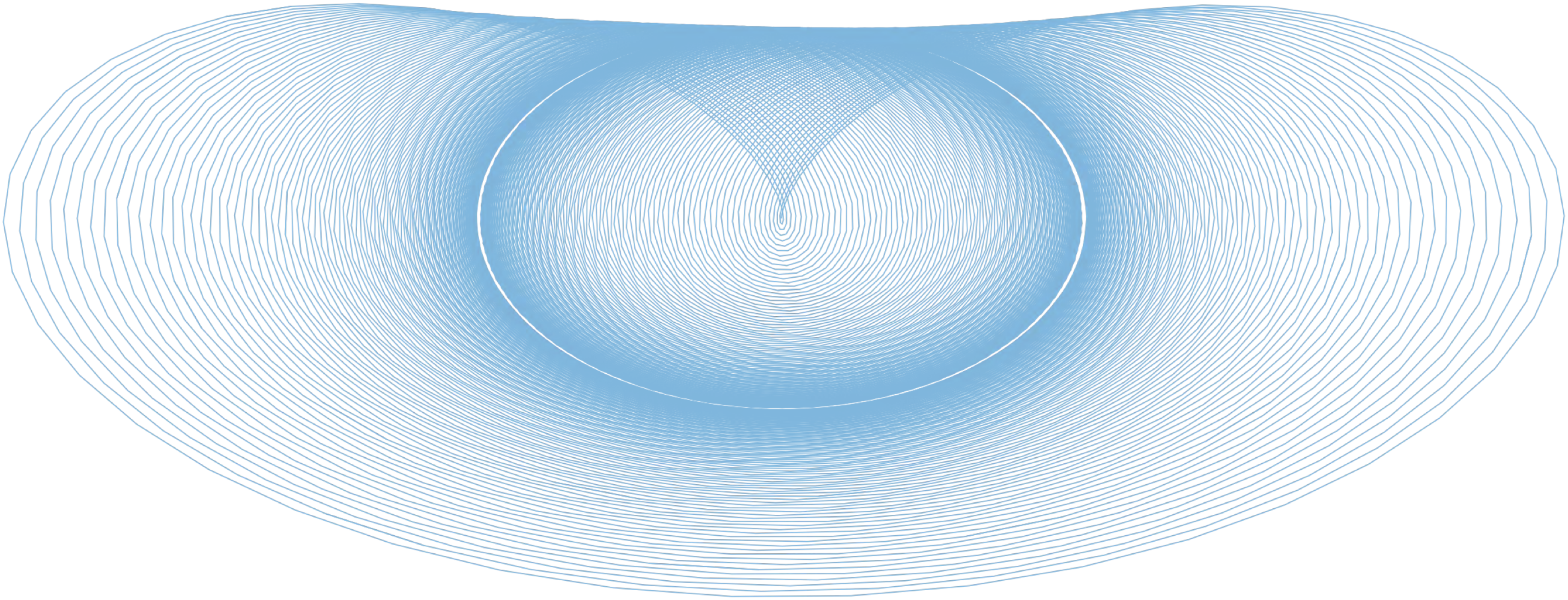
# PHYS 1420 (F19)
# Physics with Applications to Life Sciences



**2019.12.02**
Final Lecture

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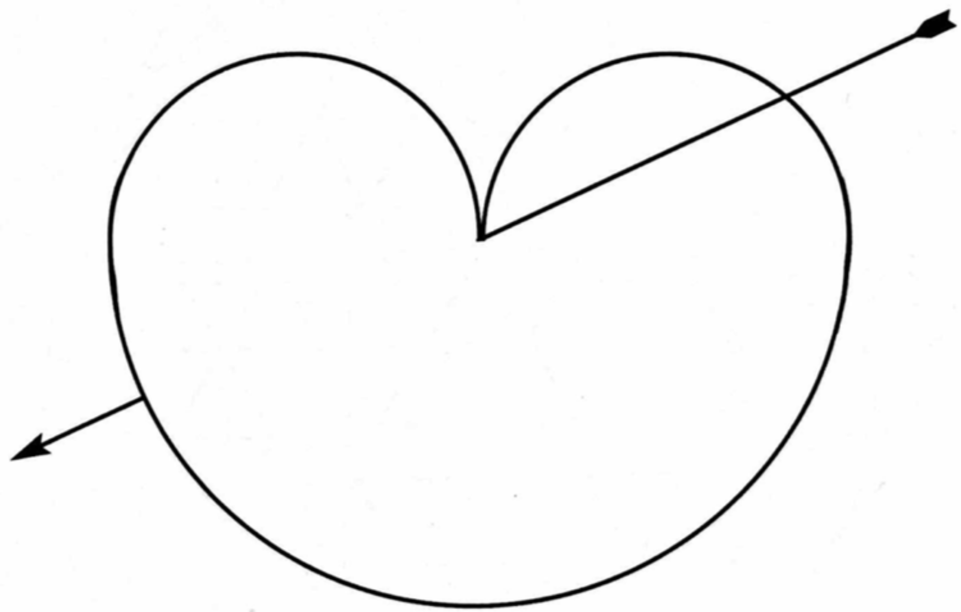
Ref. (re images):
Wolfson (2007), Knight (2017)

