sum_{all\ objects} p_{y_i} = \sum_{all\ objects} p_{y_f}$$

- Therefore, for collision problems, as long as we can identify the objects in the system, we get one equation for each dimension.

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Collisions

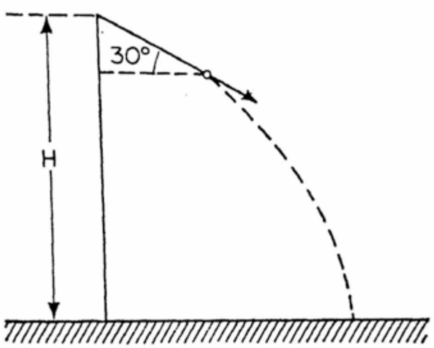


FIG. 43

81. Starting from a height H, a ball slips without friction, down a smooth plane inclined at an angle of 30° to the horizontal (Fig. 43). The length of the plane is $H/3$. The ball then falls on to a horizontal surface with an impact that may be taken as perfectly elastic. How high does the ball rise after striking the horizontal plane?

A tennis ball collides with a racket. Notice that the right side of the ball is flattened.

Collisions

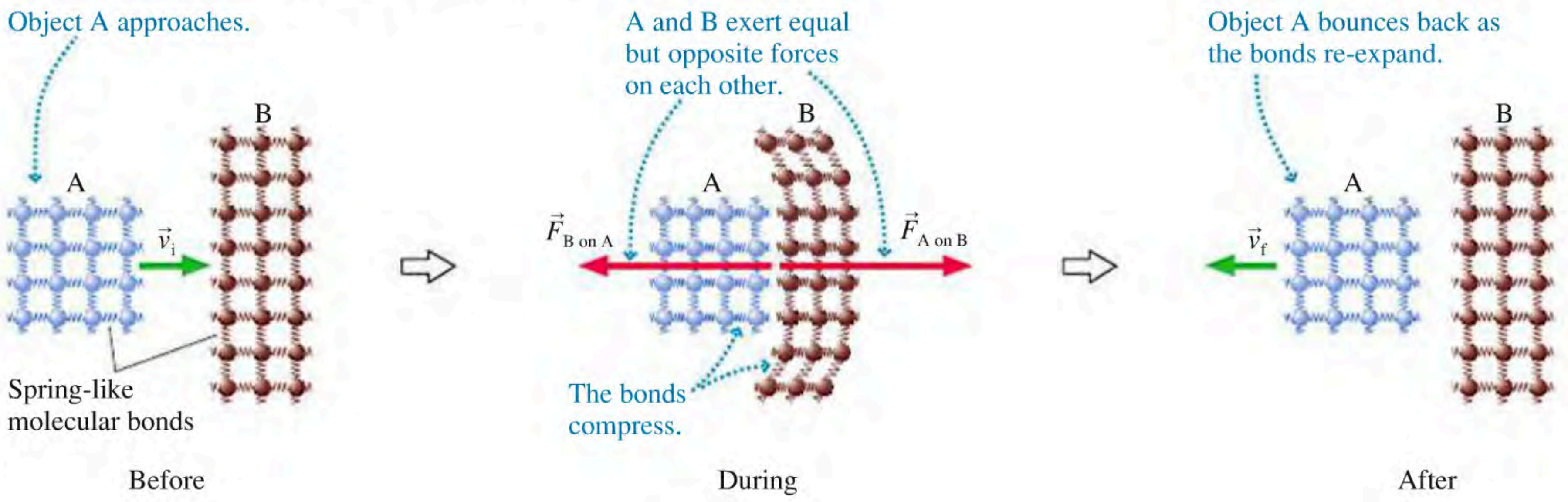
- **Elastic**: objects collide and bounce sharply off one another with no permanent deformation.
  - momentum is conserved
  - mechanical energy is conserved (kinetic+potential)
- **Inelastic**: objects collide and bounce off each other but there is some permanent deformation of the object.
  - momentum is conserved
  - mechanical energy is not conserved (lost to deformation and heat.)
- **Completely Inelastic**: objects collide an stick together, and travel along a common path after the collision.
  - momentum is conserved
  - mechanical energy is not conserved (lost to deformation and heat.)

