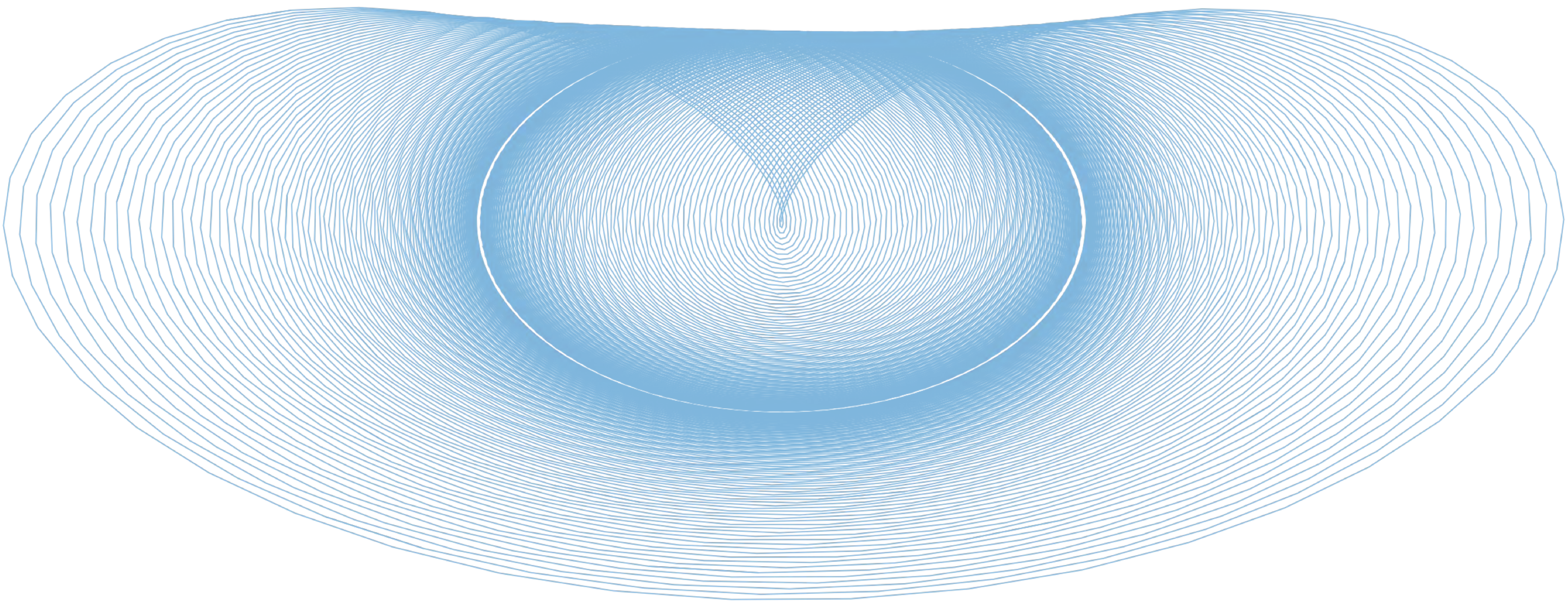
PHYS 1420 (F19)
Physics with Applications to Life Sciences



2019.11.11

Relevant reading:

Kesten & Tauck ch. N/A

Christopher Bergevin
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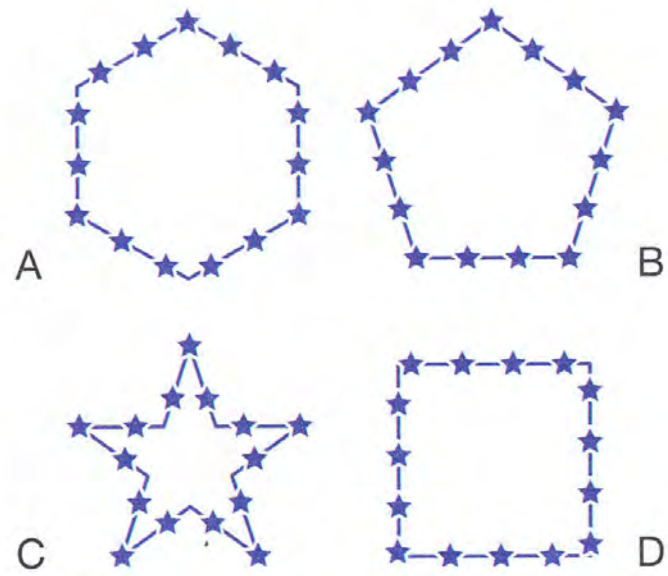
Office: Petrie 240   Lab: Farq 103
cberge@yorku.ca

Ref. (re images):
Wolfson (2007), Knight (2017),
Kesten & Tauck (2012)

81. Constellation Shapes



Which constellation has the most stars?

A B C D

Announcements & Key Concepts (re Today)

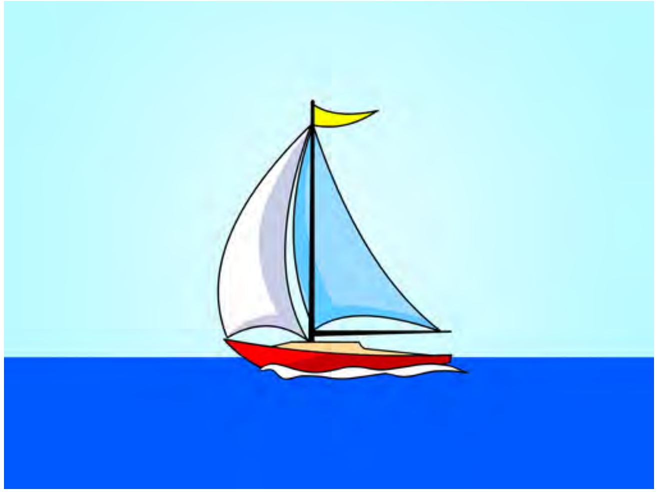
→ Written HW #2: Posted and due Friday 11/15 in class

→ Midterm exams: Grades posted on Moodle and exams handed back

Some relevant underlying concepts of the day…

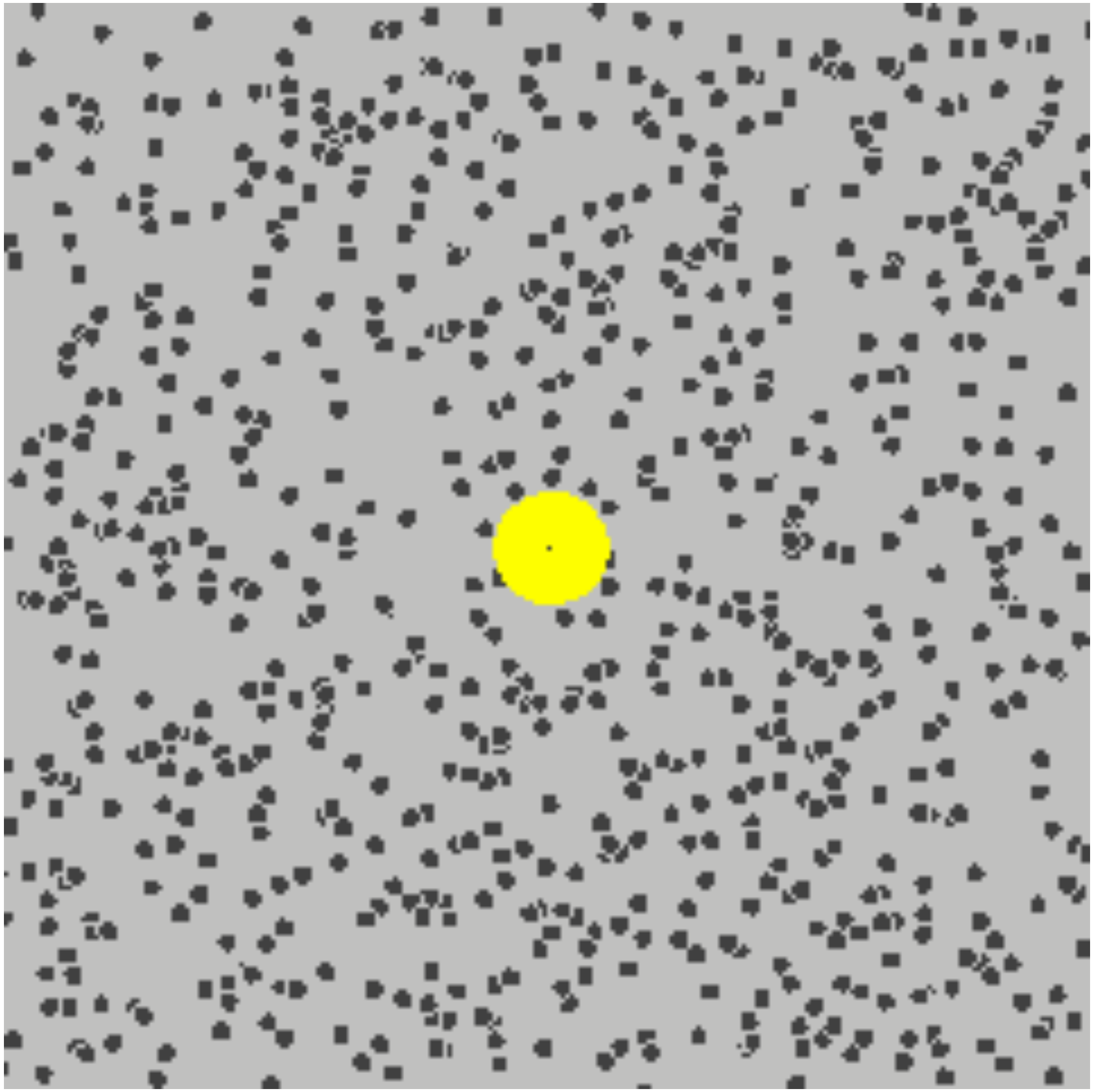
- Diffusion
- Random walkers
- Multivariable functions

Passive versus active movement



Brownian Motion

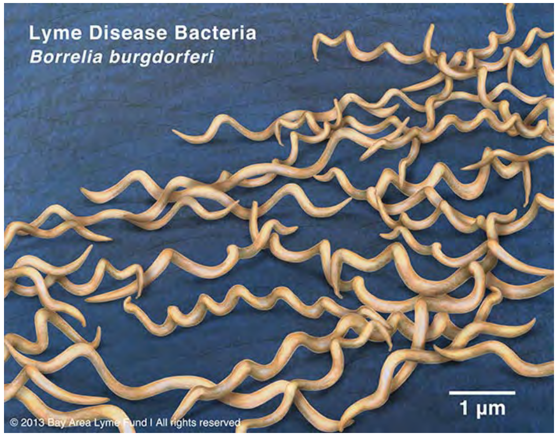
Random motion of large object (yellow circle) due to interaction with many little objects (black circles)

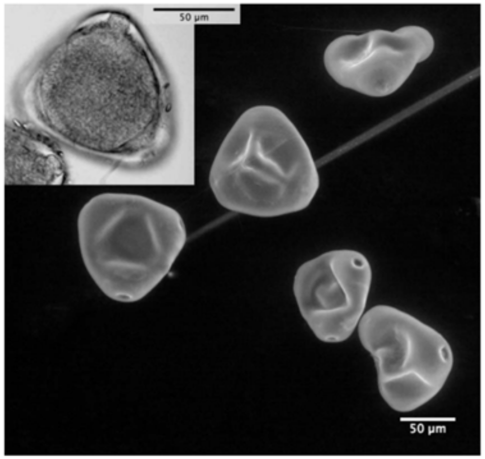


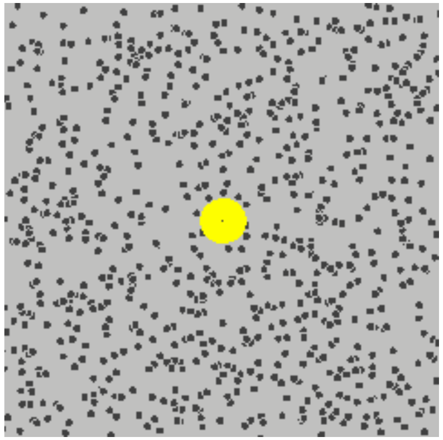
Seems to be "jostled" around...

Physical Idea:
At any moment, there is a net non-zero force (from all the black balls) pushing in the large yellow ball in some random direction

wikipedia (Brownian motion)







→ So a more general question emerges:

What are the basic mechanisms by which "stuff" moves around?

Heat-transfer mechanisms

When two objects are in direct contact, such as the soldering iron and the circuit board, heat is transferred by *conduction*.

Air currents near a warm glass of water rise, taking thermal energy with them in a process known as *convection*.

The lamp at the top shines on the lambs huddled below, warming them. The energy is transferred by *radiation*.

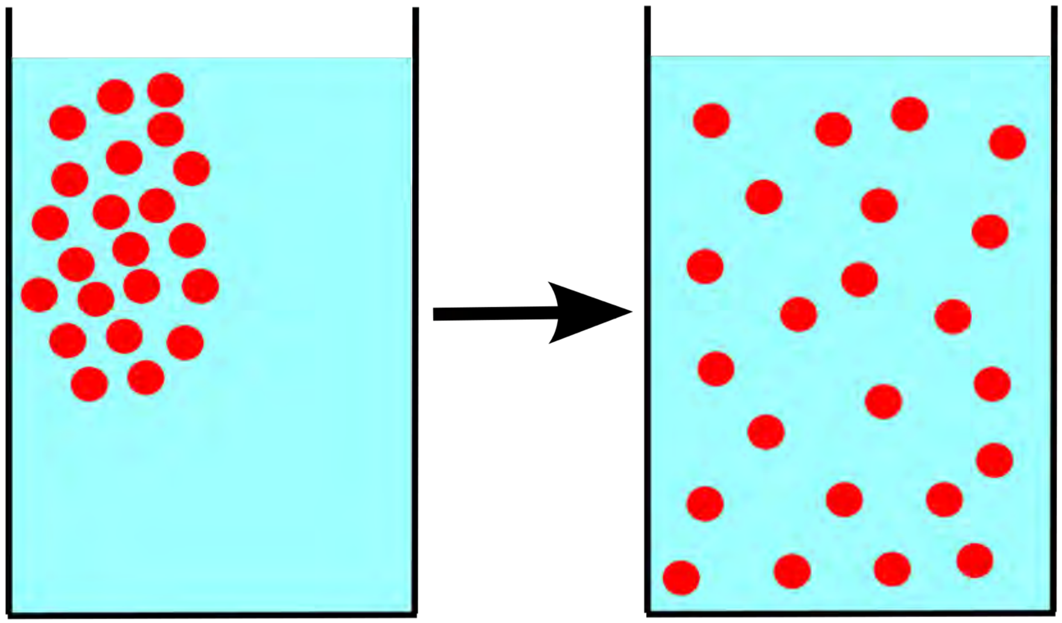
Blowing on a hot cup of tea or coffee cools it by *evaporation*.