# Shot Through the Heart

This (okay, somewhat misshapen) Valentine heart consists of one large semicircle beneath two smaller semicircles. The arrow passes right through the point at which the two smaller semicircles meet.



Which part of the heart's perimeter is the longer: that lying above the line of the arrow, or that lying below?

# Aside → Differentiating multivariable functions

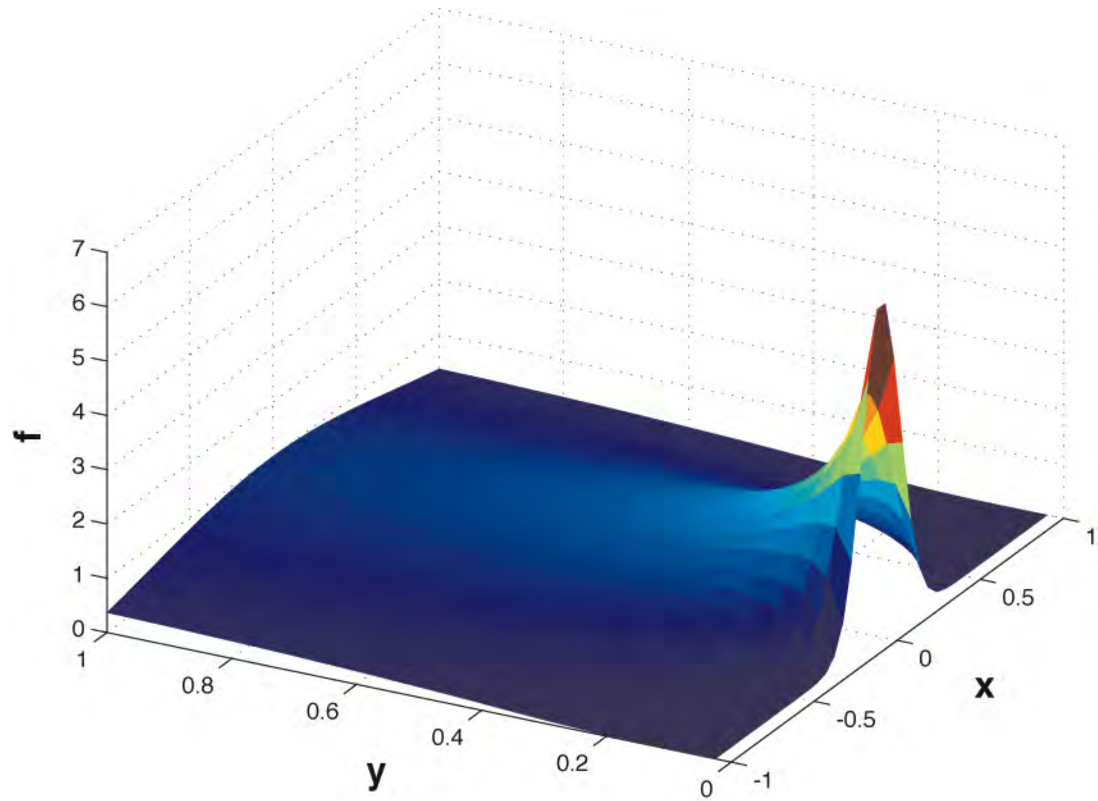
- Can take partial derivative with respect to partial derivative

