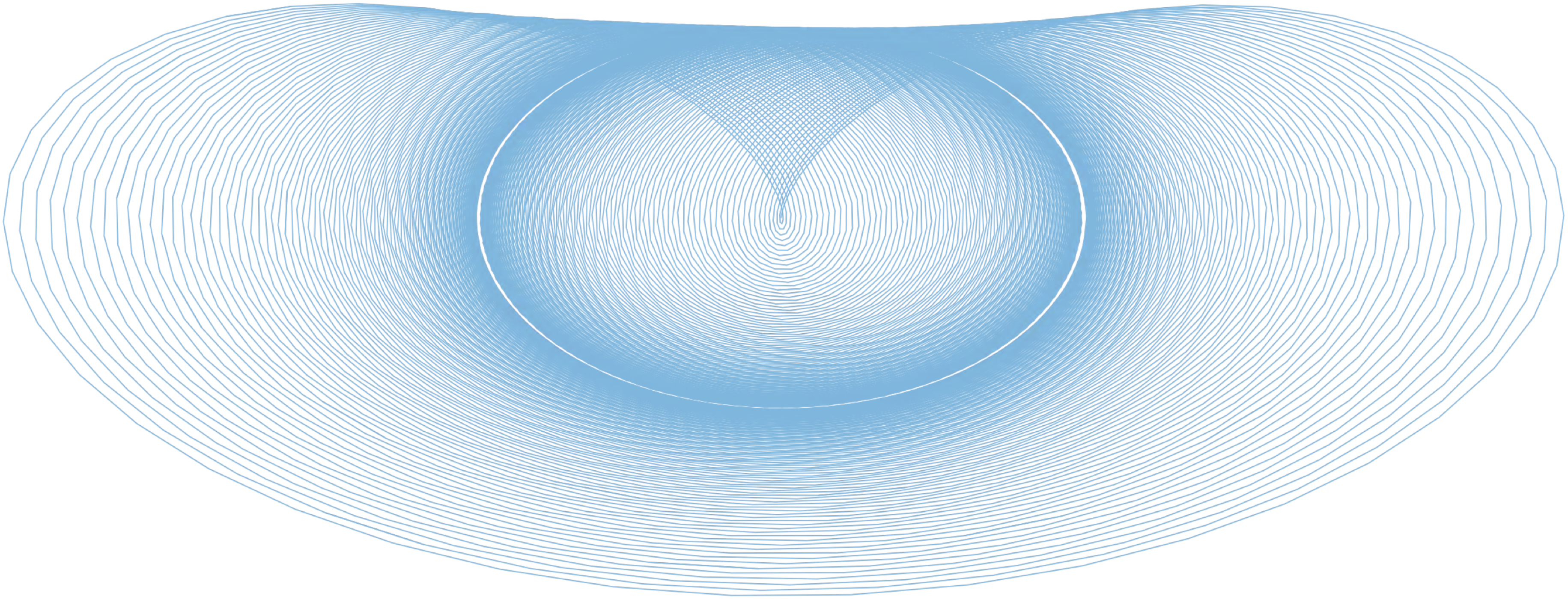
PHYS 1420 (F19)
Physics with Applications to Life Sciences



2019.11.22

Relevant reading:

Kesten & Tauck ch. 12.5-12.8

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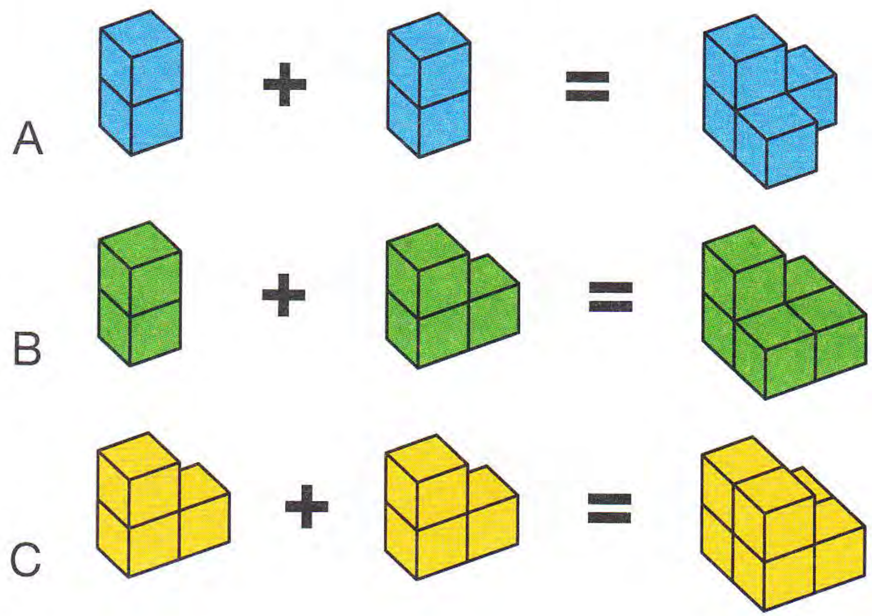
Ref. (re images):
Wolfson (2007), Knight (2017), Kesten & Tauck (2012)

78. Cubic Sums



Which equation is not correct?

A | B | C

Announcements & Key Concepts (re Today)

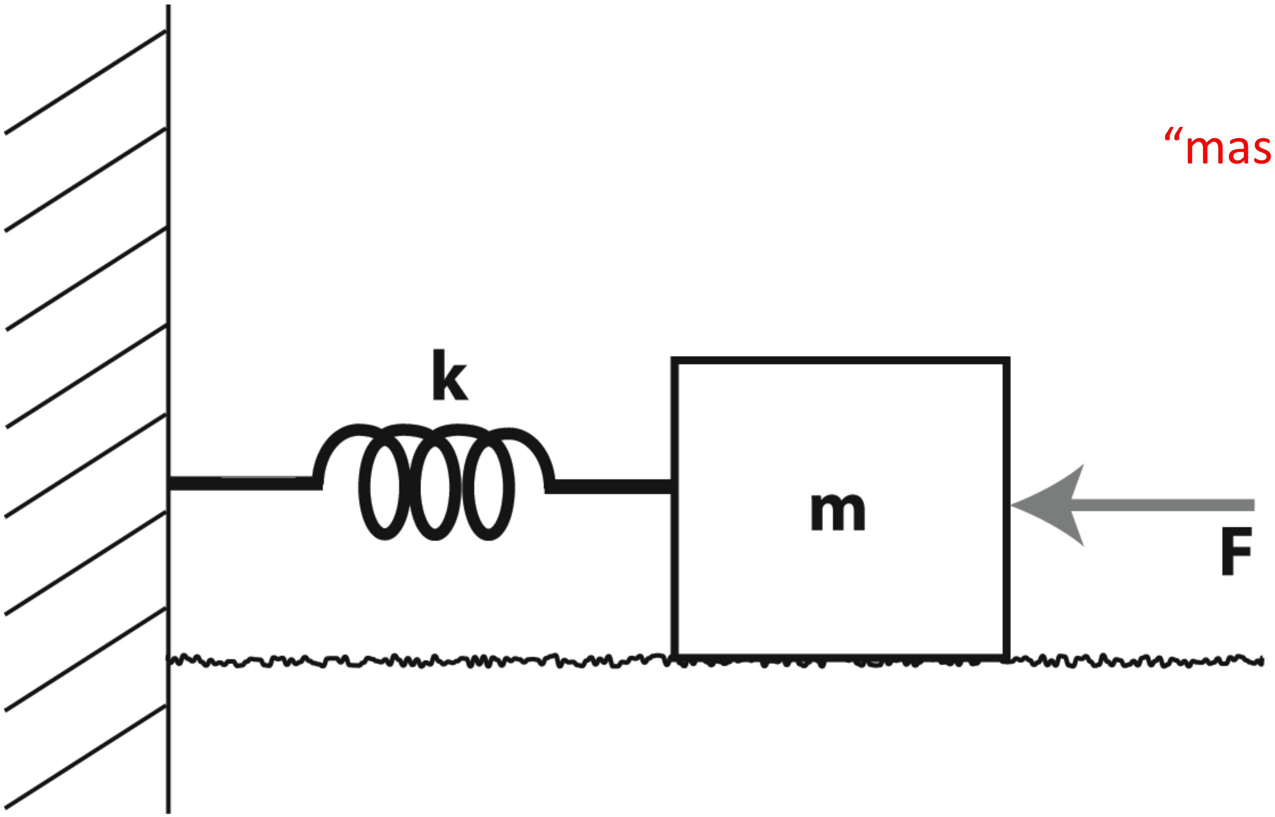
→ Online HW #8 (re fluids): Posted and due TODAY (11/22)

→ Final exam: Saturday, Dec. 14 (start preparing!)

Some relevant underlying concepts of the day…

- Harmonic oscillator
- Frog ears
- Resonance
- Damped situation
- Complex #s(!?!)