Goal now is to build up a theme focusing on one of these in particular....

... and that is a key principle underlying *conduction*

Knight

Diffusion



Note: Lots of "objects in direct contact" here!

→ Lots of collisions!

Recall (re collisions)

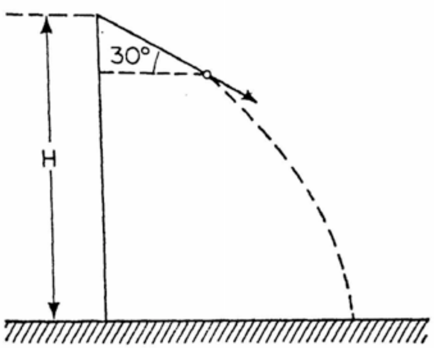
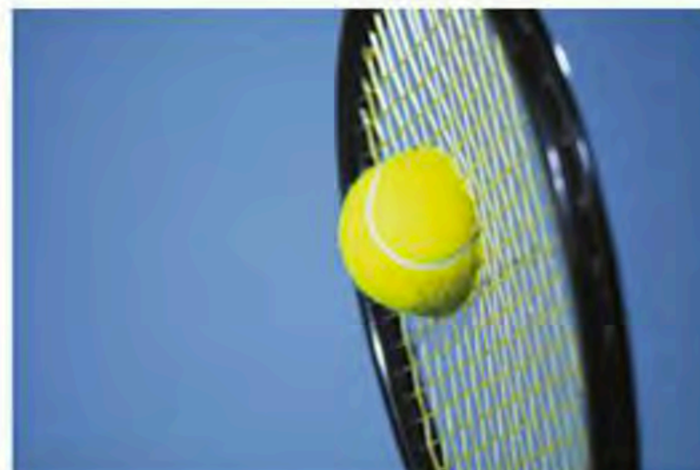


FIG. 43

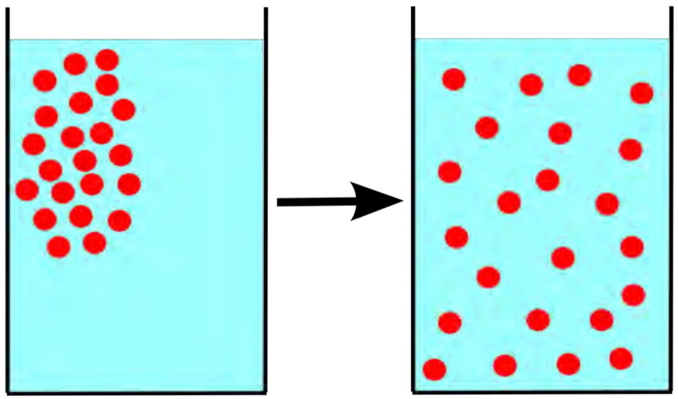
81. Starting from a height H, a ball slips without friction, down a smooth plane inclined at an angle of 30° to the horizontal (Fig. 43). The length of the plane is $H/3$. The ball then falls on to a horizontal surface with an impact that may be taken as perfectly elastic. How high does the ball rise after striking the horizontal plane?



A tennis ball collides with a racket. Notice that the right side of the ball is flattened.

Recall (re collisions)

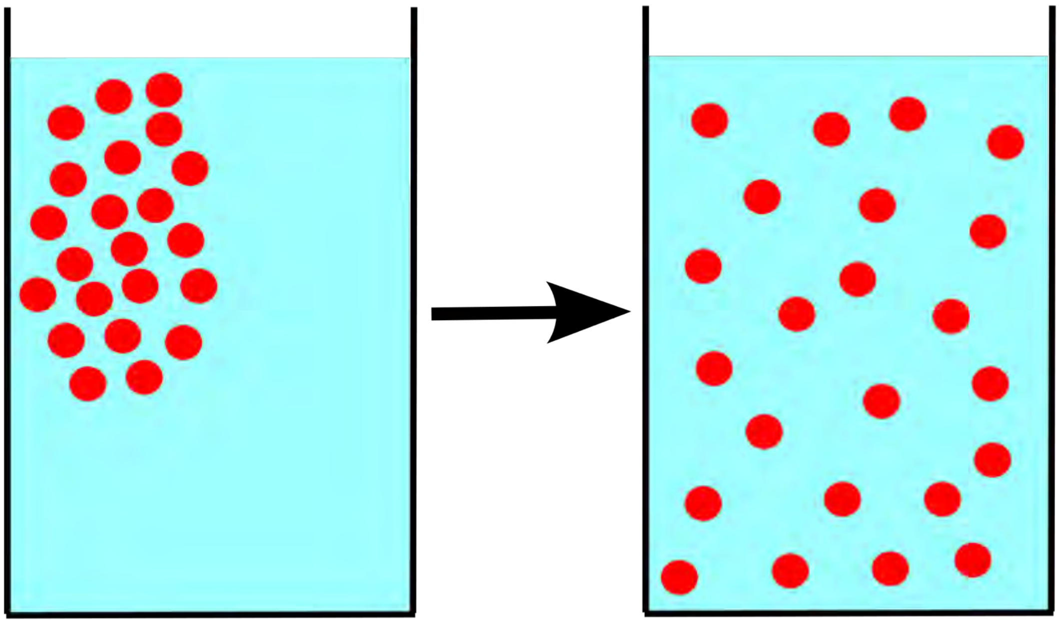
- Elastic: objects collide and bounce sharply off one another with no permanent deformation.
  - momentum is conserved
  - mechanical energy is conserved (kinetic+potential)
- Inelastic: objects collide and bounce off each other but there is some permanent deformation of the object.
  - momentum is conserved
  - mechanical energy is not conserved (lost to deformation and heat.)
- Completely Inelastic: objects collide an stick together, and travel along a common path after the collision.
  - momentum is conserved
  - mechanical energy is not conserved (lost to deformation and heat.)



→ All this stuff is still at play in terms of how the *solute* (red circles) interacts with the *solvent* (blue stuff)

M. George

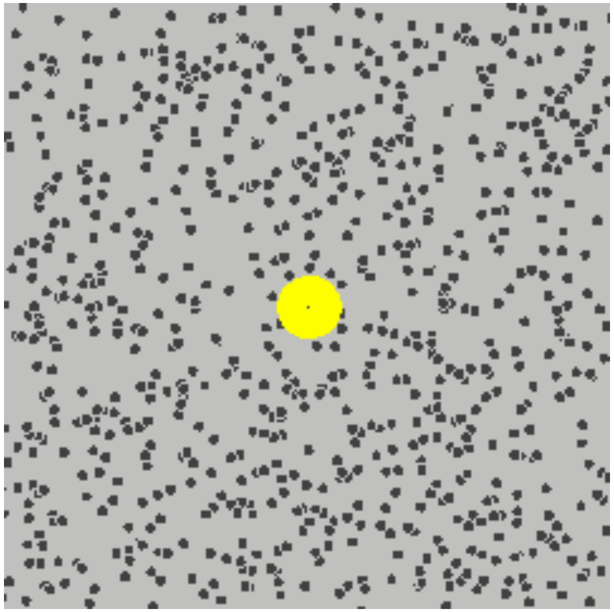
Diffusion



Note: Lots of "objects in direct contact" here!

→ Lots of collisions!

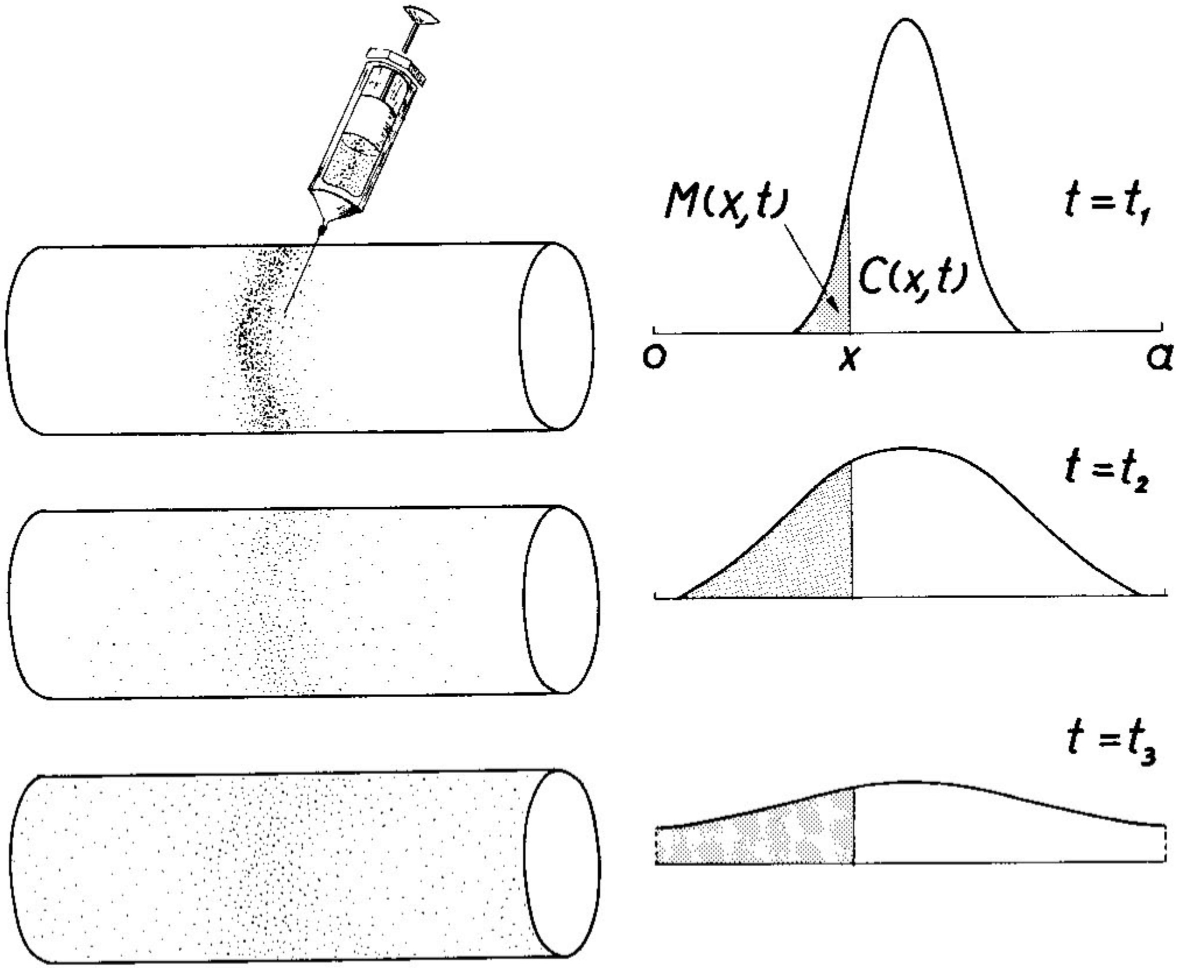
" ...how the *solute* (red circles) interacts with the *solvent* (blue stuff)"



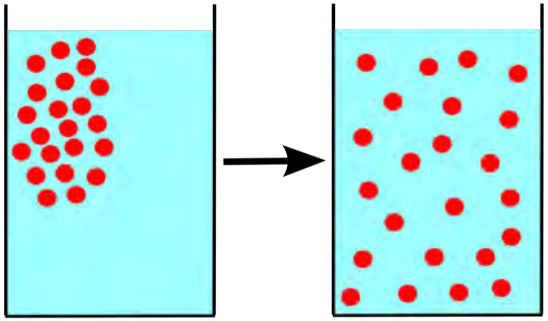
→ These two are connected (i.e., yellow circle on lower right represents behavior of one of the red circles above)

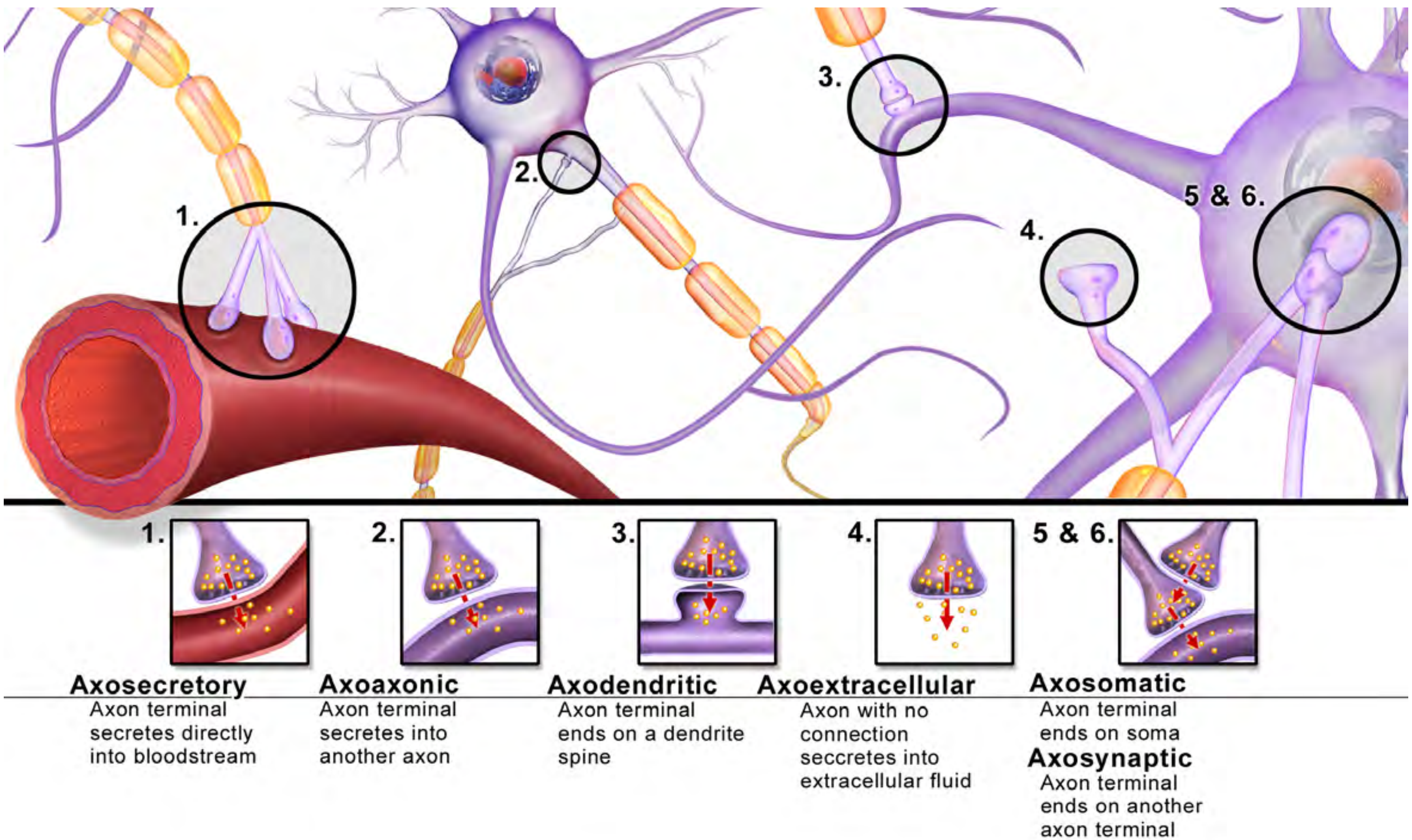
Diffusion Processes

→ You have intuition for this already....