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$$

- Simplified notation:

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}\left(f_x\right) = \left(f_x\right)_y = f_{xy}$$

"Wave math" → Multivariable functions




$$f(x,y) = \frac{1}{\sqrt{y}} e^{-x^2/y}$$

Solution to
diffusion equation
(or *heat eqn.*)

# "Wave math" → The wave equation

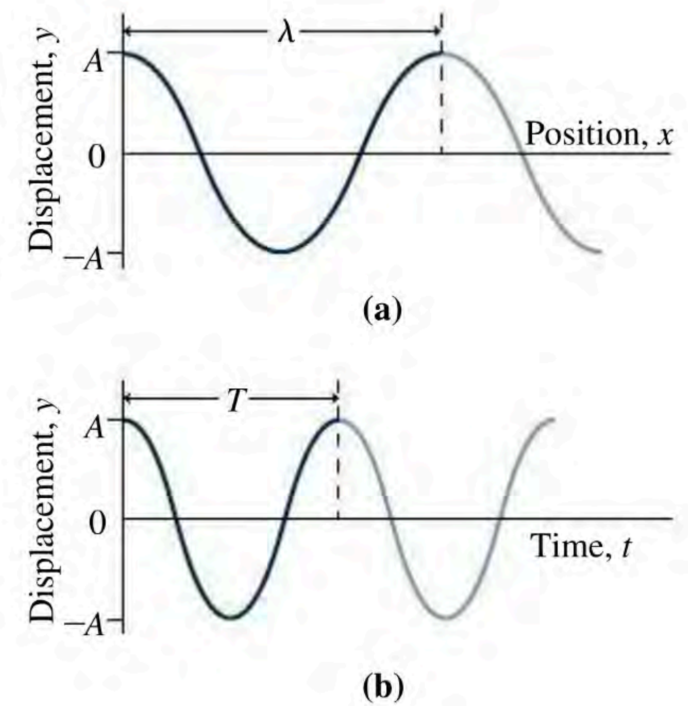
➢ A wave's dependence upon space and time are interrelated via a PDE commonly referred to as the *wave equation*

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

Note: One can readily derive this via combining Newton's 2nd Law and conservation of mass

$$y(x, t) = A\cos(kx \pm \omega t)$$

Possible solution to wave eqn. ("sinusoidal wave")




$$k = \frac{2\pi}{\lambda} \qquad \text{(wave number)}$$

$$v = \frac{\lambda}{T} = \frac{2\pi/k}{2\pi/\omega} = \frac{\omega}{k}$$

# "Wave math" → Multivariable functions (REVISITED)

Relevant derived quantities:

$$D(x, t) = D(x - vt, t = 0)$$

$$v = \lambda f = \lambda / T$$

So we rewrite as:

$$D(x, t) = A \sin\left(2\pi \frac{x - vt}{\lambda} + \phi_0\right) = A \sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + \phi_0\right)$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

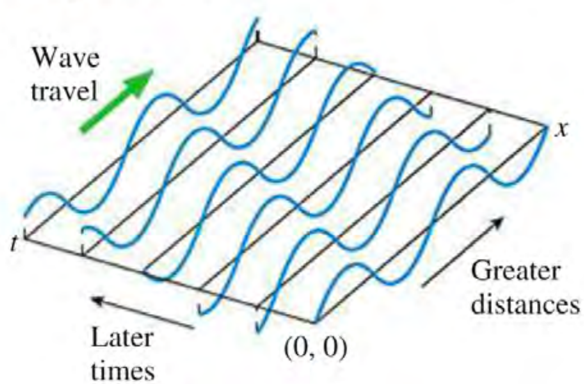
$$k = \frac{2\pi}{\lambda}$$

$$D(x, t) = A \sin(kx - \omega t + \phi_0)$$

(sinusoidal wave traveling in the positive $x$-direction)

$$v = \lambda f = \frac{2\pi}{k} \frac{\omega}{2\pi} = \frac{\omega}{k}$$

A sinusoidal wave moving along the $x$-axis.