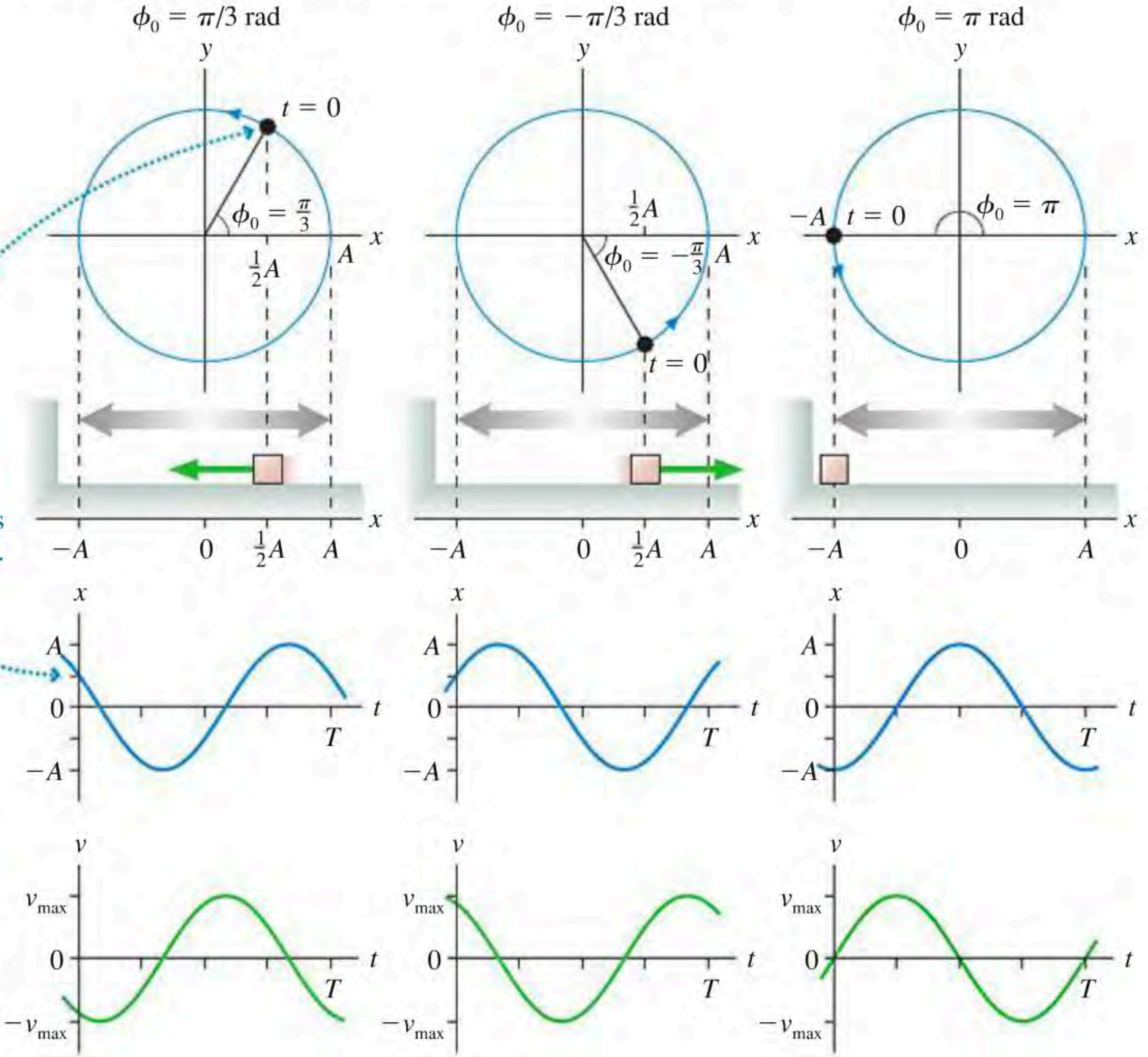
Harmonic oscillator



"mass-on-a-spring"

- One of the most fundamental/canonical problems in physics

Oscillations described by the phase constants $\phi_0 = \pi/3$ rad, $-\pi/3$ rad, and π rad.



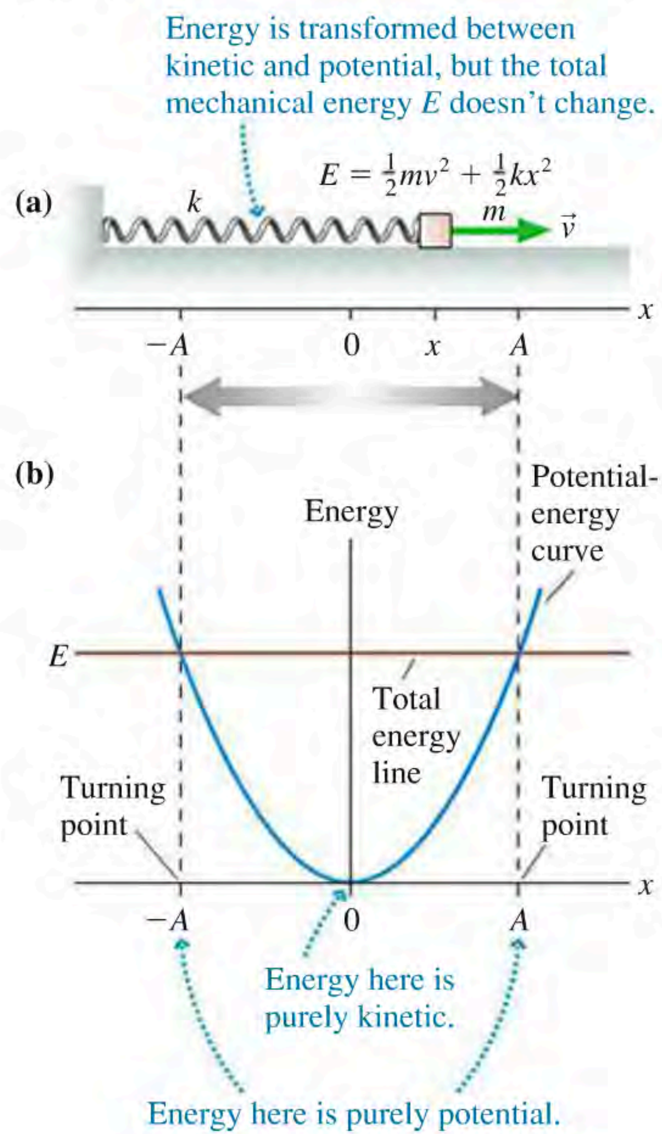
The starting point of the oscillation is shown on the circle and on the graph.

The graphs each have the same amplitude and period. They are *shifted* relative to the $\phi_0 = 0$ rad graphs of Figure 14.5 because they have different initial conditions.

Knight

Harmonic oscillator: Energy

The energy is transformed between kinetic energy and potential energy as the object oscillates, but the mechanical energy $E = K + U$ doesn't change.



Knight

Ex. "Frog's eardrum"

Example 12-1 Finding the Phase

Sound causes a frog's eardrum to move back and forth as shown in the plot of displacement versus time in Figure 12-7. In this case, displacement is given as a percentage of the amplitude of the oscillations. At the start of the measurements, the displacement is about 12% of the maximum in the negative direction. Find thephase angle ϕ if the displacement obeys $x(t) = A\cos(\omega_0 t + \phi)$ (Equation 12-4). (We will address the fact that the amplitude of the oscillations changes from one cycle to the next in Section 12-7.)

Note: This example would be better placed in sec.12.8 (as it is both a *forced* and *damped* oscillator situation)

Displacement from equilibrium of a frog's eardrum

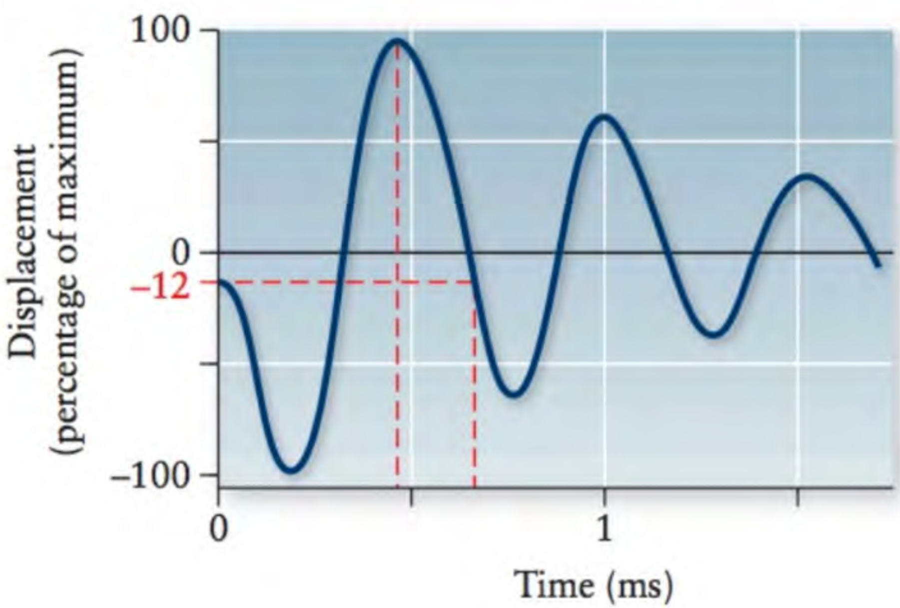


Figure 12-7 Sound causes a frog's eardrum to move back and forth, in (approximately) harmonic motion. The vertical dashed red line marks the beginning of a cosine cycle; the horizontal dashed red line marks −12% of the maximum displacement from equilibrium. The time interval between the two red arrows is just slightly more than one-quarter of a full cycle. (After Chung, Pettigrew, and Anson, (1981) *Proc. Royal Soc. London Ser. B Biol. Sci.* 212, No. 1189, pp. 459–485.)