Batschelet Fig.12.5




1.
2.
3.
4.
5 & 6.
Axosecretory
Axon terminal secretes directly into bloodstream
Axoaxonic
Axon terminal secretes into another axon
Axodendritic
Axon terminal ends on a dendrite spine
Axoextracellular
Axon with no connection seccretes into extracellular fluid
Axosomatic
Axon terminal ends on soma
Axosynaptic
Axon terminal ends on another axon terminal

Wikipedia

Neurons + "Electrodiffusion"

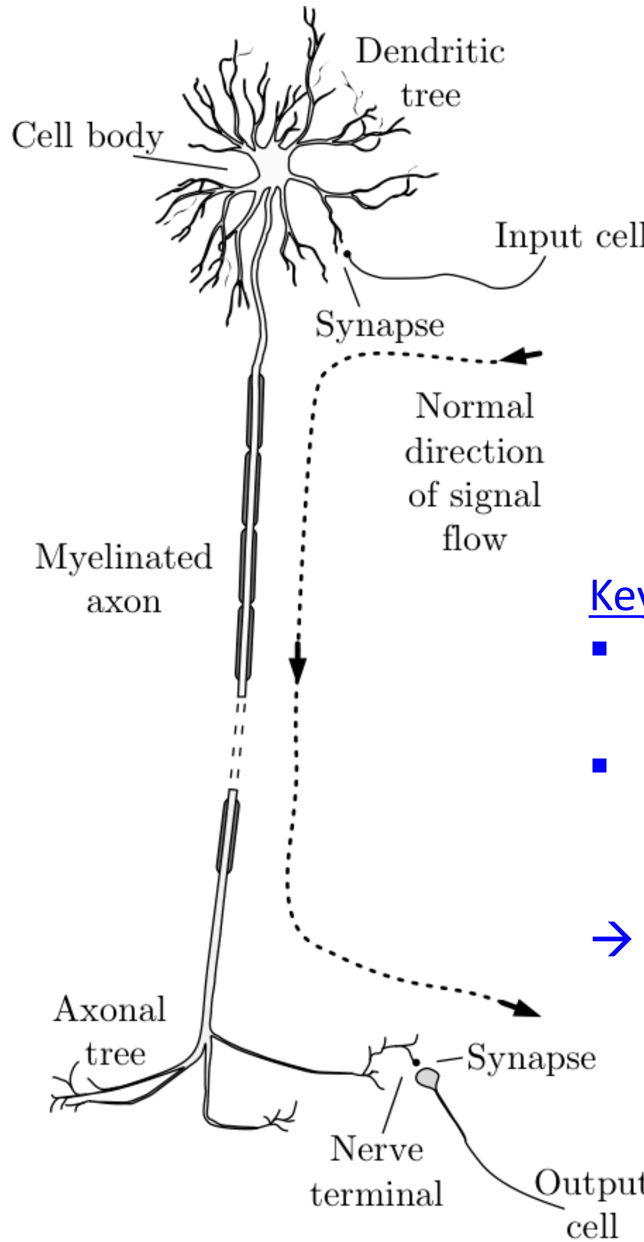


Figure 1.22

Neurons ("fibers")
= Information highway

→ Action potentials (or spikes) is a primary means for *information* to propagate through the body

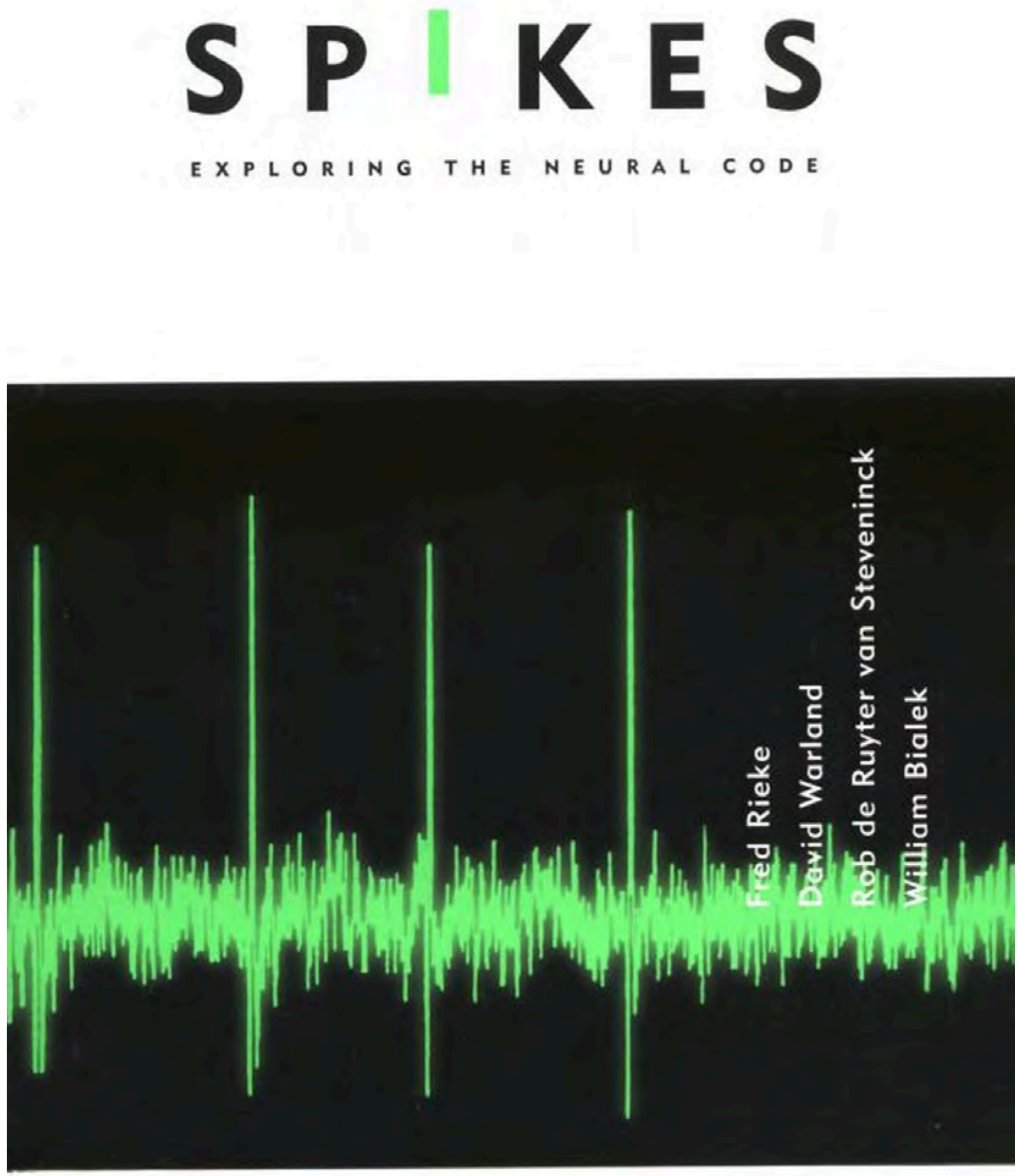
Key Ideas

- Electrical properties of cells are important
- Diffusion sets foundation for transport across cell membrane

→ *Action potentials*

Weiss (1996)

Action Potentials



"Neural code"

Aside

Is our central nervous system essentially "digitized"?

Key Idea: Membrane transport

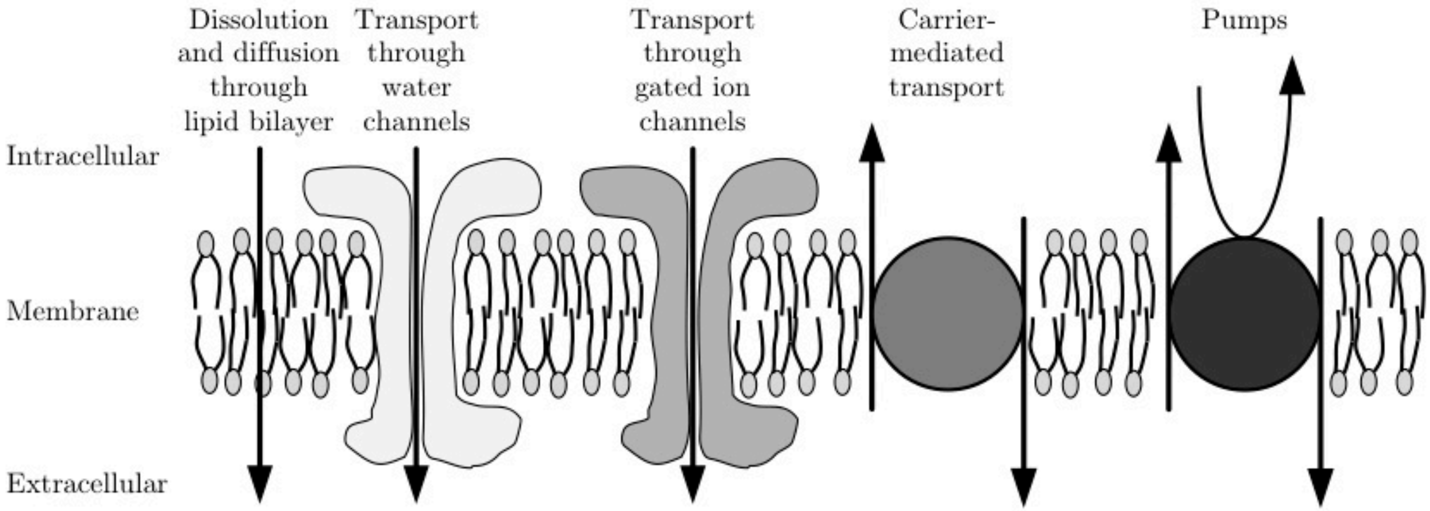


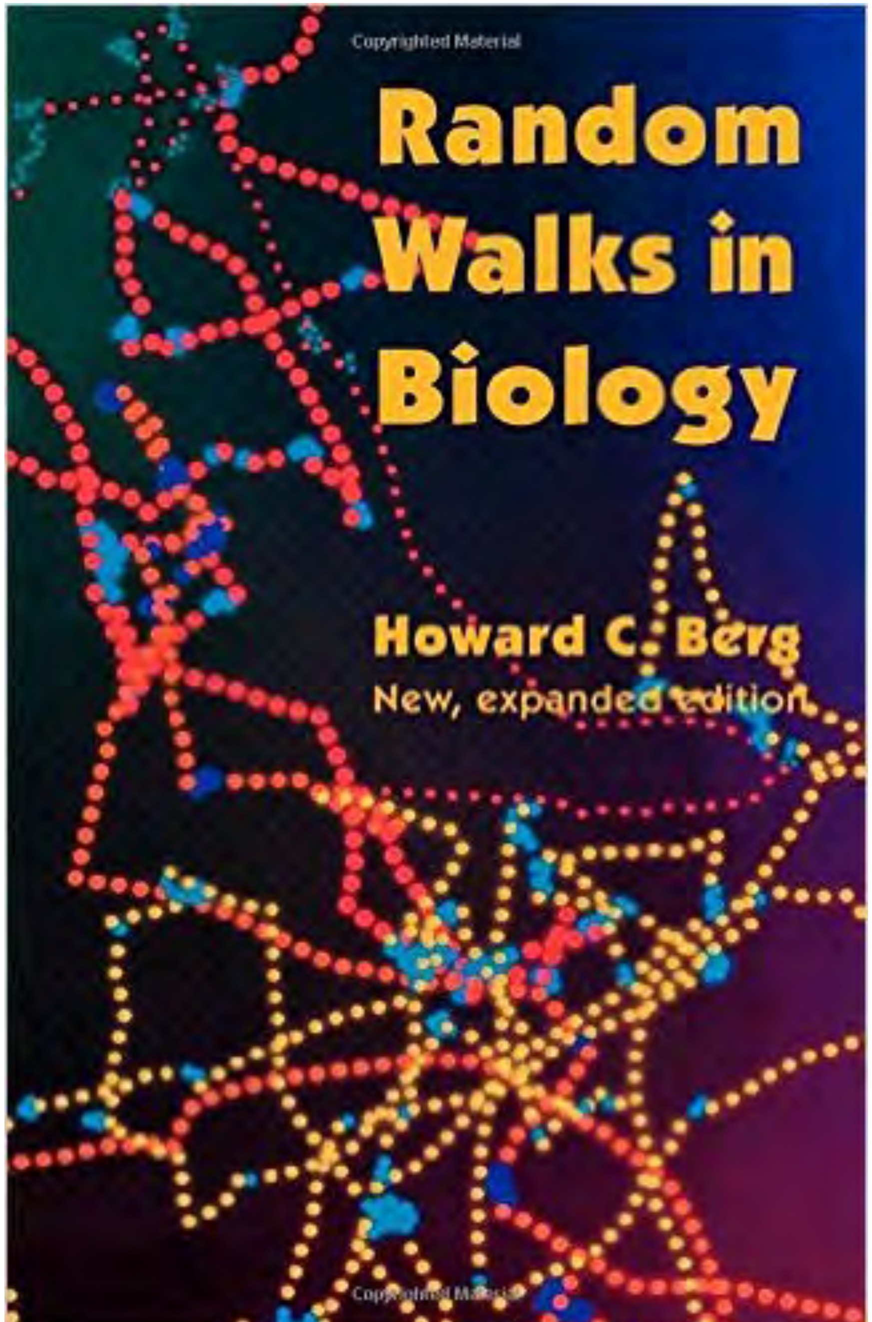
Figure 2.19

- Cell membranes separate *inside* and *outside*
- Acts as a *barrier* to diffusion
- Controls what *solutes* are on either side and means to *transport* such
- Variety of transport mechanisms: carriers, ion channels, pumps, etc.…

→ These stem to help control movement of stuff in/out of cell

Weiss (1996)

Diffusion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Howard C. Berg
New, expanded edition
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Diffusion