Standing waves

- Consider that in 1-D, there can be two waves on a string: one going *forward* and one going *backward*

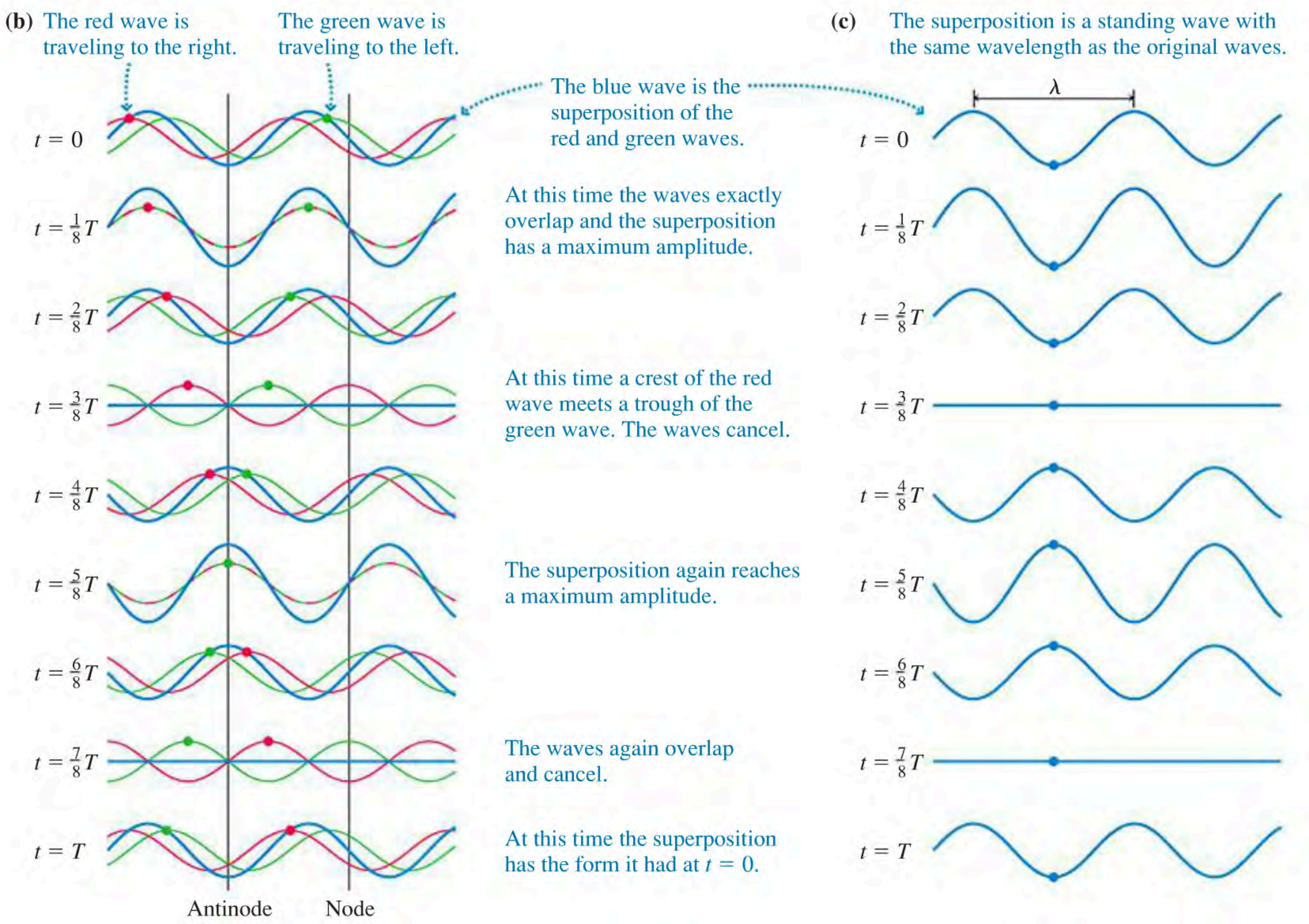
The superposition of two sinusoidal waves traveling in opposite directions.


- Their combination leads to interference (or *superposition*)

- Sometimes the waves interfere (i.e., add up) *constructively*, other times it is *destructively*

Knight

# Standing waves




→ A bit hard to see via a static picture....