Slow Dynamics of the Amphibian Tympanic Membrane

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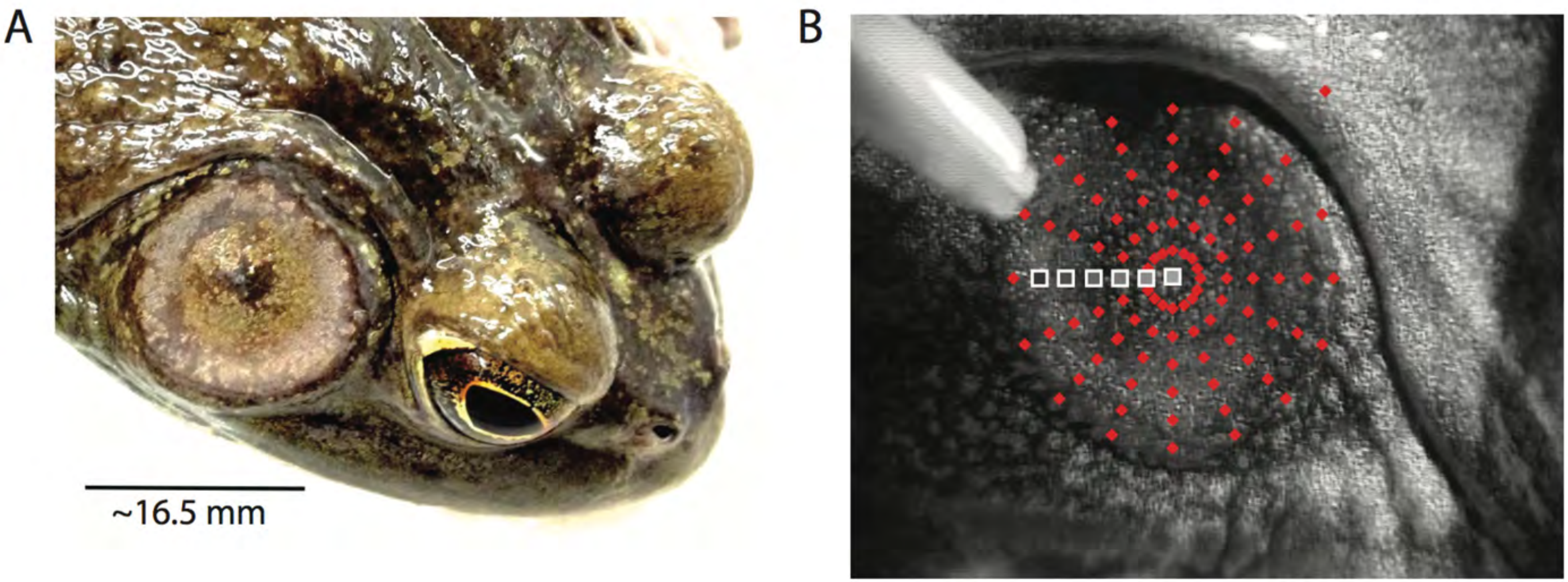
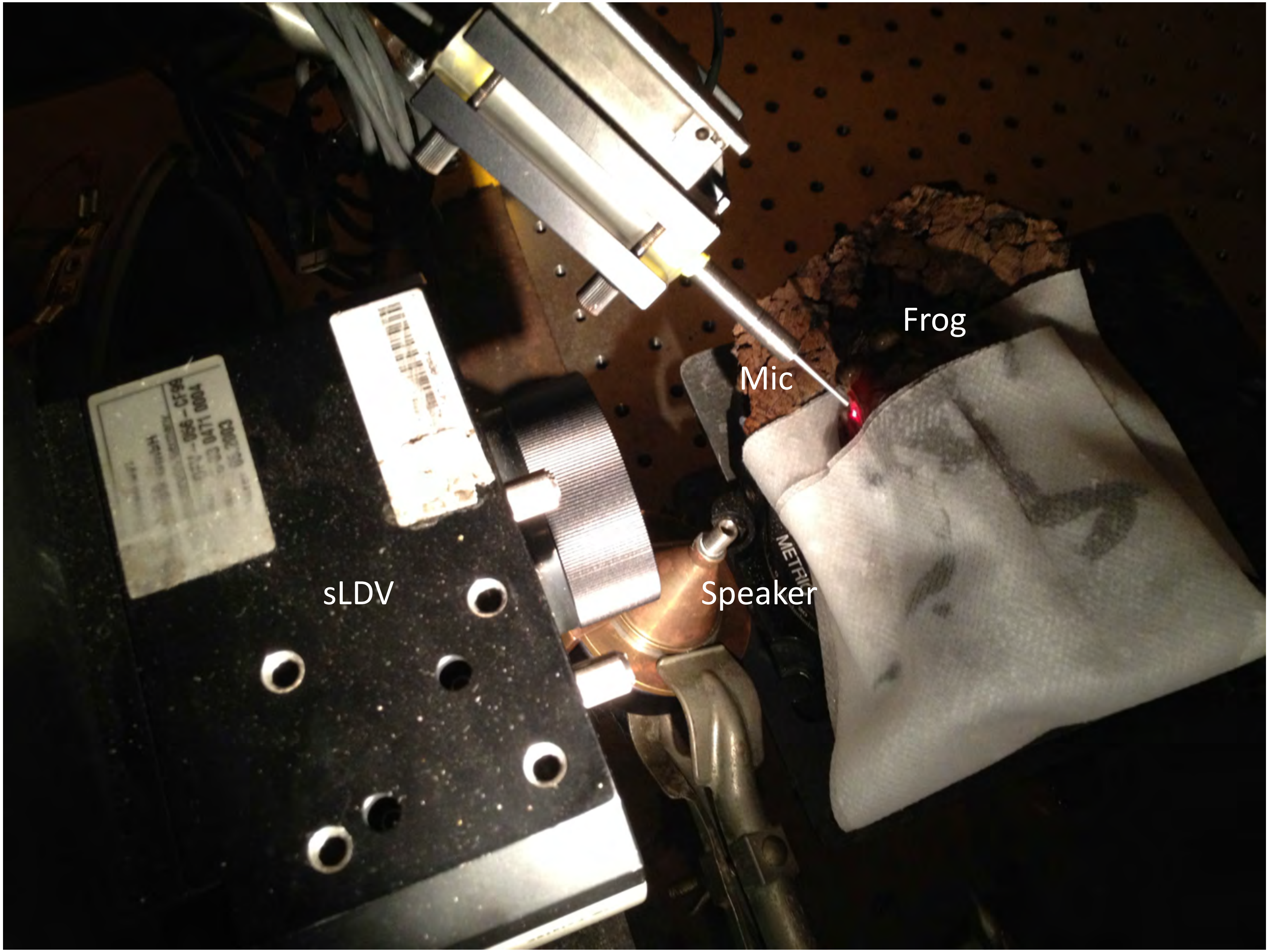
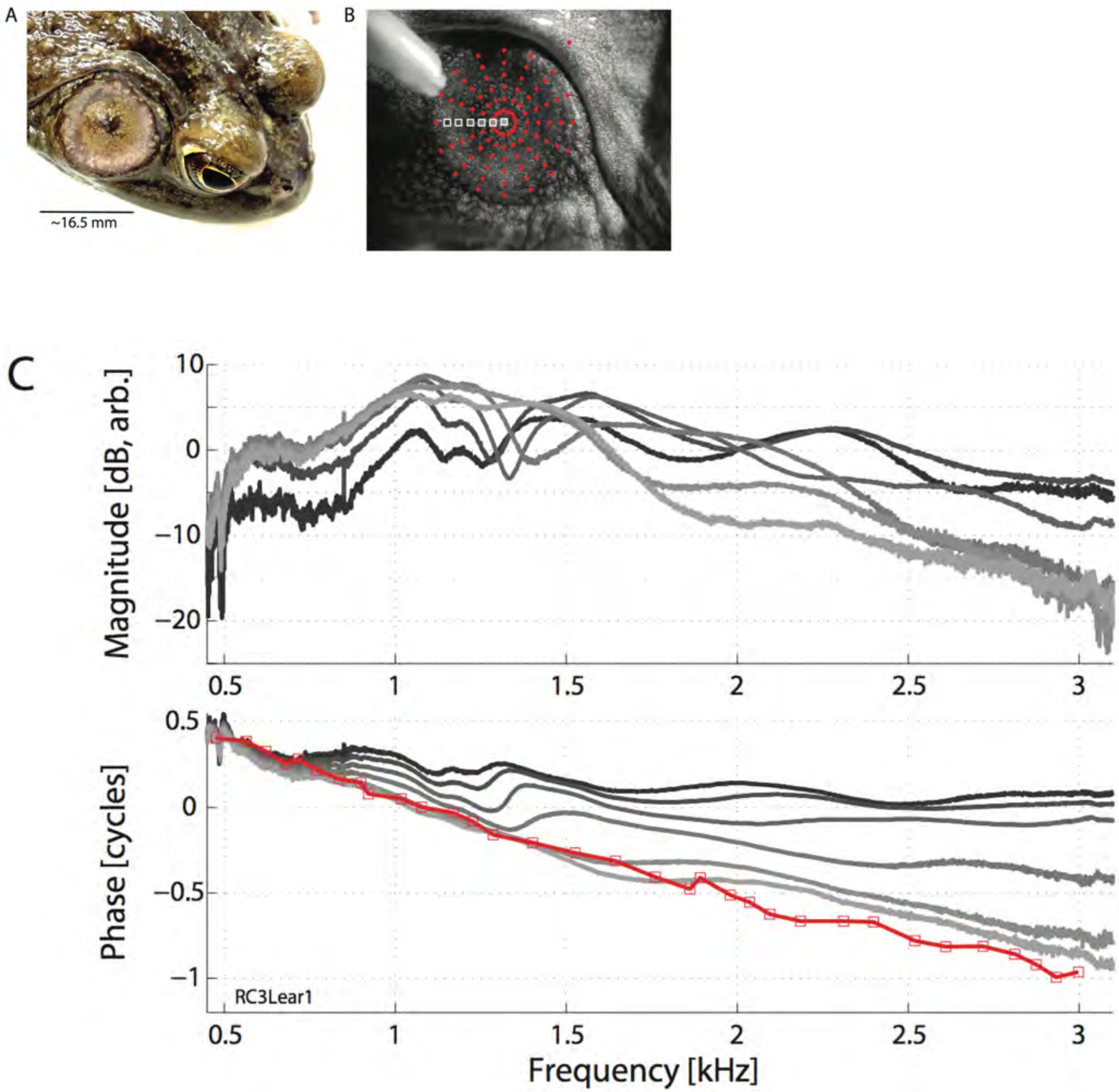
‡Department of Integrative Biology and Physiology, University of California, Los Angeles, California, USA

AIP Conference Proceedings **1703**, 060001 (2015); https://doi.org/10.1063/1.4939356



<u>Methods</u> → Scanning Laser Doppler Vibrometry (sLDV)





→ Magnitude phase, and frequency (the three key proerties of sinusoidal motion!)

However, there is more than meets the eye here...