Chapter 1

Diffusion: Microscopic Theory

Diffusion is the random migration of molecules or small particles arising from motion due to thermal energy. A particle at absolute temperature T has, on the average, a kinetic energy associated with movement along each axis of $kT/2$, where k is Boltzmann's constant. Einstein showed in 1905 that this is true regardless of the size of the particle, even for particles large enough to be seen under a microscope, i.e., particles that exhibit Brownian movement. A particle of mass m and velocity v_x on the x axis has a kinetic energy $mv_x^2/2$. This quantity fluctuates, but on the average $\langle mv_x^2/2 \rangle = kT/2$, where $\langle \; \rangle$ denotes an average over time or over an ensemble of similar particles. From this relationship we compute the mean-square velocity,

$$\langle v_x^2 \rangle = kT/m, \qquad (1.1)$$

and the root-mean-square velocity,

$$\langle v_x^2 \rangle^{1/2} = (kT/m)^{1/2}. \qquad (1.2)$$

We can use Eq. 1.2 to estimate the instantaneous velocity of a small particle, for example, a molecule of the protein lysozyme. Lysozyme has a molecular weight 1.4×10^4 g. This is the mass of one mole, or 6.0×10^{23} molecules; the mass of one molecule is $m = 2.3 \times 10^{-20}$ g. The value of kT at 300°K (27°C) is 4.14×10^{-14} g cm^2/sec^2. Therefore, $\langle v_x^2 \rangle^{1/2} = 1.3 \times 10^3$ cm/sec. This is a sizeable speed. If there were no obstructions, the molecule would cross a typical classroom in about 1 second. Since the protein is not in a vacuum but is immersed in an aqueous medium, it does not go very far before it bumps into molecules of

Some (remarkably deep) ideas right off the bat:

- Random walkers
- Temperature, Boltzmann's constant
- Einstein and 1905
- Mean-squared velocity, "ensemble"
- "Brownian movement"
- "Microscopic theory" (ch.2 is "Macroscopic theory")

→ A kernel of a deep idea is here, the distinction between "lots of little things" versus "big things"

[statistical mechanics being the thread tying things together]

Berg (1993)

<u>Random walks</u> (REVISTED)

Let us consider a passive random walker(s) in **1-D** (i.e., on a line)
with **no bias** (i.e., equal probability of steeping left or right)

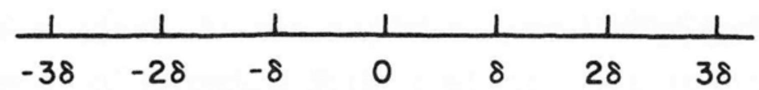


Fig. 1.2. Particles executing a one-dimensional random walk start at the origin, 0, and move in steps of length δ, occupying positions 0, $\pm\delta$, $\pm 2\delta$, $\pm 3\delta$, ...

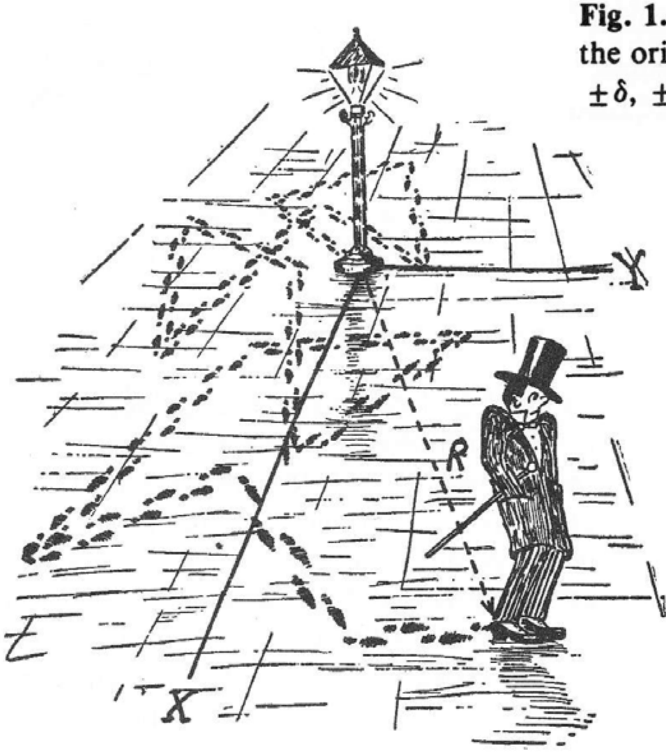


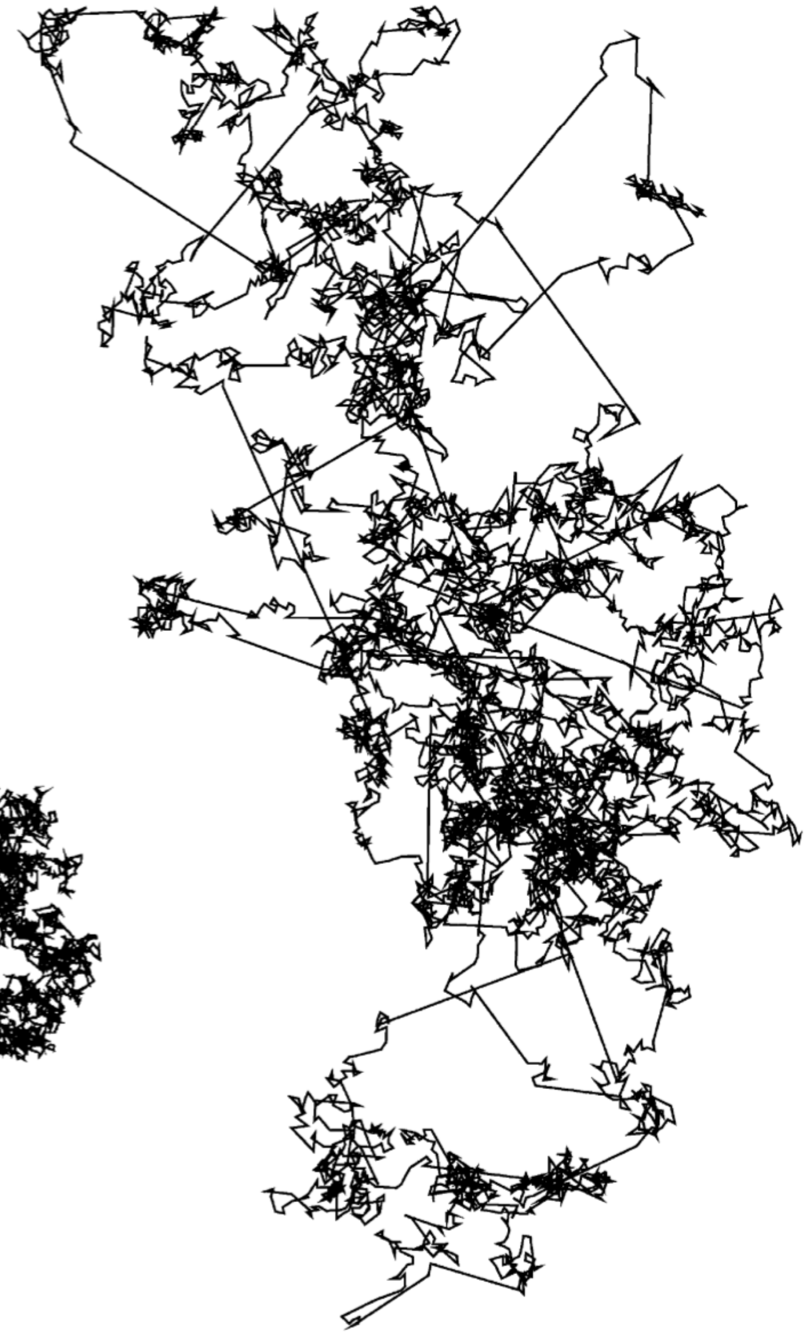
Figure 4.1: (Metaphor.) A random (or "drunkard's") walk. [Cartoon by George Gamow, from Gamow, 1961.]

Nelson (2004)

Recall: 2-D random walkers

Notes

- These are 2-D random walks
- Left one is passive, the right one is active (i.e., it swims at times)
- Both are likely *unbiased*



Metzler & Klafter (2000)

<u>Diffusion</u> (Microscopic)

- Brownian motion ⇒ 'Random Walker' (1-D)

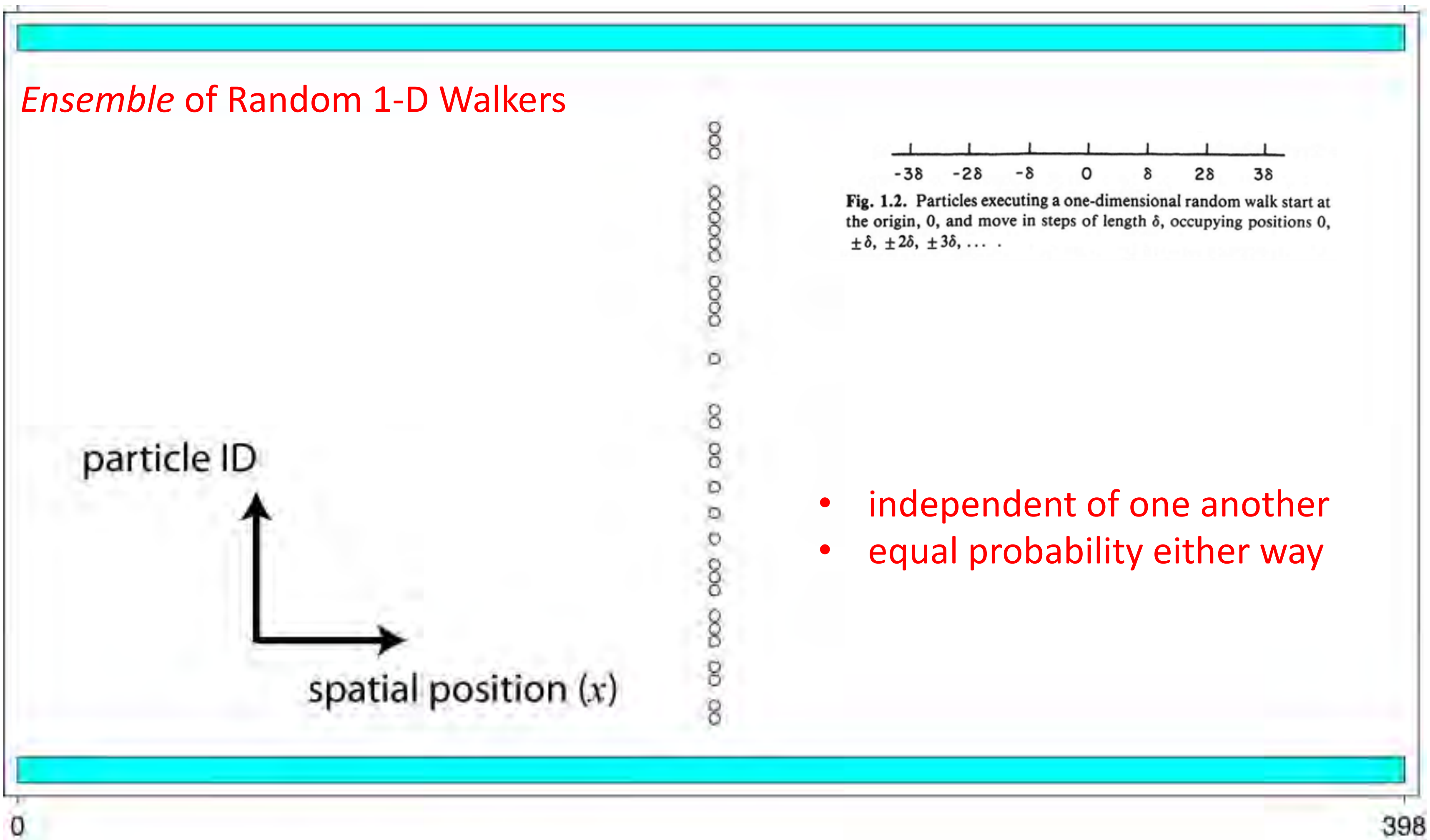
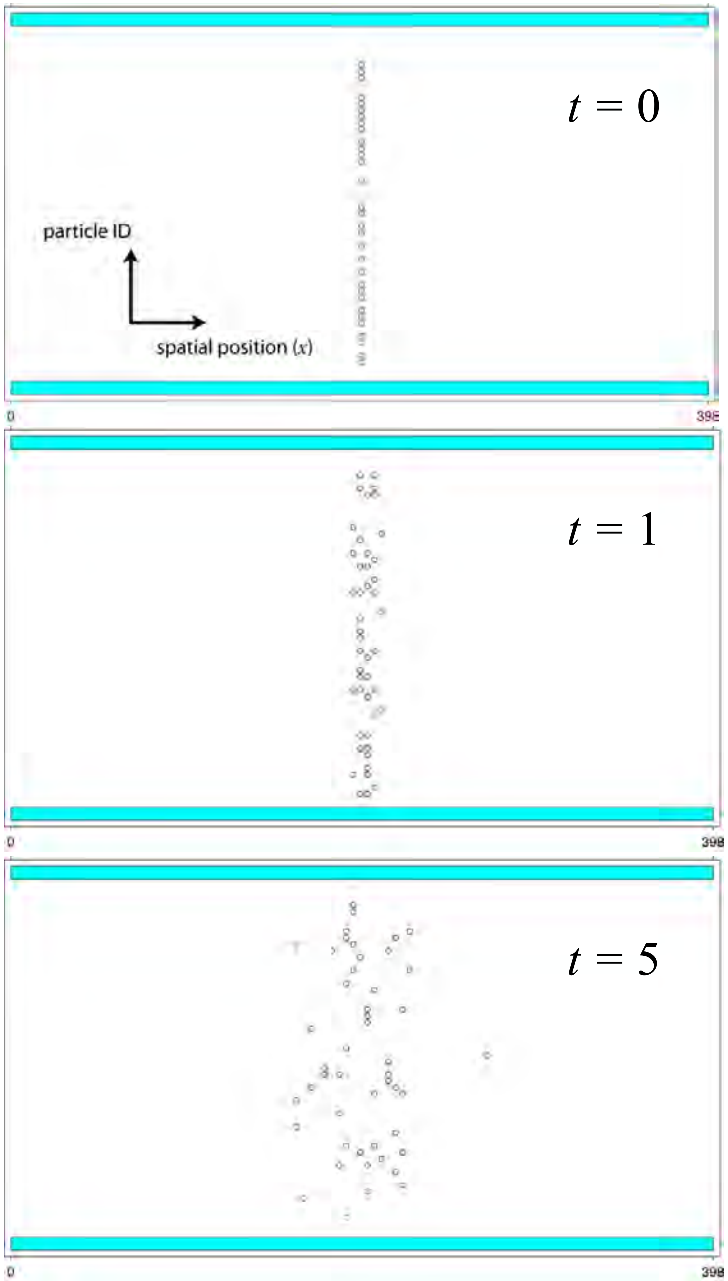
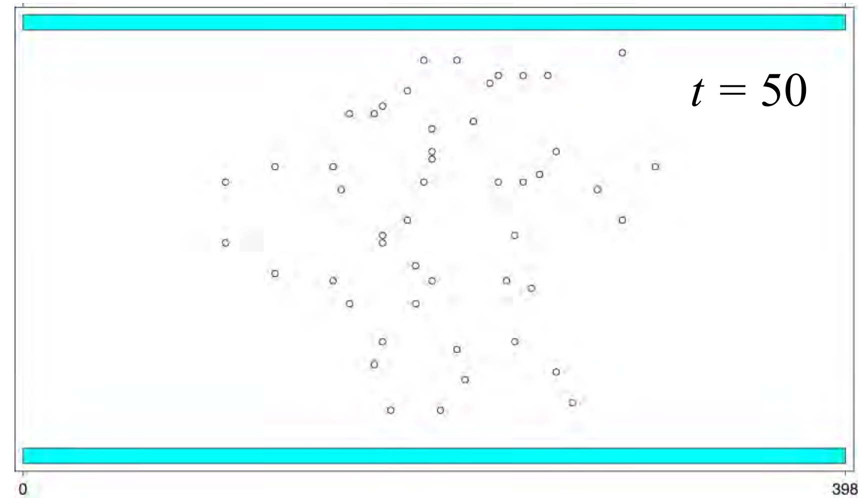
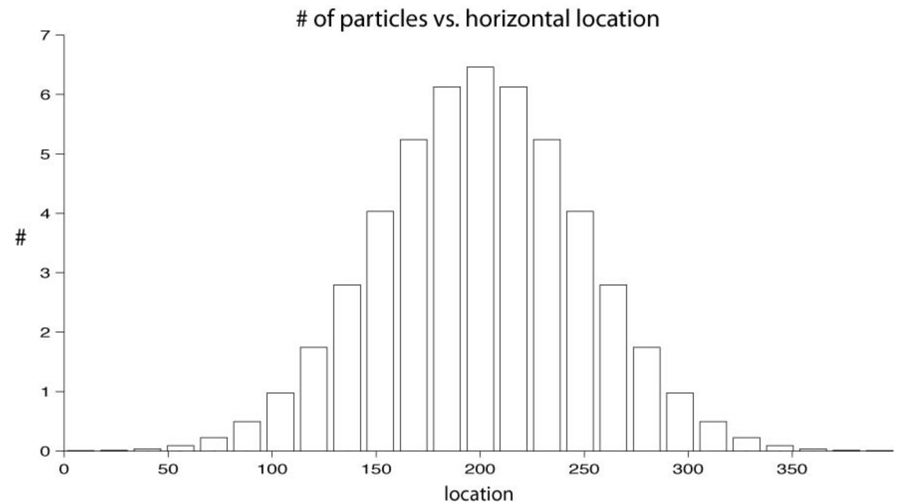