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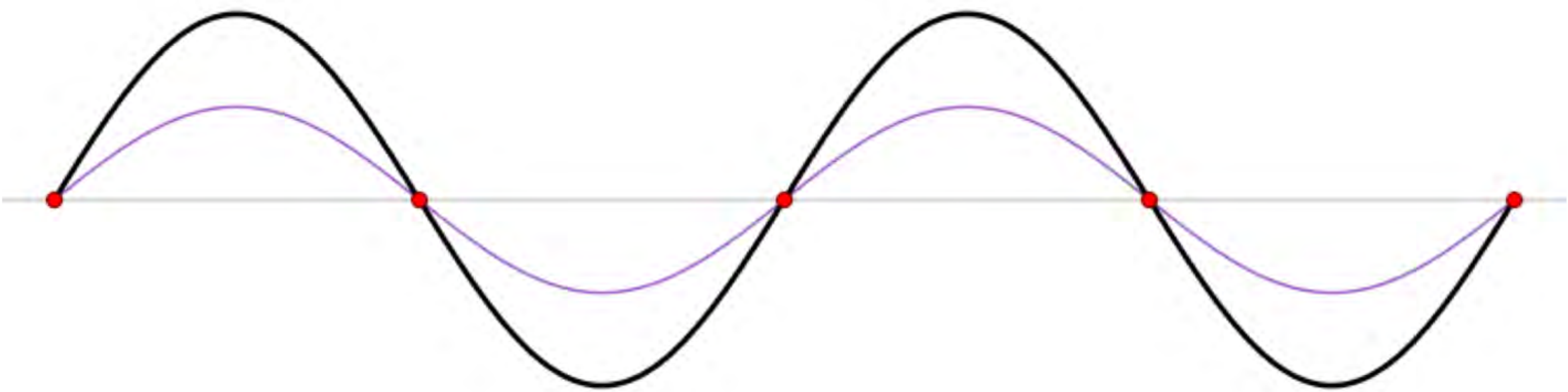
... but is much more readily apparent via a movie



Blue is the left-going wave
Red is the right-going wave
Black is the sum of the two (i.e., the “standing” wave)

Note: Locations where the amplitude stays zero are called ***nodes***

https://en.wikipedia.org/wiki/Standing_wave

# "Wave math" → Standing waves (REVISITED)

$$D_R = a\sin(kx - \omega t)$$ Right-going wave

$$D_L = a\sin(kx + \omega t)$$ Left-going wave

Note: The difference here is the sign. For "bonus" credit, look up **d´Alembert's formula**

Via superposition: $$D(x, t) = D_R + D_L = a\sin(kx - \omega t) + a\sin(kx + \omega t)$$

Relevant trig identity: $$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

Rewriting:

$$D(x, t) = a(\sin kx\cos\omega t - \cos kx\sin\omega t) + a(\sin kx\cos\omega t + \cos kx\sin\omega t)$$

$$= (2a\sin kx)\cos\omega t$$

$$D(x, t) = A(x)\cos\omega t$$

$$A(x) = 2a\sin kx$$

Note: A standing wave is not a traveling wave per se(!)

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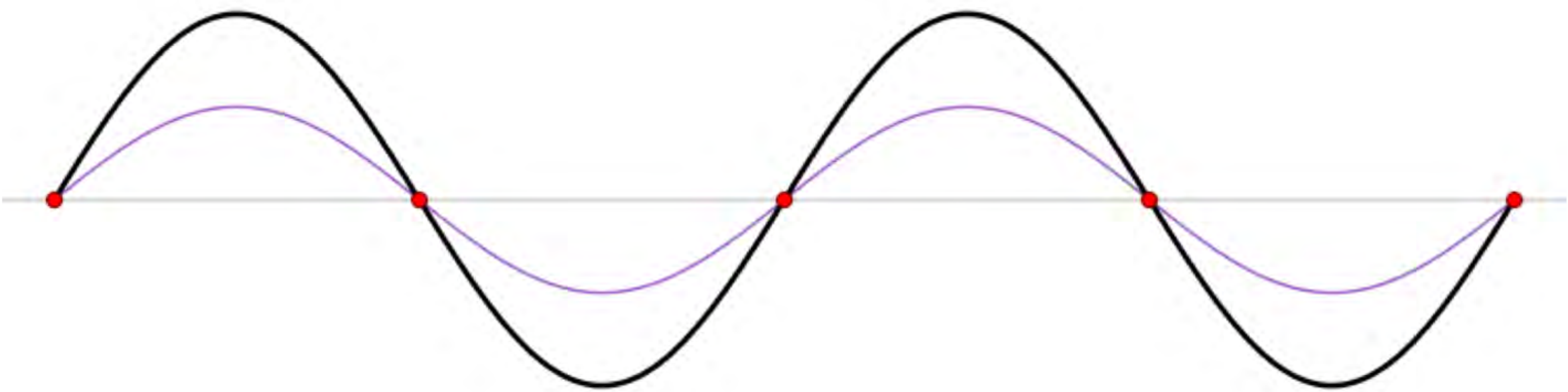
## Standing Waves (REVISITED)