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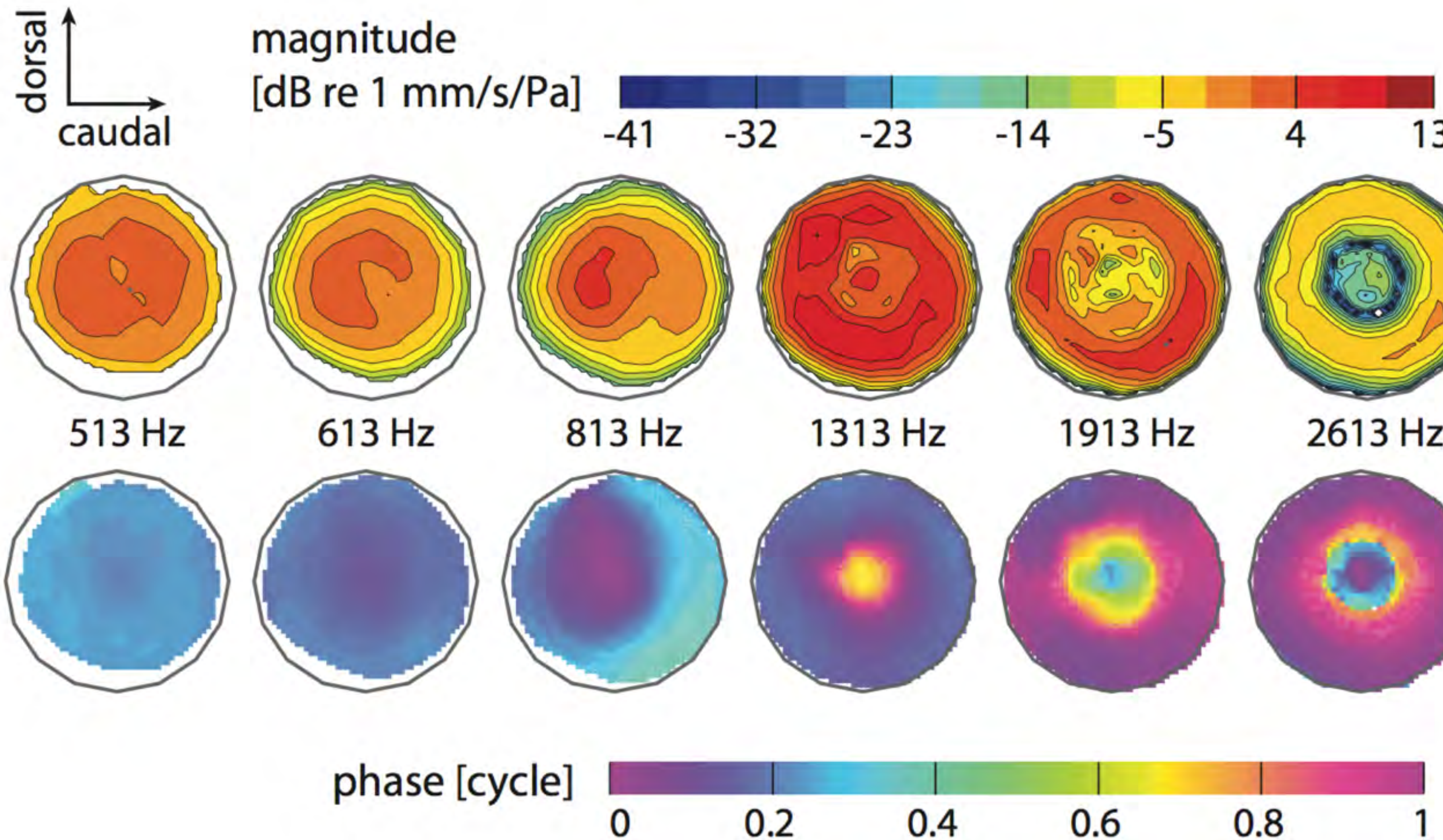

Phase accumulates across space.... → A traveling wave!

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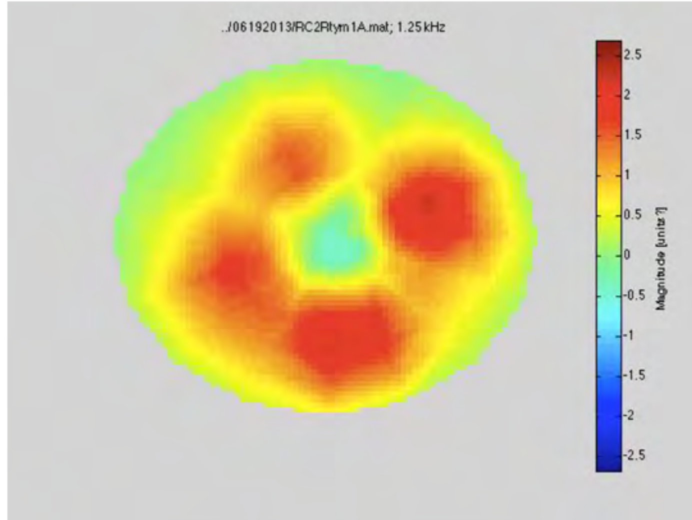
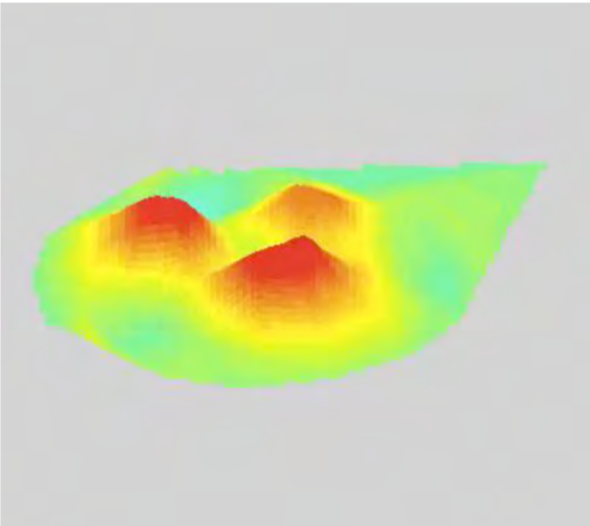
Ex. "Frog's eardrum"

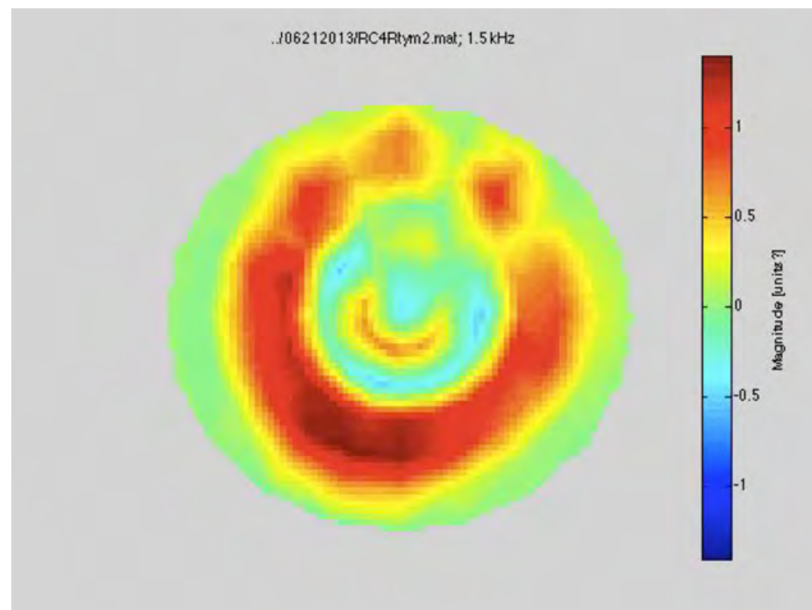
Female, 1.25 kHz



Female, 1.2 kHz



Male, 1.5 kHz



→ We will come back to waves soon....

Harmonic oscillator: Driven case (no damping)

$$\ddot{x} + \frac{k}{m}x = F_o \cos \omega t$$

Sinusoidal driving force at frequency ω

Assumption: Ignore onset behavior and that system oscillates at frequency ω

$$x(t) = B \cos(\omega t + \alpha)$$

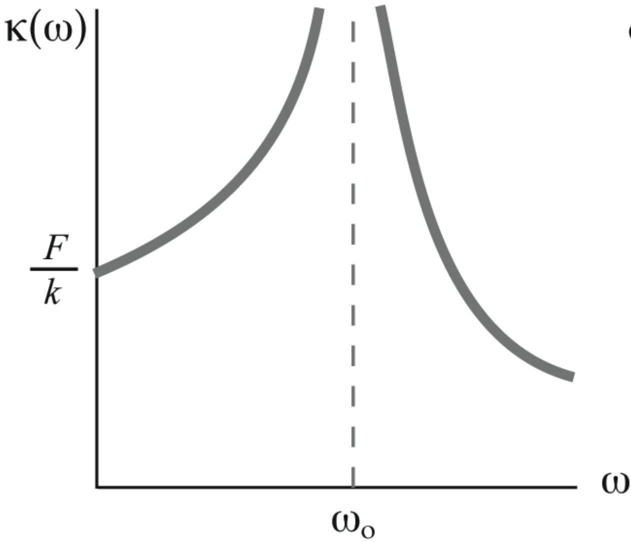
Assumed form of solution

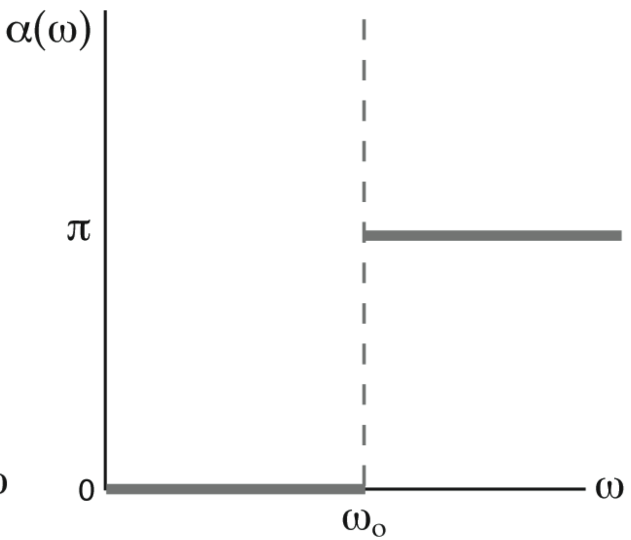
$$-m\omega^2 B \cos \omega t + kB \cos \omega t = F_o \cos \omega t$$

$$\boxed{x(t) = \frac{F_o/m}{\omega_o^2 - \omega^2} \cos(\omega t + \alpha)}$$

Harmonic oscillator: Driven case (no damping)

$$x(t) = \frac{F_o/m}{\omega_o^2 - \omega^2}\cos(\omega t + \alpha) = \kappa(\omega)\cos(\omega t + \alpha)$$





Two Important Concepts Demonstrated Here:

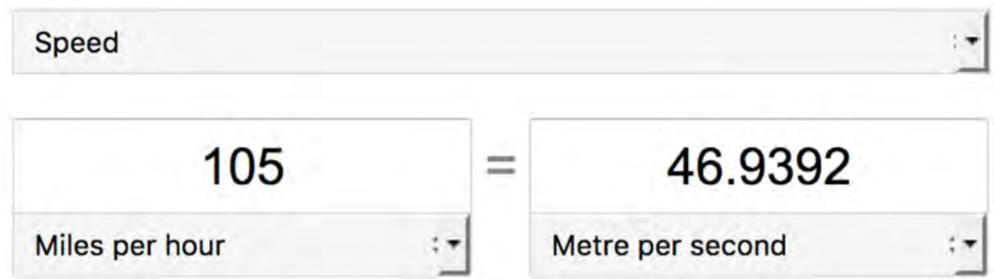
- *Resonance* when system is driven at natural frequency

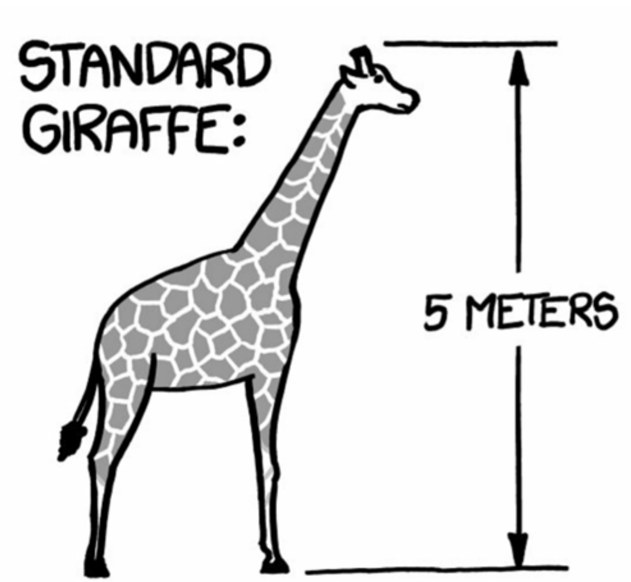
- Phase shift of 1/2 cycle about resonant frequency

<u>Recall</u>: Fact Check

$$y_{max} = \frac{V_0^2}{2g}$$

Baseball pitcher w/ 105 mph fastball = 14 giraffes(!!)