Fig. 1.2. Particles executing a one-dimensional random walk start at the origin, 0, and move in steps of length δ, occupying positions 0, ±δ, ±2δ, ±3δ, … .

<u>Diffusion</u>: Microscopic

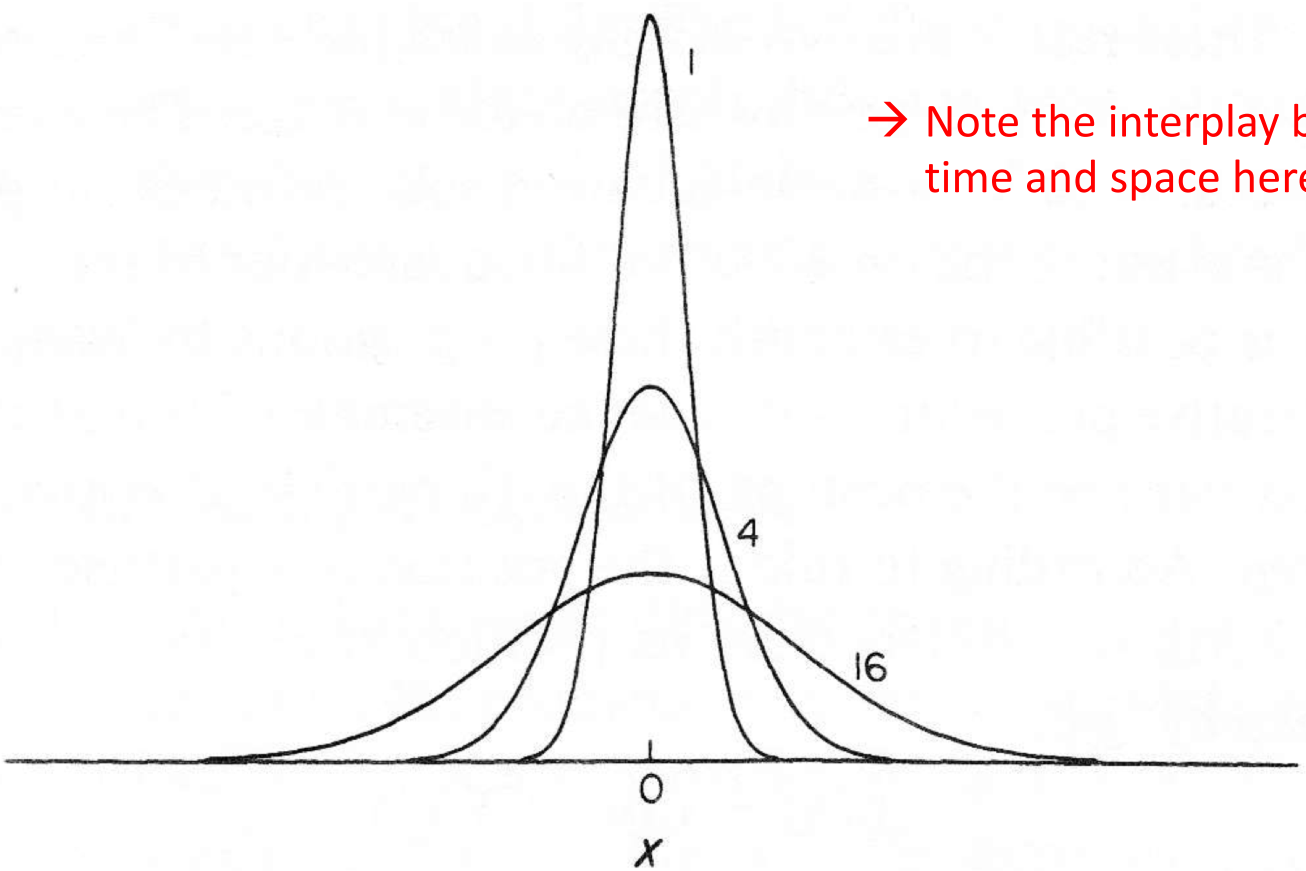






→ On average, they don't go anywhere... but they do "spread out" with time

Diffusion: Microscopic → Macroscopic

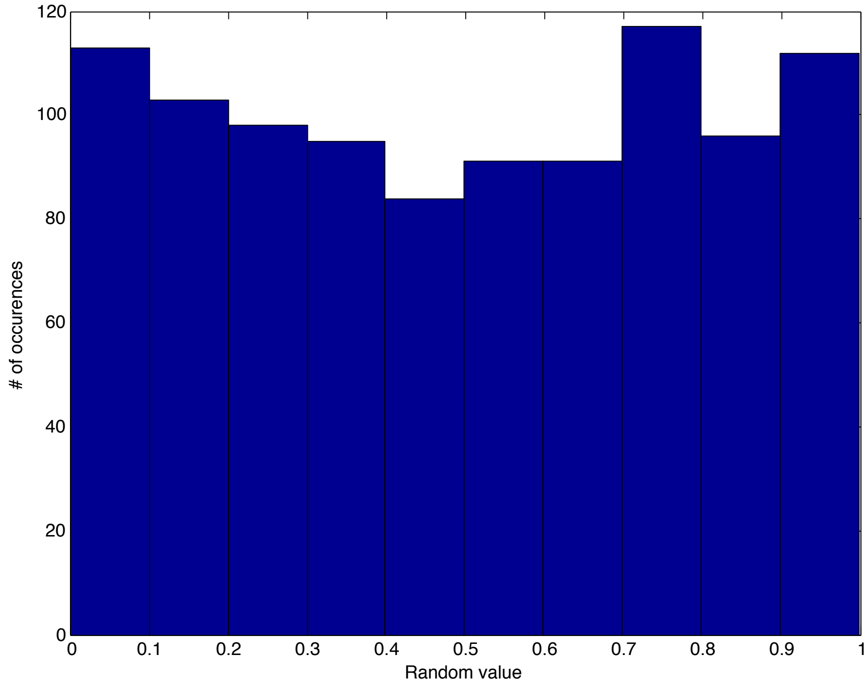


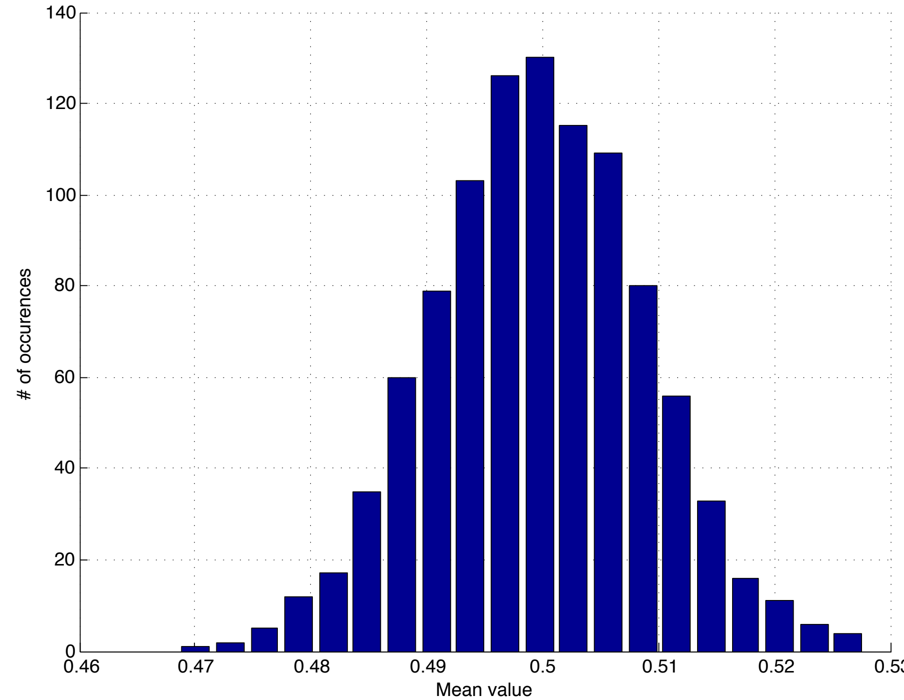
→ Note the interplay between time and space here....

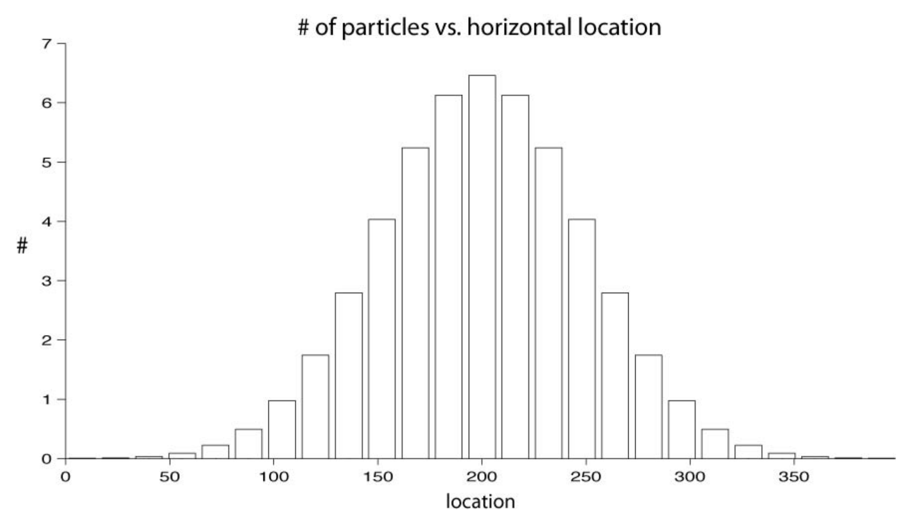
Fig. 1.3. The probability of finding particles at different points x at times $t = 1, 4$, and 16. The particles start out at position $x = 0$ at time $t = 0$. The standard deviations (root-mean-square widths) of the distributions increase with the square-root of the time. Their peak heights decrease with the square-root of the time. See Eq. 1.22.

Berg (1993)

Deep-ish connection point!







<u>Diffusion</u>: Statistics of the "microscopic"...

Let's assume:

- independent of one another
- equal probability either way

➢ Random walking (in 1-D for simplicity)

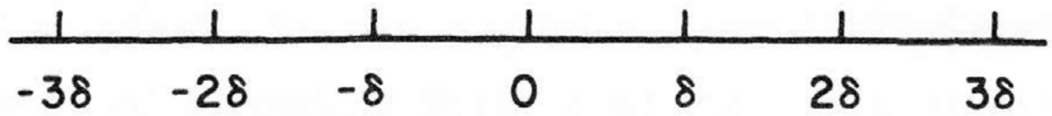


Fig. 1.2. Particles executing a one-dimensional random walk start at the origin, 0, and move in steps of length δ, occupying positions 0, $\pm\delta$, $\pm 2\delta$, $\pm 3\delta$, ...

Position of i'th walker after n'th step:

$$x_i(n) = x_i(n-1) \pm \delta.$$

Mean displacement:

$$\langle x(n) \rangle = \frac{1}{N} \sum_{i=1}^{N} x_i(n)$$

$$\langle x(n) \rangle = \frac{1}{N} \sum_{i=1}^{N} [x_i(n-1) \pm \delta]$$

$$= \frac{1}{N} \sum_{i=1}^{N} x_i(n-1) = \langle x(n-1) \rangle$$

→ On average, they don't go anywhere(!)