<u>Interdisciplinary connection</u> (re chemistry)

- Interactions between objects....

A tennis ball collides with a racket. Notice that the right side of the ball is flattened.

Atomic model of a collision.



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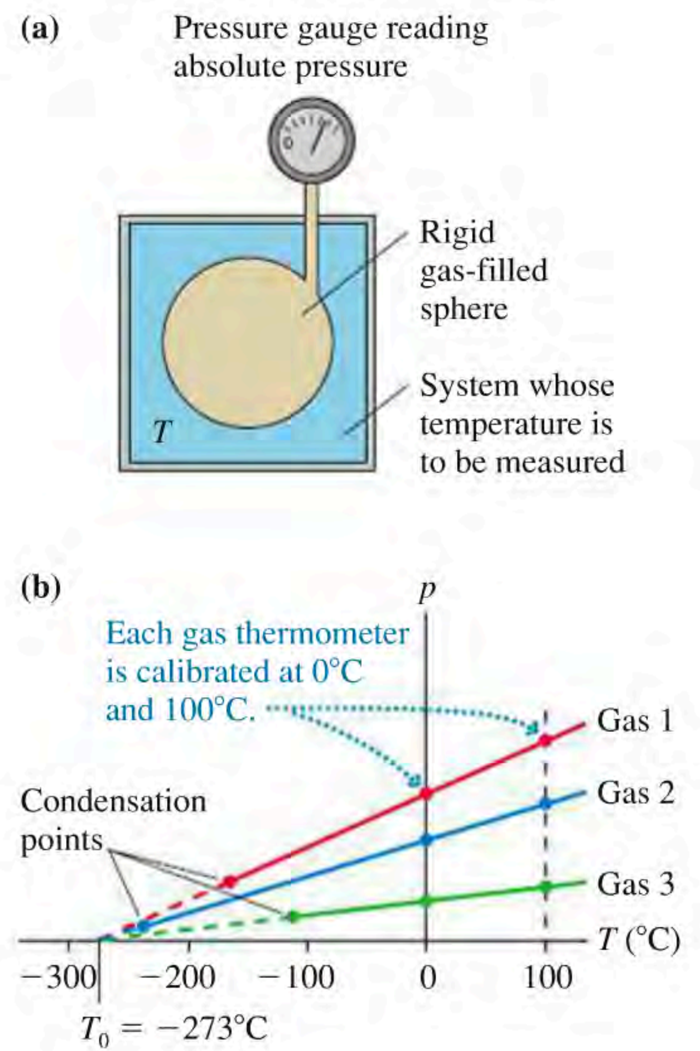
<u>Interdisciplinary connection</u> (re chemistry)

- .... helps lead towards the *ideal gas law*

$$pV = nRT \qquad \text{(ideal-gas law)}$$

$$pV = Nk_BT \qquad \text{(ideal-gas law)}$$

The pressure in a constant-volume gas thermometer extrapolates to zero at $T_0 = -273°C$. This is the basis for the concept of absolute zero.



Knight

Interdisciplinary connection (re chemistry)

OVERVIEW

It's All About Energy

Thermodynamics—the science of energy in its broadest context—arose hand in hand with the industrial revolution as the systematic study of converting heat energy into mechanical motion and work. Hence the name *thermo* + *dynamics*. Indeed, the analysis of engines and generators of various kinds remains the focus of engineering thermodynamics. But thermodynamics, as a science, now extends to all forms of energy conversions, including those involving living organisms. For example:

- **Engines** convert the energy of a fuel into the mechanical energy of moving pistons, gears, and wheels.
- **Fuel cells** convert chemical energy into electrical energy.
- **Photovoltaic cells** convert the electromagnetic energy of light into electrical energy.
- **Lasers** convert electrical energy into the electromagnetic energy of light.
- **Organisms** convert the chemical energy of food into a variety of other forms of energy, including kinetic energy, sound energy, and thermal energy.

Knight