- $(46.9)^2/(2*9.8) = 112.2$ m
- 112.2 m ~ 22.4 giraffes

22.4 giraffes > 14 giraffes, so what gives?

→ Air resistance? “Aerodynamics”?

Munroe (2014)

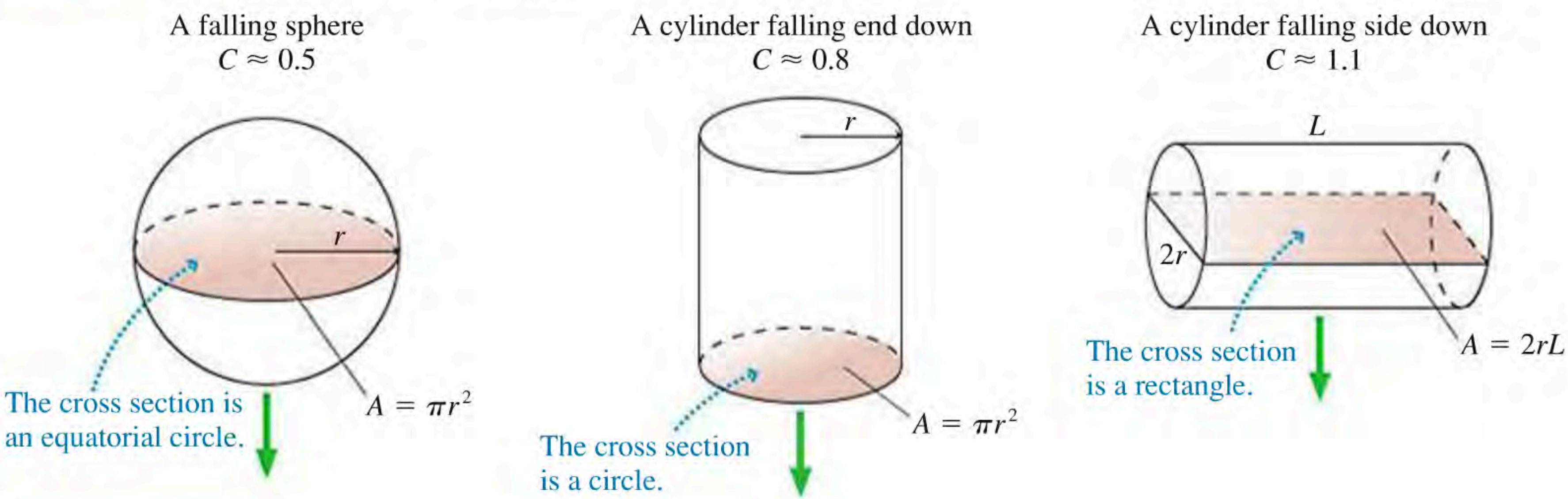
Review: Drag

$$\vec{D} = (\tfrac{1}{2} C\rho A v^2, \text{direction opposite the motion})$$

Notice that the drag force is proportional to the *square* of the object's speed. The symbols in Equation 6.16 are:

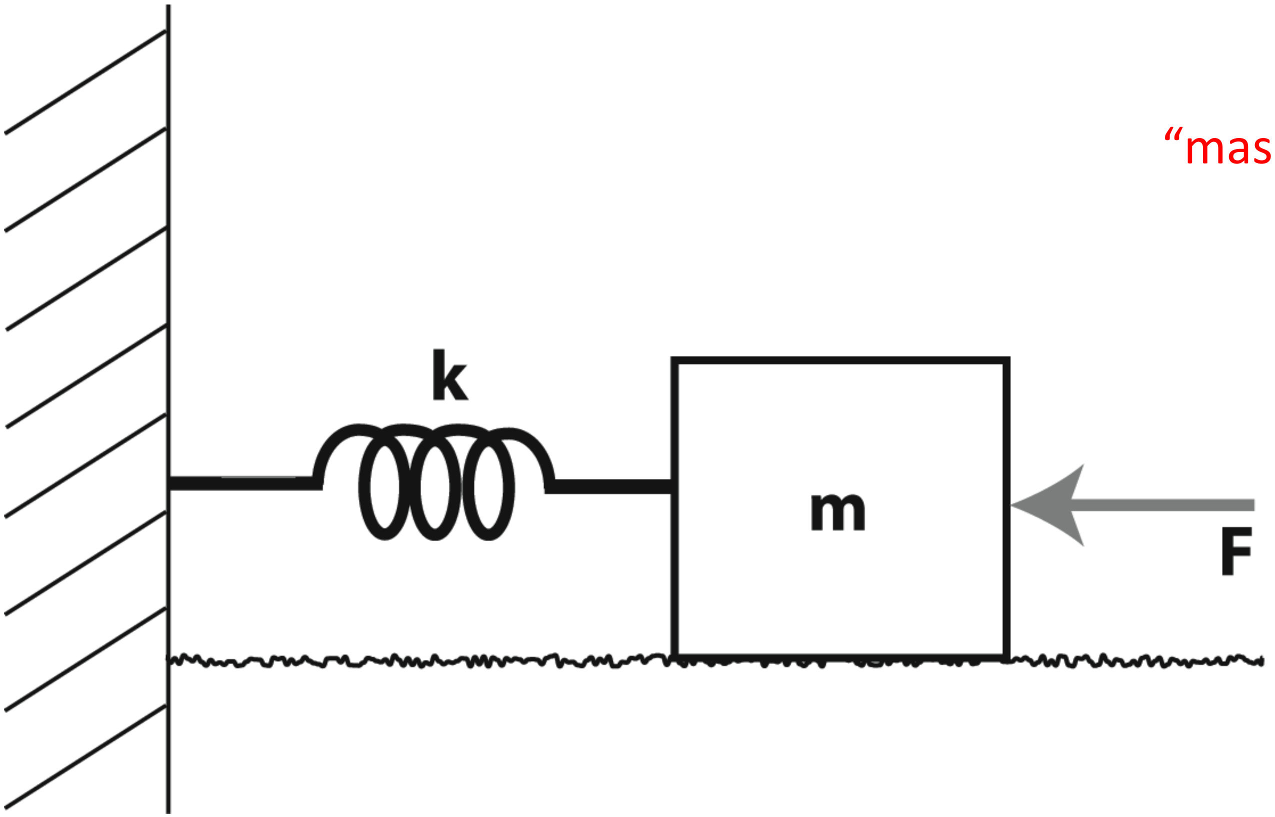
- A is the *cross-section area* of the object as it "faces into the wind," as illustrated in FIGURE 6.20.
- ρ is the density of the air, which is 1.2 kg/m^3 at atmospheric pressure and room temperature.
- C is the **drag coefficient.** It is smaller for aerodynamically shaped objects, larger for objects presenting a flat face to the wind. Figure 6.20 gives approximate values for a sphere and two cylinders.

FIGURE 6.20 Cross-section areas for objects of different shape.



Knight (2013)

Harmonic oscillator



"mass-on-a-spring"

- Let us now factor damping in as well (as any "real" system must have!)
- Will assume damping is proportional to velocity

Harmonic oscillator: Undriven case (w/ damping)

- Will assume damping is proportional to velocity

$$m\ddot{x} + b\dot{x} + kx = 0$$

Purely sinusoidal solution no longer works!

$$\ddot{x} + \gamma\dot{x} + \omega_o^2 x = 0$$

Change variables

Assumption: Form of solution is a complex exponential

Recall (re exponentials as solutions)

Solution

Exponential growth/decay

$$\frac{dP}{dt} = kP$$

$$P = P_0 e^{kt}$$

e.g., Nuclear decay, 1st order chemical reaction, bacterial growth

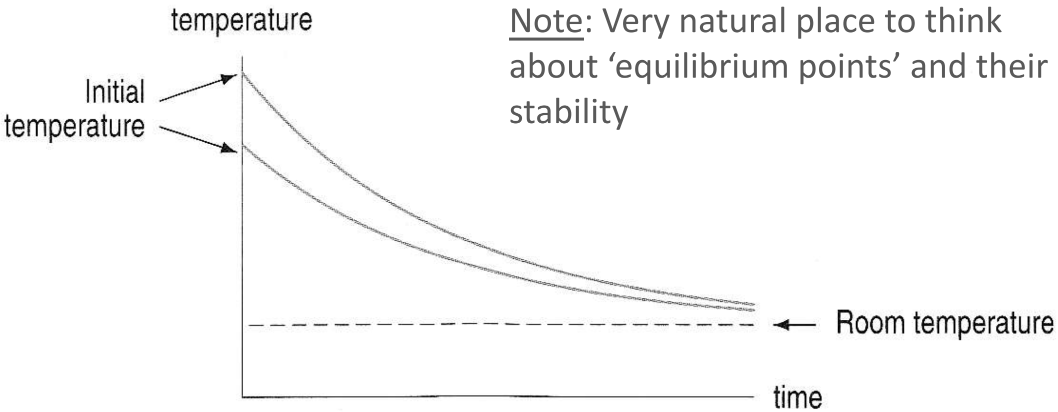
Newton's law of heating/cooling

"Newton proposed that the temperature of a hot object decreases at a rate proportional to the difference between its temperature and that of its surroundings. Similarly, a cold object heats up at a rate proportional to the temperature difference between the object and its surroundings."

$$\frac{dT}{dt} = \alpha(T_o - T)$$

Solution

$$T(t) = T_0 + Ce^{-\alpha t}$$



Note: Very natural place to think about 'equilibrium points' and their stability

Hughes-Hallett et al. (2005)

Recall (re exponentials as solutions)

Problem 1. A first-order, linear differential equation with constant coefficients and a constant inhomogeneous (drive or input) term has an exponential solution. Therefore, the solution can be written in the form

$$n(t) = n_\infty + \left(n_0 - n_\infty\right) e^{-t/\tau},$$

where $n_0 = n(0)$ is the initial value of $n(t)$ and $n_\infty = \lim_{t\to\infty} n(t)$ is the final value of $n(t)$. The form of this solution can be verified by evaluating $n(t)$ at $t = 0$ and $t \to \infty$. Substitution into the differential equation shows that this solution satisfies the differential equation. The solutions for cases i-vi are shown in Figure 1. The solutions for part a (i and ii) have the same initial and final values but different time constants (by $t = 10$ s, curve ii is just above 6 and has not yet reached its final value of 10). The solutions for part b (iii and iv) have the same initial values and different final values. Although curve iv was calculated with the same time constant as in iii, it doesn't make sense to compare the time constants of the curves, since curve iv isn't changing. The solutions for part c (v and vi) have different initial and final values and the same time constants.