Berg (1993)

Diffusion: Microscopic → Macroscopic

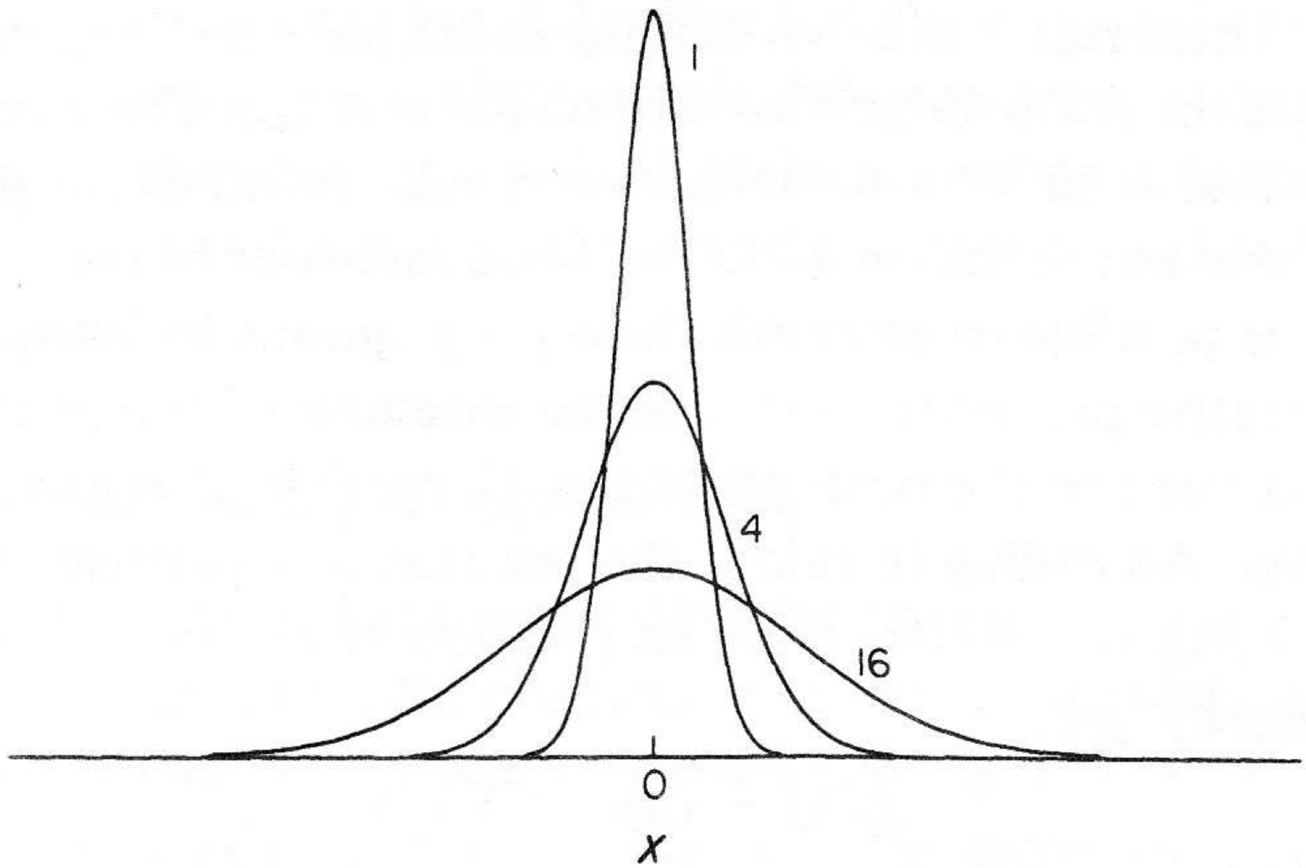
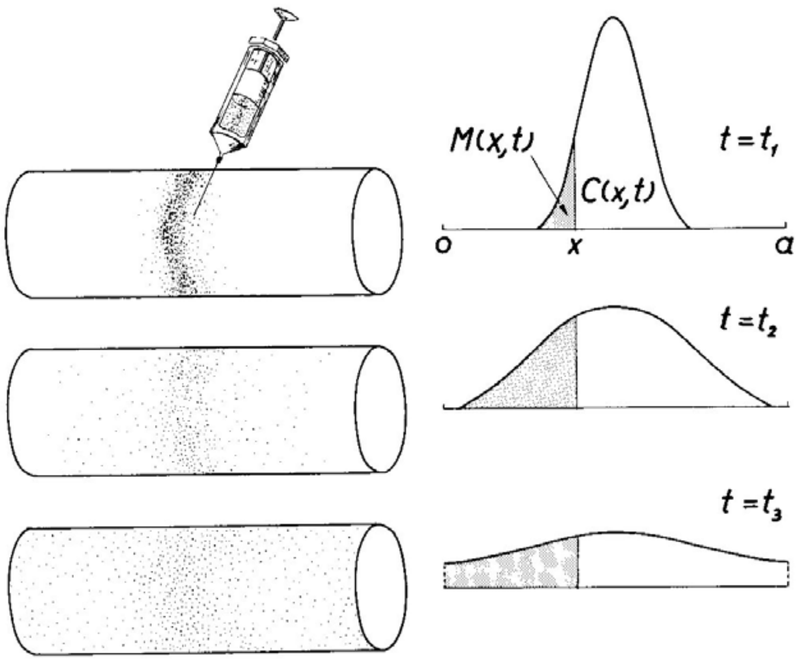


Fig. 1.3. The probability of finding particles at different points x at times $t = 1, 4$, and 16. The particles start out at position $x = 0$ at time $t = 0$. The standard deviations (root-mean-square widths) of the distributions increase with the square-root of the time. Their peak heights decrease with the square-root of the time. See Eq. 1.22.

Berg (1993)

"Diffusion math" → Multivariable functions



Note: Concentration of a solute in a solution (*c*) depends upon both spatial location (*x*) and time (*t*)

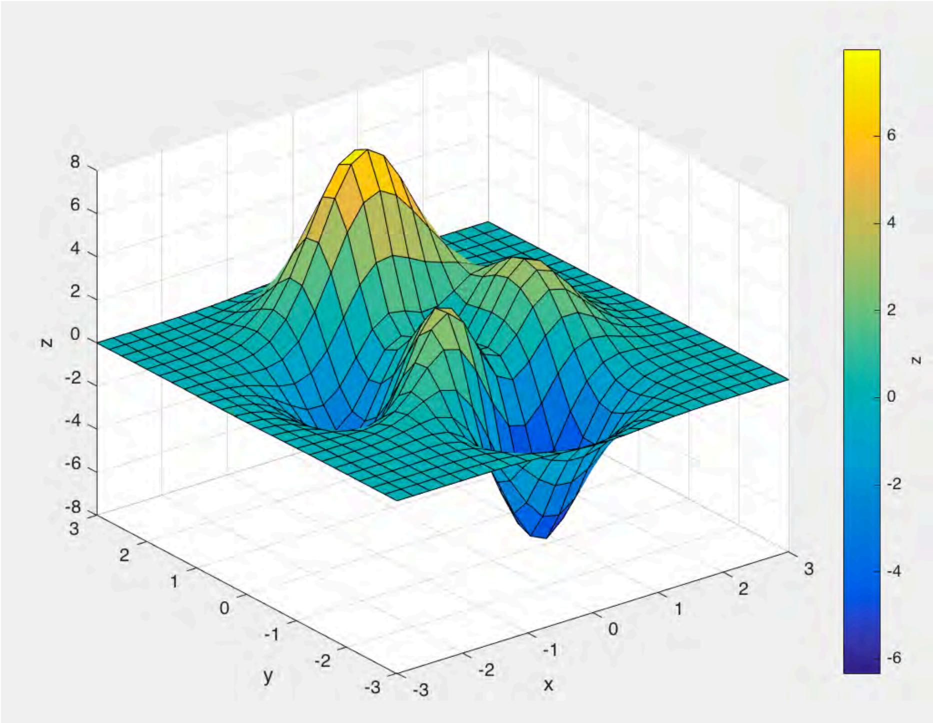
Multivariable function

$$f = f(x, y)$$

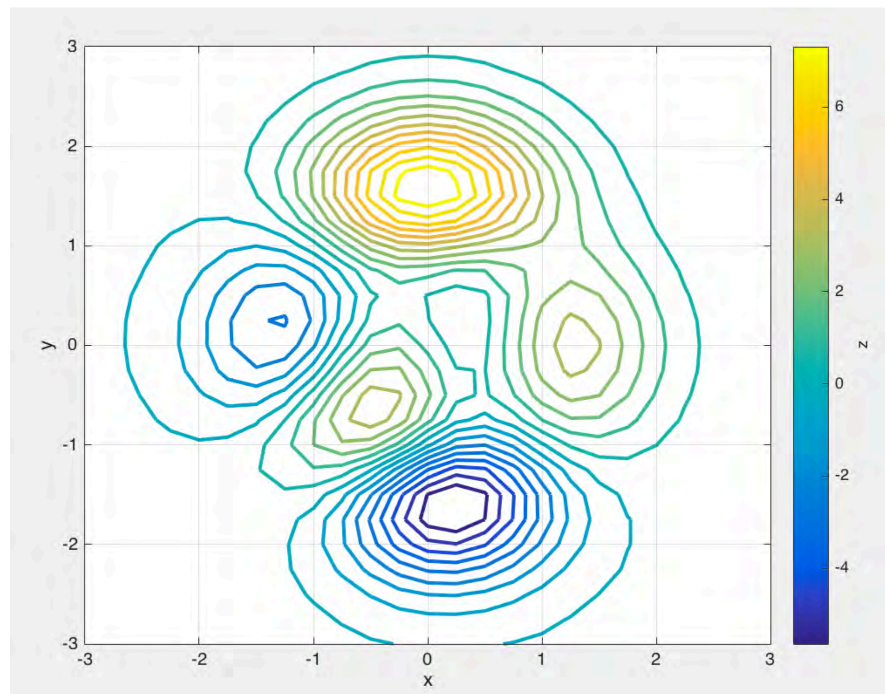
f - dependent variable

x, y - independent variables

- Multivariate functions are important in many various contexts throughout science



z
x
y

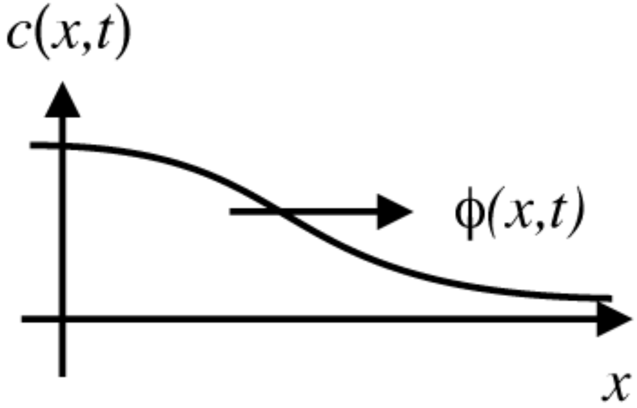


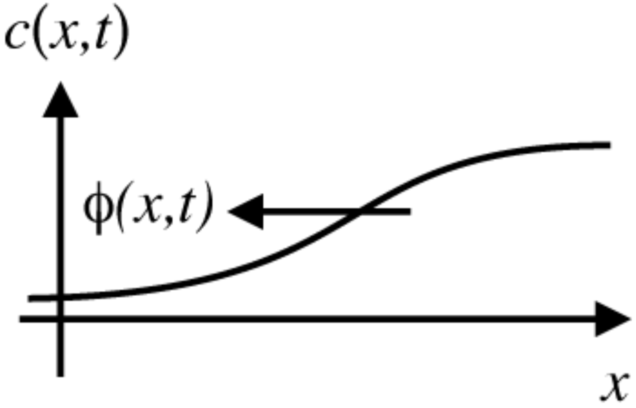
z
x
y

Diffusion: Macroscopic

Note: This is a multi-variable function(!!)

From Graham's observations (~1830):





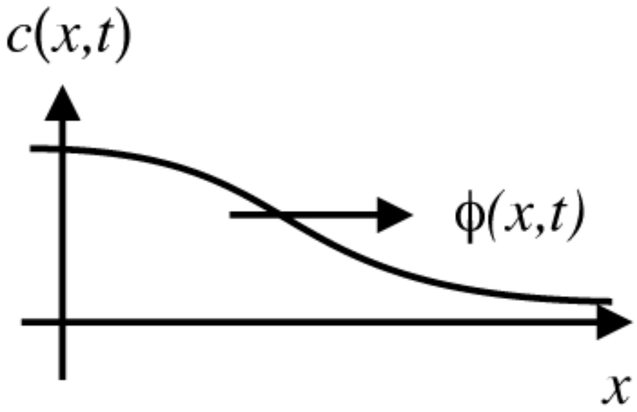
" A few years ago, Graham published an extensive investigation on the diffusion of salts in water, in which he more especially compared the diffusibility of different salts. It appears to me a matter of regret, however, that in such an exceedingly valuable and extensive investigation, the development of a fundamental law, for the operation of diffusion in a single element of space, was neglected, and I have therefore endeavoured to supply this omission."