Harmonic oscillator: Undriven case (w/ damping)

- Will assume damping is proportional to velocity

$$m\ddot{x} + b\dot{x} + kx = 0$$

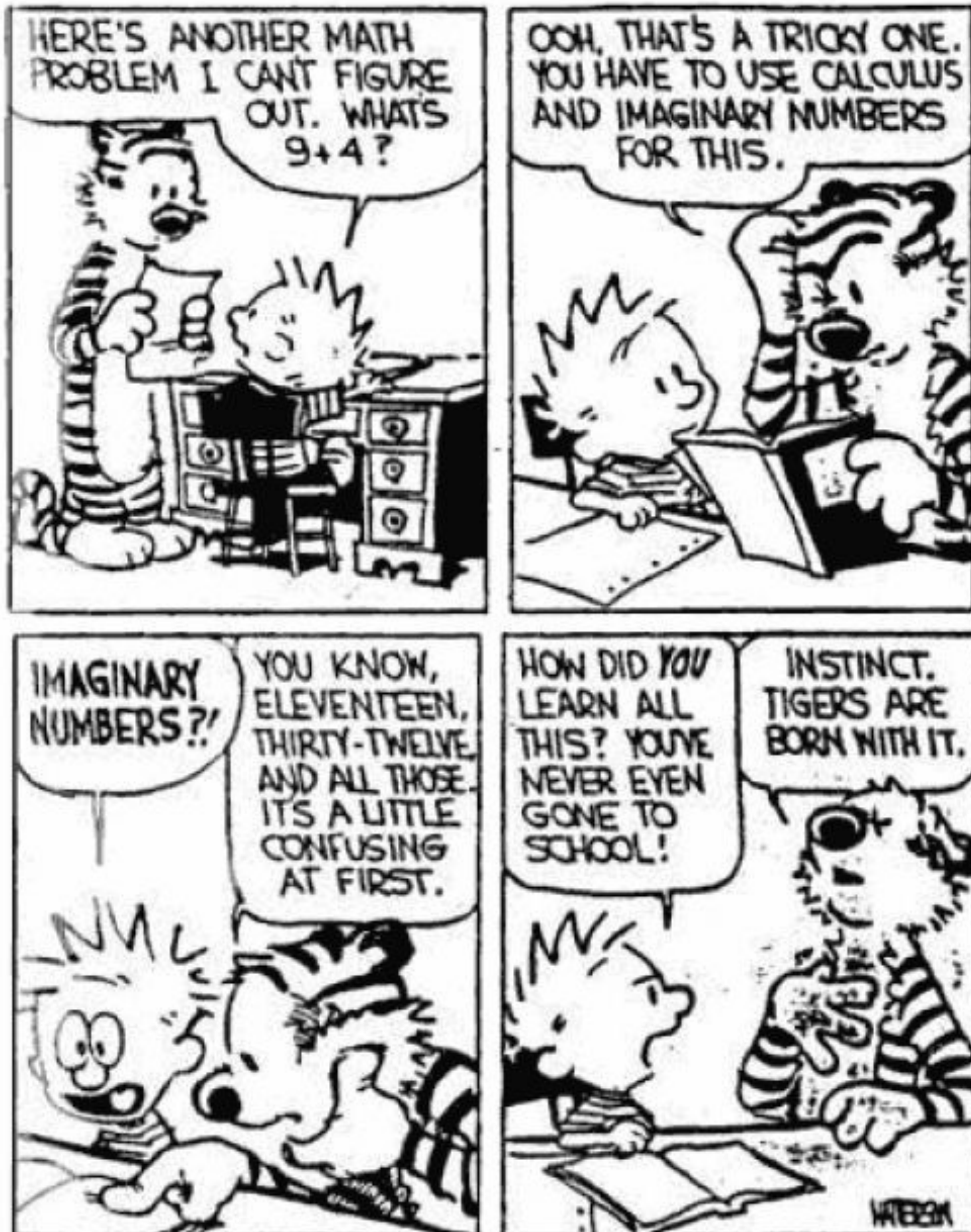
Purely sinusoidal solution no longer works!

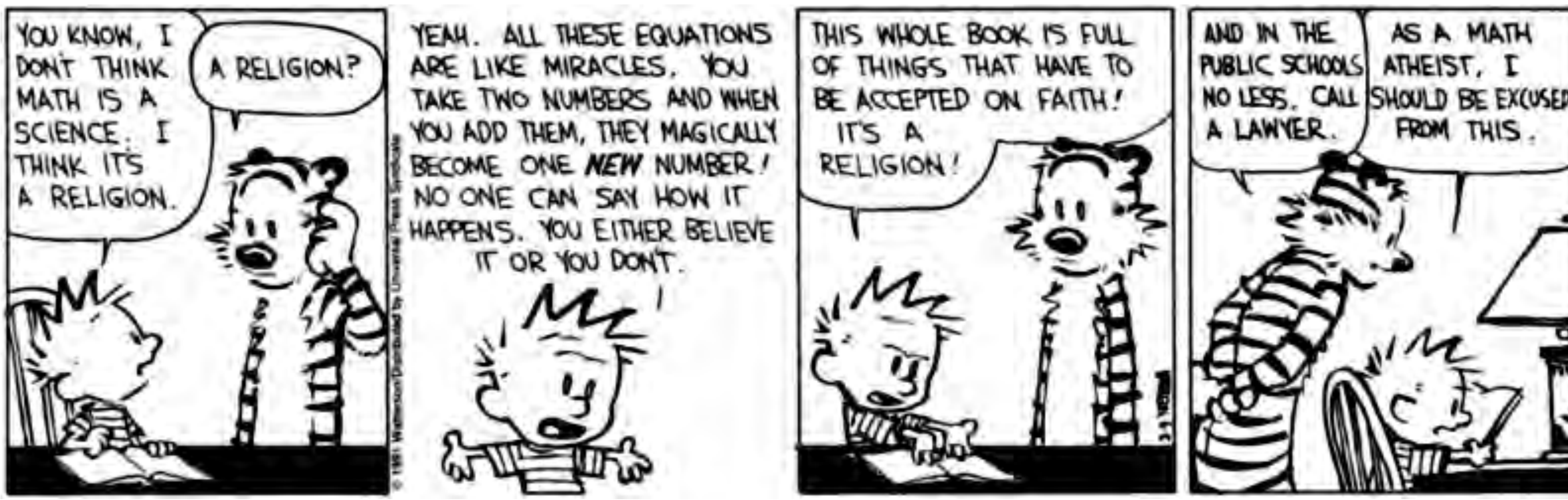
$$\ddot{x} + \gamma\dot{x} + \omega_o^2 x = 0$$

Change variables

Assumption: Form of solution is a **complex** exponential

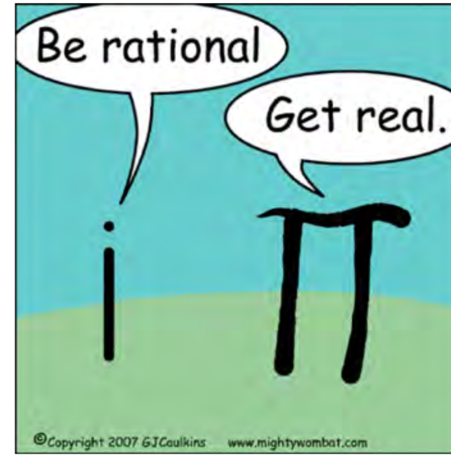
$$x(t) = Ae^{i(\omega t + \delta)}$$


HERE'S ANOTHER MATH PROBLEM I CAN'T FIGURE OUT. WHAT'S 9+4?
OOH, THAT'S A TRICKY ONE. YOU HAVE TO USE CALCULUS AND IMAGINARY NUMBERS FOR THIS.
IMAGINARY NUMBERS?!
YOU KNOW, ELEVENTEEN, THIRTY-TWELVE, AND ALL THOSE. IT'S A LITTLE CONFUSING AT FIRST.
HOW DID YOU LEARN ALL THIS? YOU'VE NEVER EVEN GONE TO SCHOOL!
INSTINCT. TIGERS ARE BORN WITH IT.
WATERSON


YOU KNOW, I DON'T THINK MATH IS A SCIENCE. I THINK IT'S A RELIGION.
A RELIGION?
YEAH. ALL THESE EQUATIONS ARE LIKE MIRACLES. YOU TAKE TWO NUMBERS AND WHEN YOU ADD THEM, THEY MAGICALLY BECOME ONE *NEW* NUMBER! NO ONE CAN SAY HOW IT HAPPENS. YOU EITHER BELIEVE IT OR YOU DON'T.
THIS WHOLE BOOK IS FULL OF THINGS THAT HAVE TO BE ACCEPTED ON FAITH! IT'S A RELIGION!
AND IN THE PUBLIC SCHOOLS NO LESS. CALL A LAWYER.
AS A MATH ATHEIST, I SHOULD BE EXCUSED FROM THIS.

<u>Trigonometry Review</u>: Complex #s

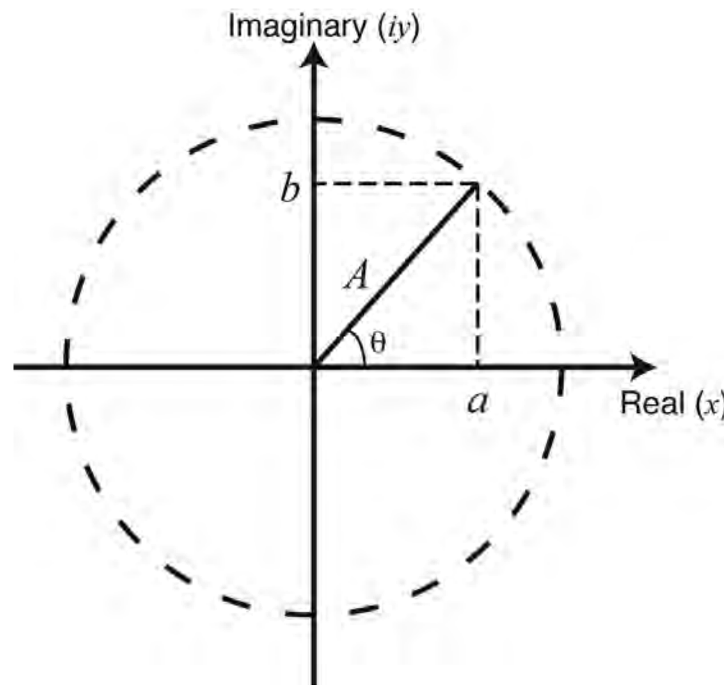
$$i^2 = -1, i = \sqrt{-1}$$



Euler's Formula

$$a + ib = Ae^{i\theta}$$

$$= A(\cos\theta + i\sin\theta)$$



Cartesian Form

$$a = A\cos(\theta)$$

$$b = A\sin(\theta)$$

$\Leftrightarrow$

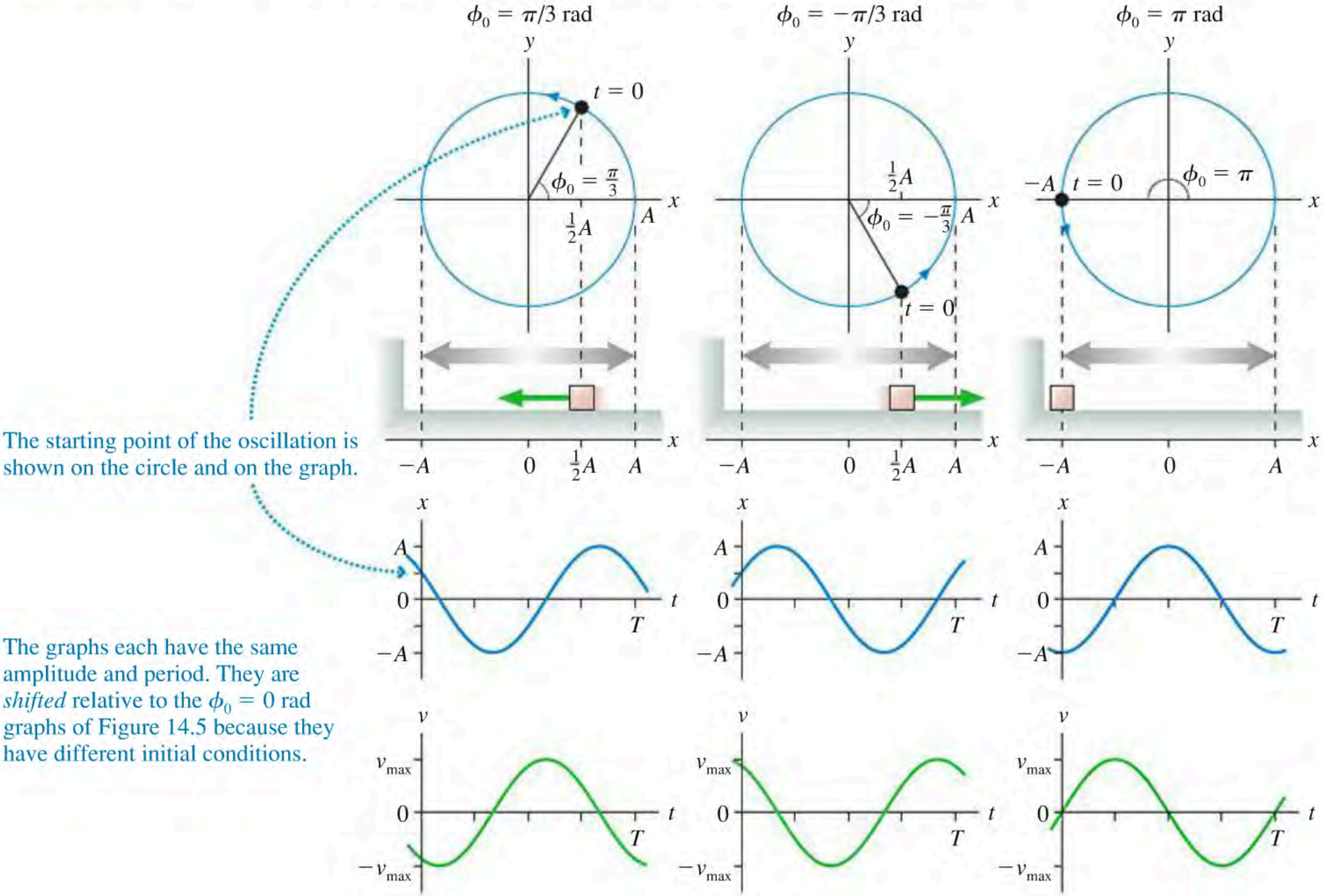
Polar Form

$$A = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$\Rightarrow$ Complex solution contain both magnitude and phase information

Oscillations described by the phase constants $\phi_0 = \pi/3$ rad, $-\pi/3$ rad, and π rad.



Knight

Harmonic oscillator: Undriven case (w/ damping)

$$\ddot{x} + \gamma\dot{x} + \omega_o^2 x = 0$$

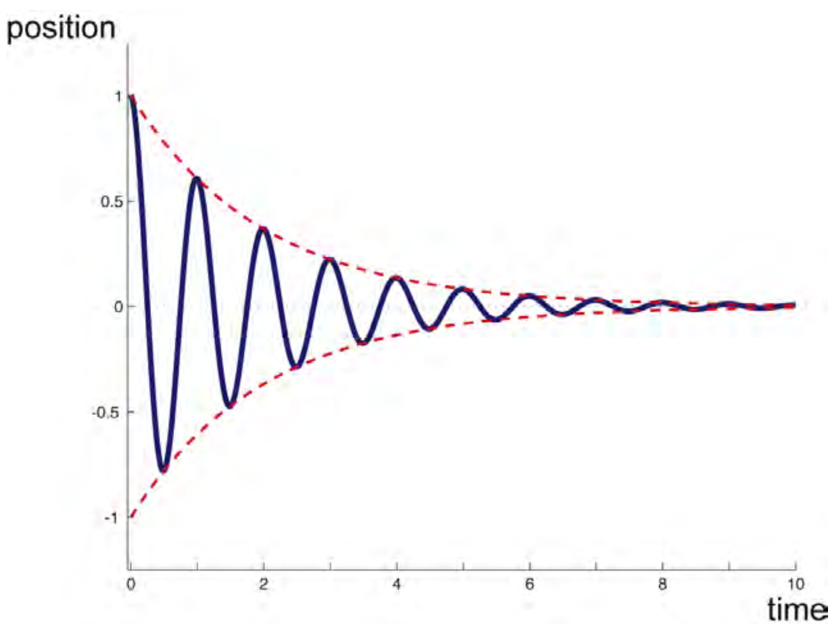
$$x(t) = Ae^{i(\omega t+\delta)}$$

$$x(t) = Ae^{-\gamma t/2}\ e^{i(\omega t+\alpha)}$$

$$\omega^2 = \omega_o^2 - \frac{\gamma^2}{4}$$

(slightly lower frequency of oscillation due to damping)

[A and α are constants of integration, depending upon initial conditions]

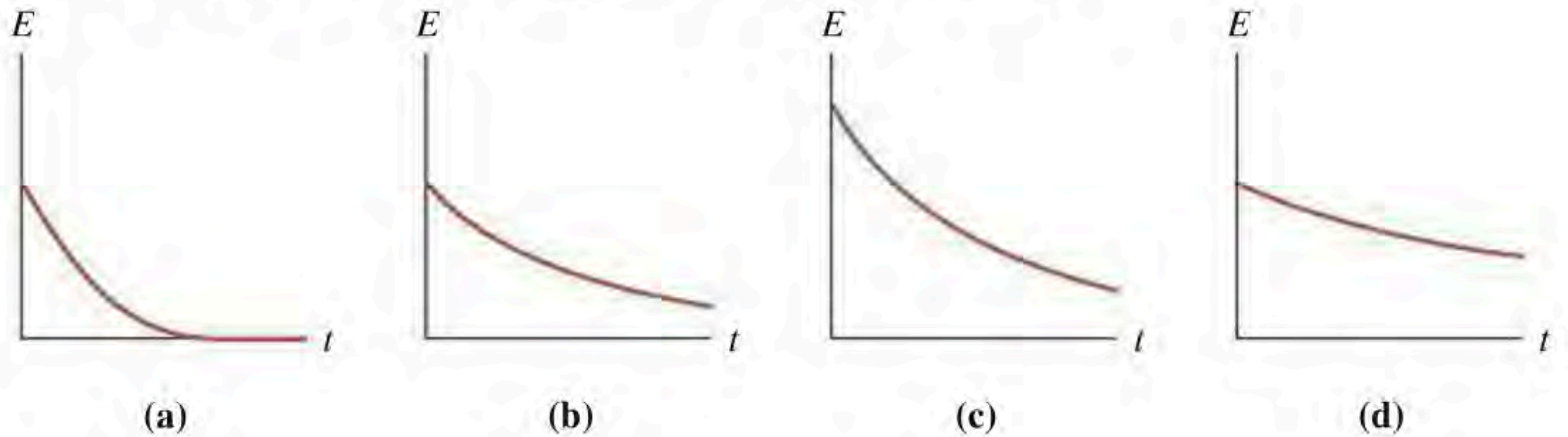


$\Rightarrow$ Damping causes energy loss from system

Note: Sometimes the "time constant" is denoted τ ($=1/\gamma$)

Ex.