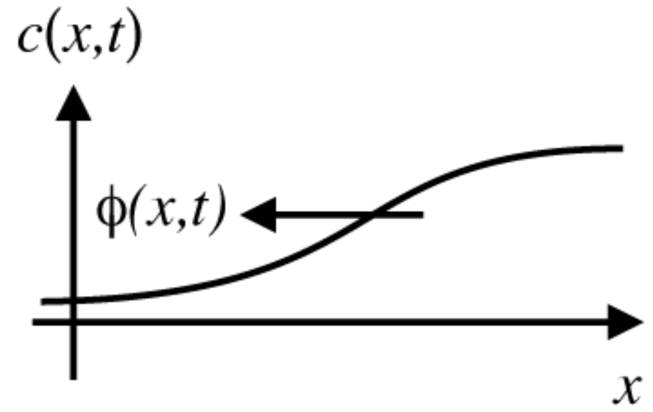
- A. Fick (1855)

Diffusion (1-D)

Note: We are starting to deal w/ multivariable functions here
(we will come back to this in more detail soon re waves)

From Graham's observations (~1830):





$$c(x,t)$$

Concentration - of solute in solution [*mol/m*^{3}]

$$\phi(x,t)$$

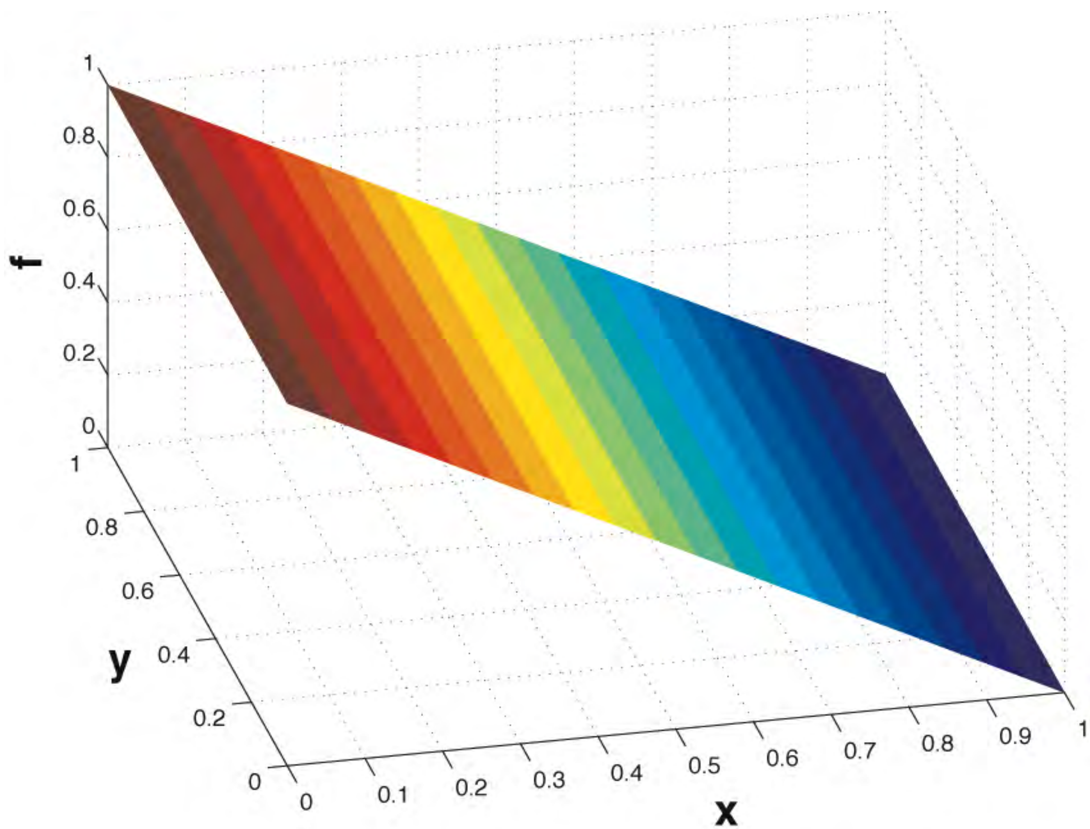
Flux - net # of moles crossing per unit time t through a unit area perpendicular to the x-axis [*mol/m*^{2}·*s*]

Note: flux is a vector!

$$x,t$$

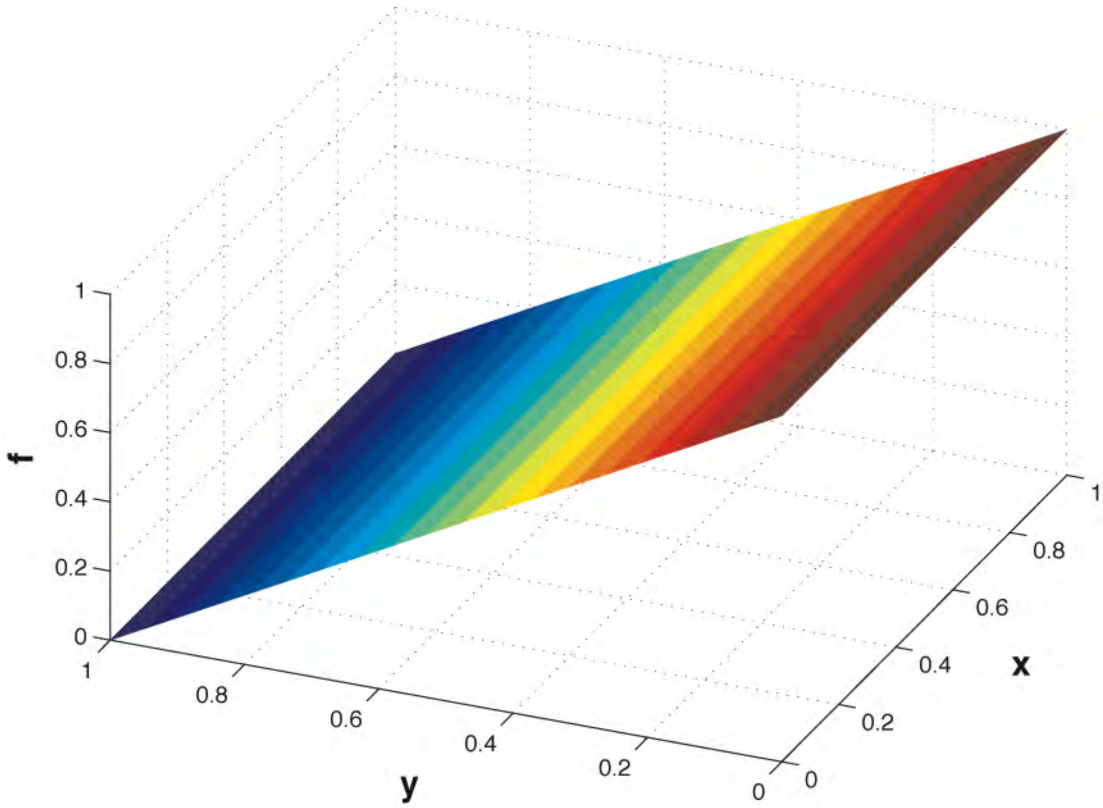
Position [*m*], Time [*s*]

“Diffusion math” → Multivariable functions



$$f(x, y) = (1 - x) = f(x)$$

“Diffusion math” → Multivariable functions



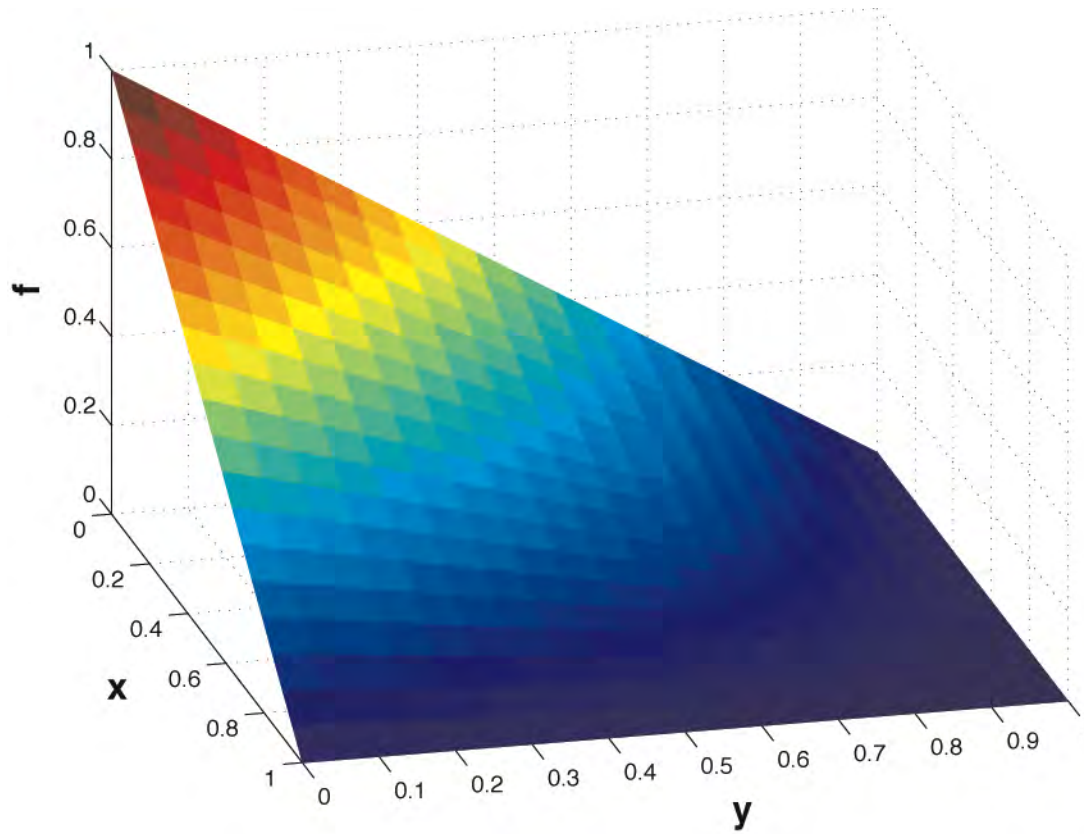
$$f(x, y) = (1 - y) = f(y)$$

“Diffusion math” → Multivariable functions

Biological Context

f - reaction rate [*mol/s*]

x, y - concentration of inhibitor agents [*mol*]



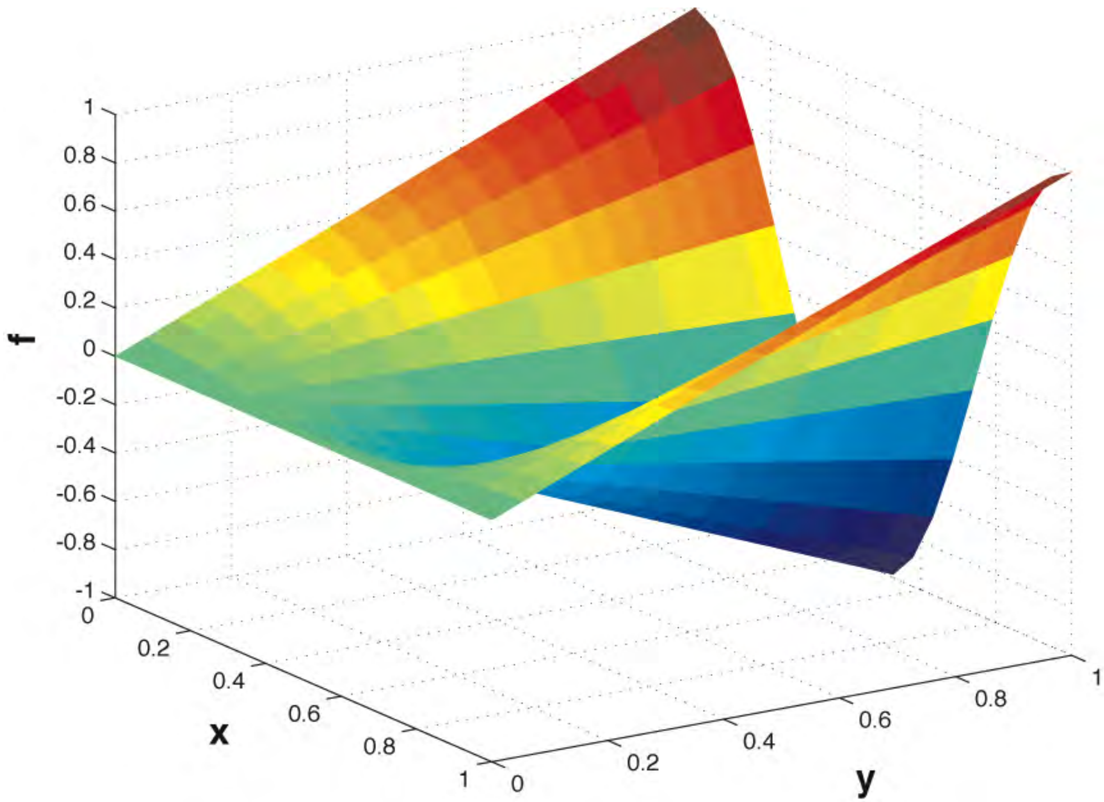
~~$f(x,y) = (1-y)(1-x)$~~

don’t forget about units!

$$f(x,y) = k(1-x)(1-y)$$

$$[k] = \frac{1}{mol \cdot s}$$

“Diffusion math” → Multivariable functions

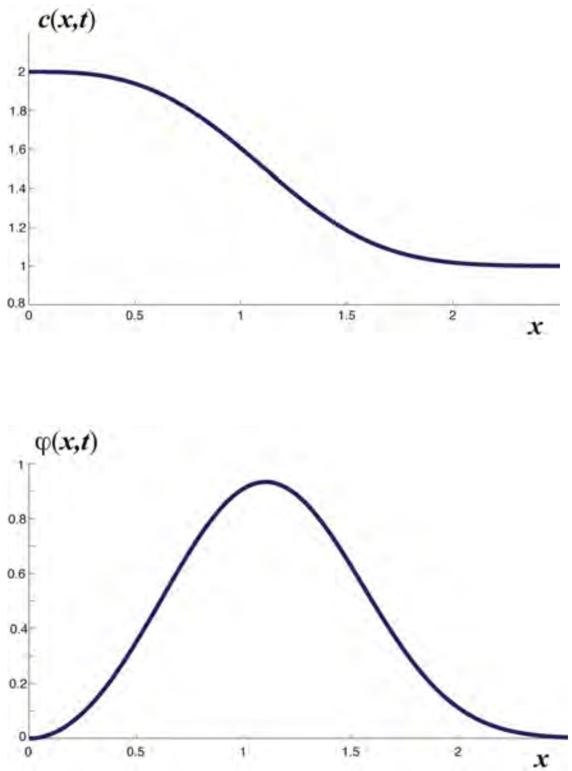


$$f(x, y) = y\cos(2\pi x)$$

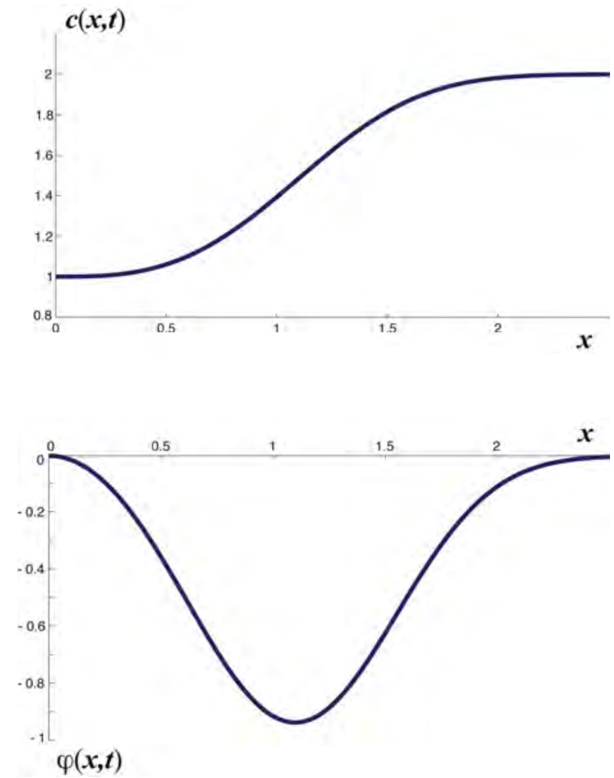
<u>Fick's 1st Law</u> (1-D)

<u>Note</u>: Here time (t) is "fixed"

Profile 1



Profile 2



$$\phi(x,t) \propto -\frac{\partial c(x,t)}{\partial x}$$

Diffusion Constant (D)

$$\phi(x,t) \propto -\frac{\partial c(x,t)}{\partial x}$$

constant of proportionality?