STOP TO THINK 14.6 Rank in order, from largest to smallest, the time constants τ_a to τ_d of the decays shown in the figure. All the graphs have the same scale.



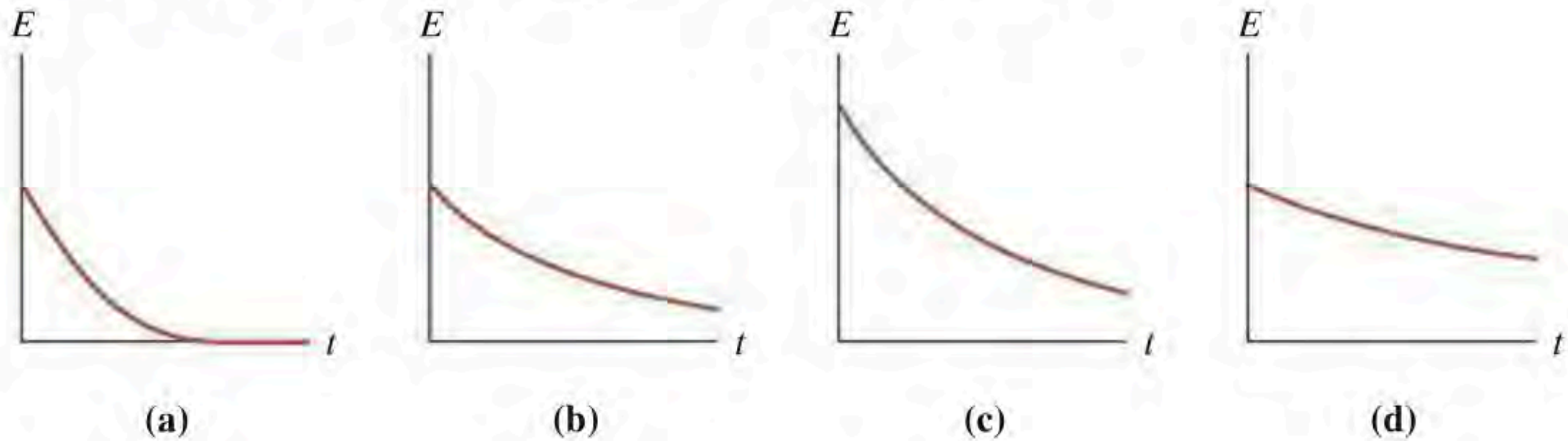
$$x(t) = Ae^{-\gamma t/2} \; e^{i(\omega t+\alpha)}$$

Caution! Here $\tau=1/\gamma$

Knight

Ex. (SOL)

STOP TO THINK 14.6 Rank in order, from largest to smallest, the time constants τ_a to τ_d of the decays shown in the figure. All the graphs have the same scale.



$$x(t) = Ae^{-\gamma t/2}\ e^{i(\omega t+\alpha)}$$

Caution! Here $\tau=1/\gamma$

$\tau_d > \tau_b = \tau_c > \tau_a$. The time constant is the time to decay to 37% of the initial height. The time constant is independent of the initial height.

Note: For further study, where in the world did the # "37%" come from?!? [Hint: e^{-1}]

Knight

Harmonic oscillator: Driven case (w/ damping)

$$\ddot{x} + \gamma\dot{x} + \omega_o^2 x = \frac{F_o}{m} e^{i\omega t}$$

Sinusoidal driving force at frequency ω

Assumption: Ignore onset behavior and that system oscillates at frequency ω

$$x(t) = Ae^{-i(\omega t+\delta)}$$

Assumed form of solution

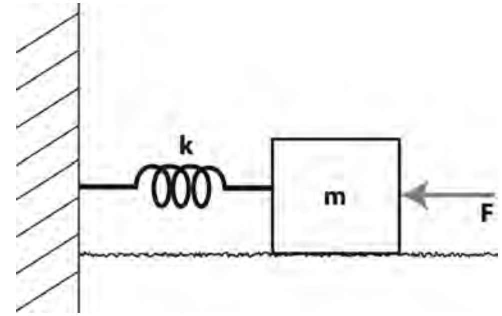
$$A(\omega) = \frac{F_o/m}{[(\omega_o^2 - \omega^2)^2 + (\gamma\omega)^2]^{1/2}}$$

(magnitude)

$$\delta(\omega) = \arctan\left(\frac{\gamma\omega}{\omega^2 - \omega_o^2}\right)$$

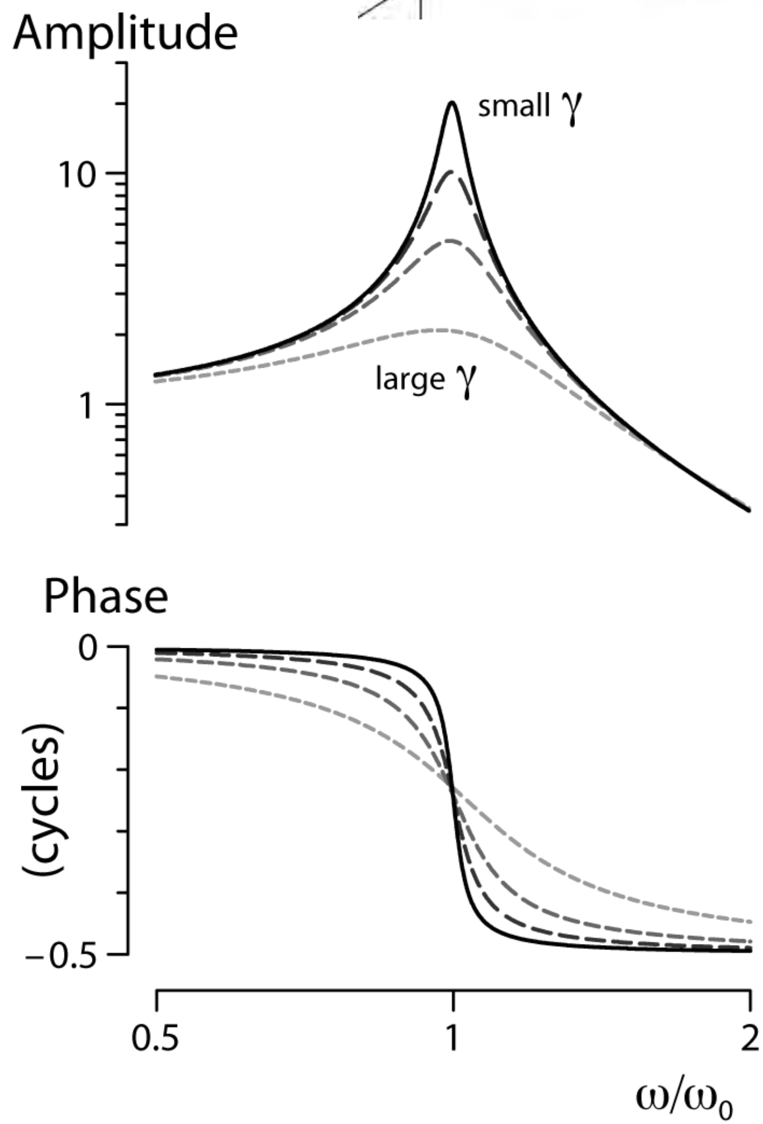
(phase)

Harmonic oscillator: Driven case (w/ damping)



$$A(\omega) = \frac{F_o/m}{[(\omega_o^2 - \omega^2)^2 + (\gamma\omega)^2]^{1/2}}$$

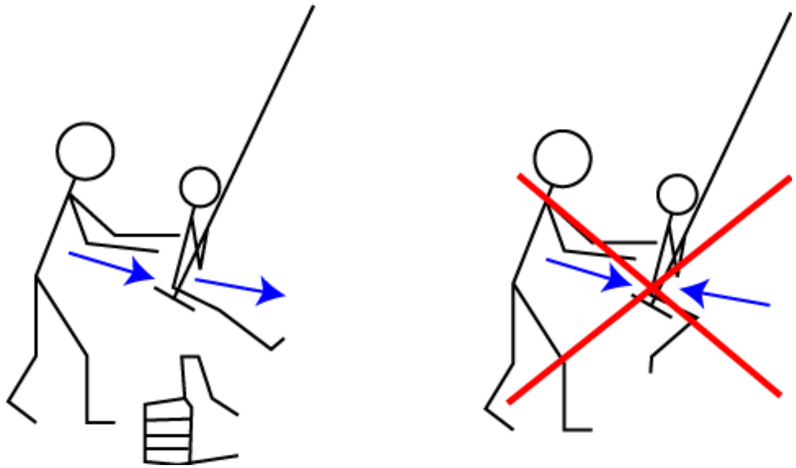
$$\delta(\omega) = \arctan\left(\frac{\gamma\omega}{\omega^2 - \omega_o^2}\right)$$



Resonance

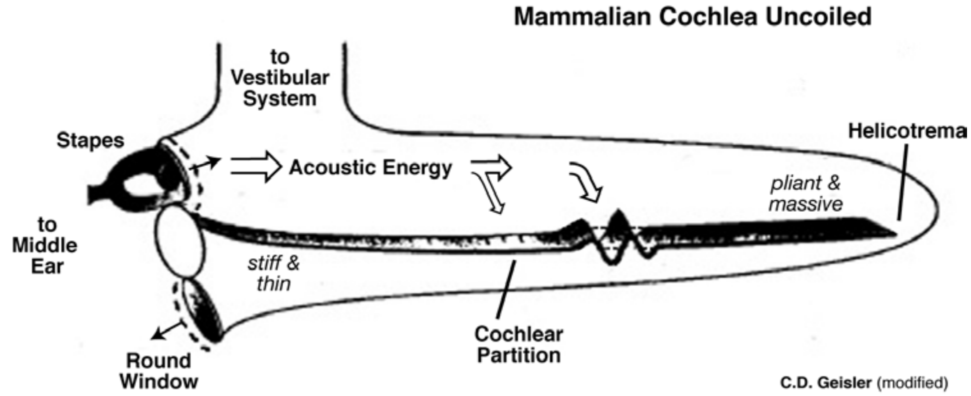
⇒ Second-order oscillator behaves as a "*band-pass filter*"

Resonance - Examples



http://physics.stackexchange.com/questions/159728/forced-oscillations-resonance

"Tonotopy" of the inner ear



Slightly different type of "resonance"...

MRI