$$\phi(x,t) = -D\frac{\partial c(x,t)}{\partial x}$$

In short, there is a net movement down a concentration gradient

- diffusion constant is always positive (i.e., $D > 0$)
- D determines time it takes solute to diffuse a given distance in a medium
- D depends upon both solute and medium (solution)
- *Stokes-Einstein relation* predicts that D is inversely proportional to solute molecular radius

Diffusion

$$\phi(x,t) = -D\frac{\partial c(x,t)}{\partial x}$$

(Fick's Law)

- diffusion constant is always positive (i.e., $D > 0$)
- D determines time it takes solute to diffuse a given distance in a medium
- D depends upon both solute and medium (solution)
- *Stokes-Einstein relation* predicts that D is inversely proportional to solute molecular radius

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$$

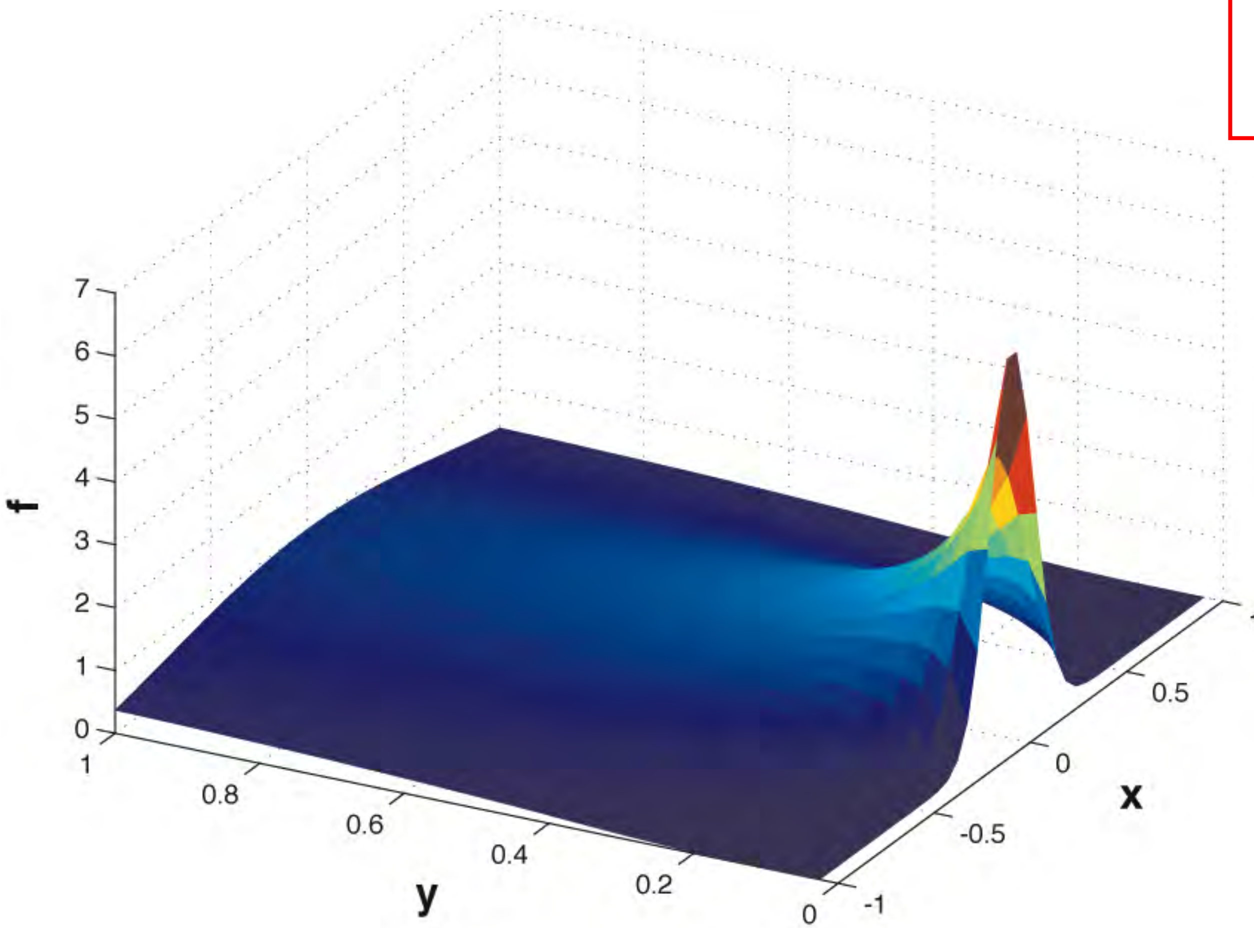
Diffusion equation

(combo of Fick's Law and continuity equation)

Note: This is a PDE(!!) → PDEs are beyond the scope of 1420

“Diffusion math” → Multivariable functions

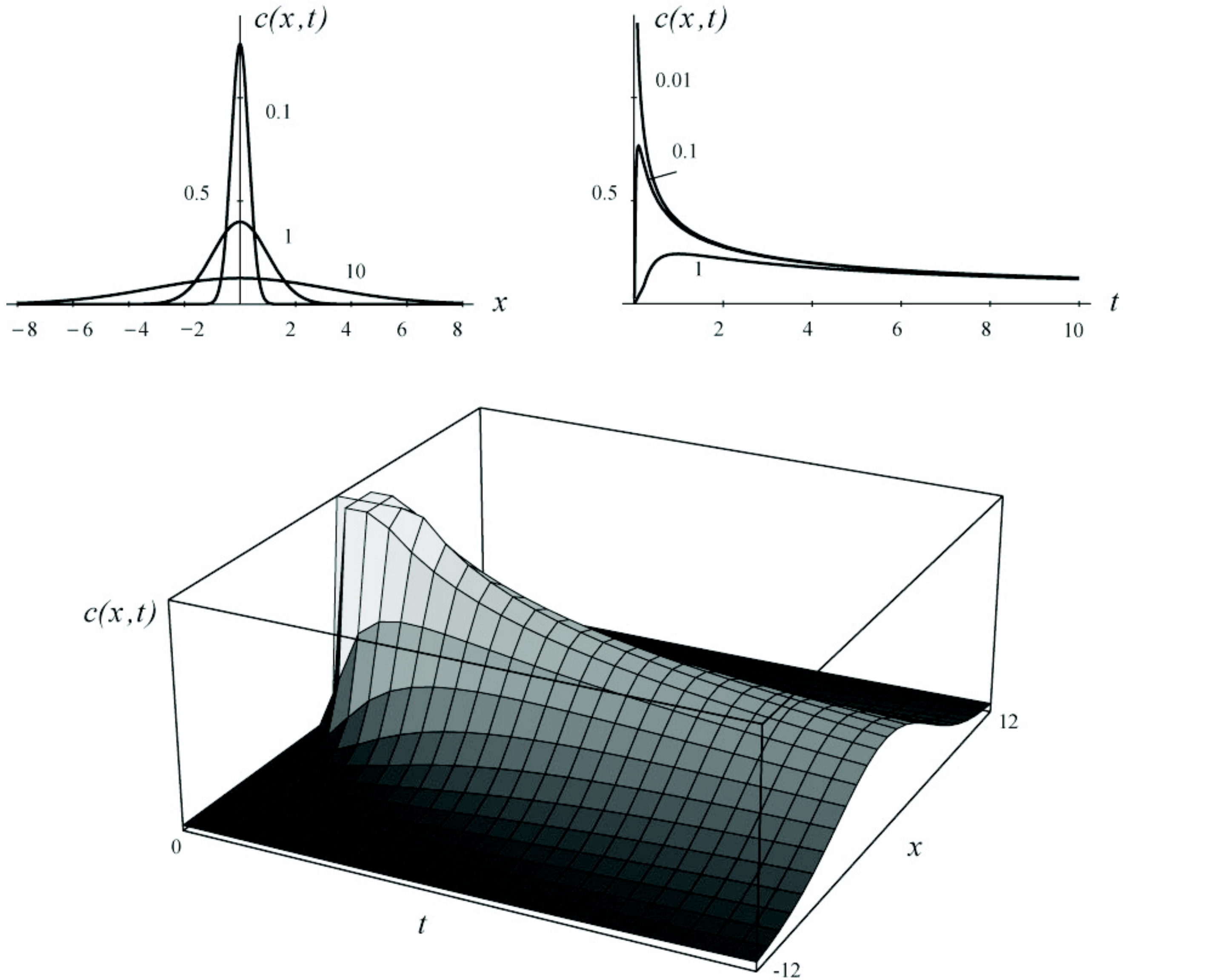
$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$$



$$f(x,y) = \frac{1}{\sqrt{y}}e^{-x^2/y}$$

Solution to diffusion equation

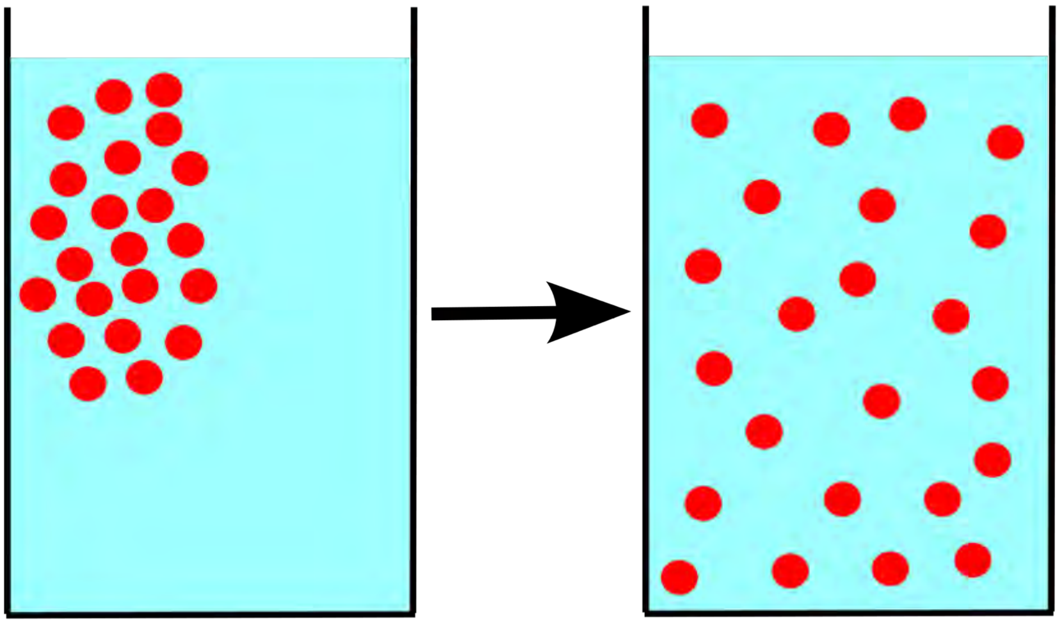
Diffusion



Weiss Fig.3.14 (modified)

Summary (re Diffusion)

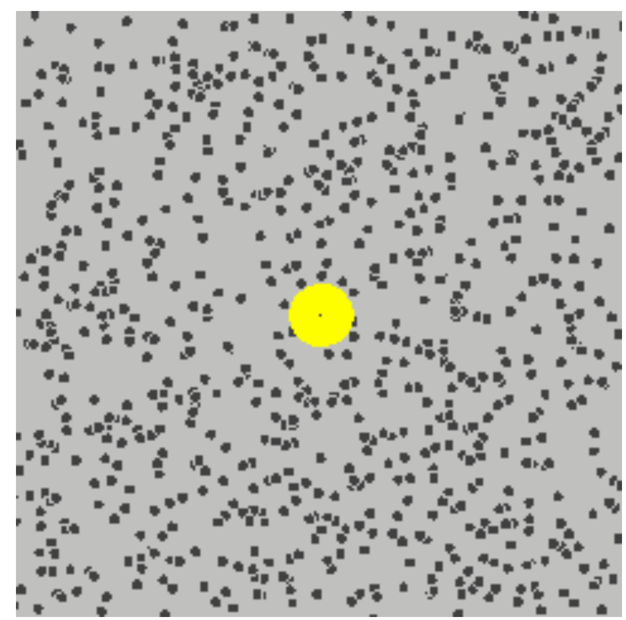
Note: Lots of "objects in direct contact" here!



Heat-transfer mechanisms

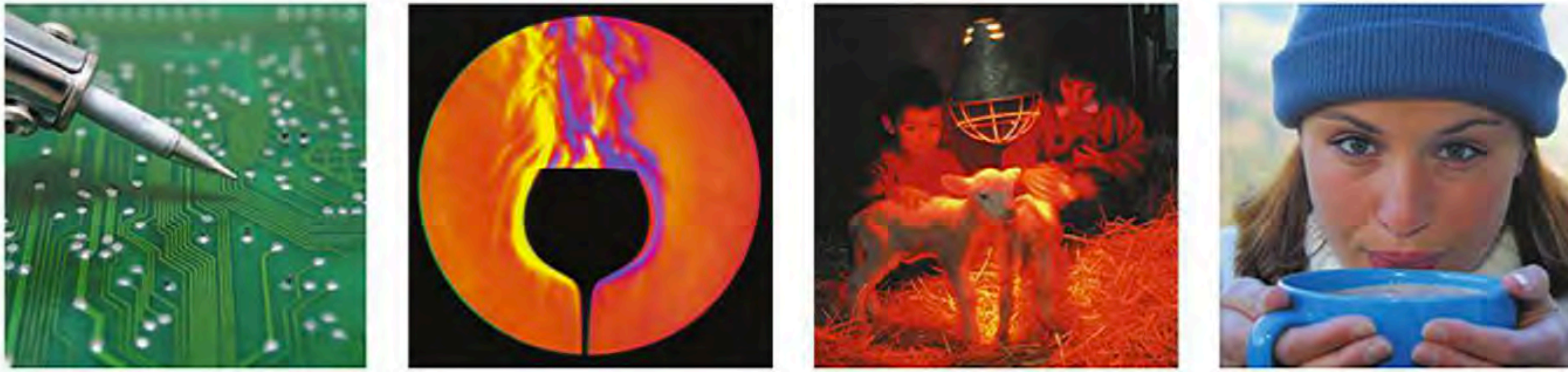
When two objects are in direct contact, such as the soldering iron and the circuit board, heat is transferred by *conduction*.

→ Diffusion is a macroscopic movement of stuff stemming from lots of random walks at the microscopic level



Summary (re Diffusion)

Heat-transfer mechanisms



When two objects are in direct contact, such as the soldering iron and the circuit board, heat is transferred by *conduction*.

Air currents near a warm glass of water rise, taking thermal energy with them in a process known as *convection*.

The lamp at the top shines on the lambs huddled below, warming them. The energy is transferred by *radiation*.

Blowing on a hot cup of tea or coffee cools it by *evaporation*.

Goal now is to build up a theme focusing on one of these in particular....

... and that is a key principle underlying *conduction*

→ We have delved into a physical means by which conduction occurs

Knight