$$D(x, t) = A(x) \cos \omega t$$

$$A(x) = 2a \sin kx$$



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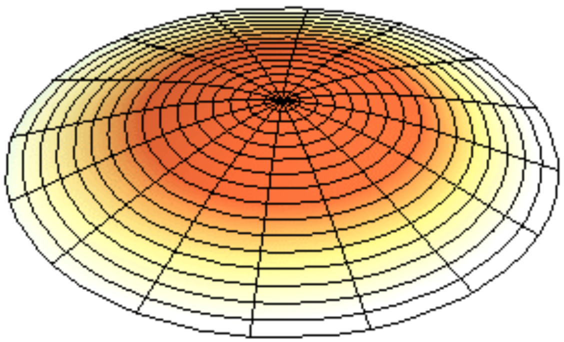
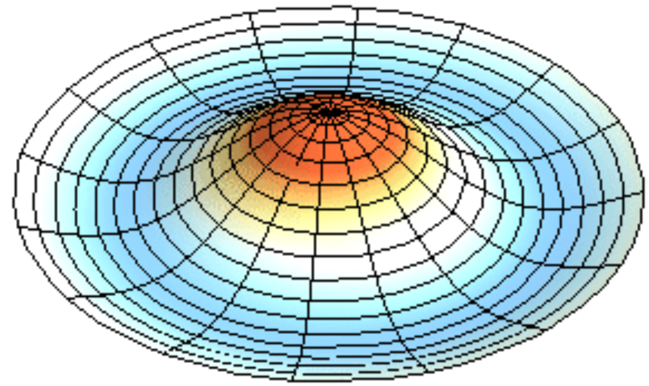
# Standing waves

- Standing waves can arise in 2-D as well (e.g., drumhead)

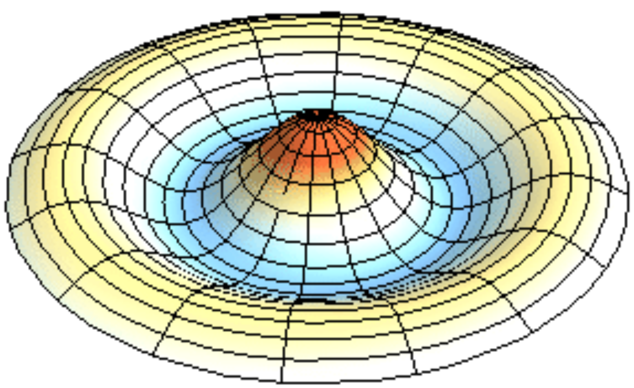
(0,1) mode



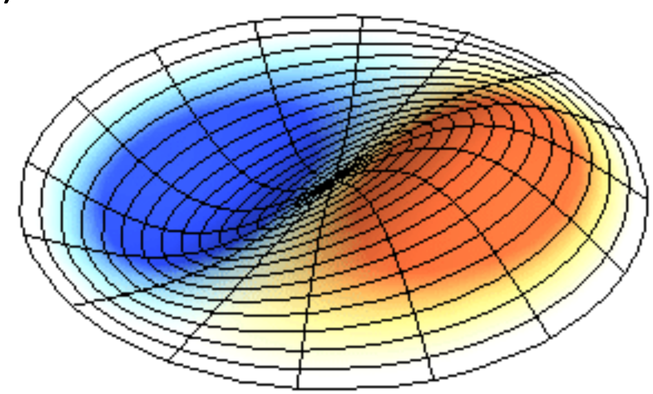
(0,2) mode



(0,3) mode



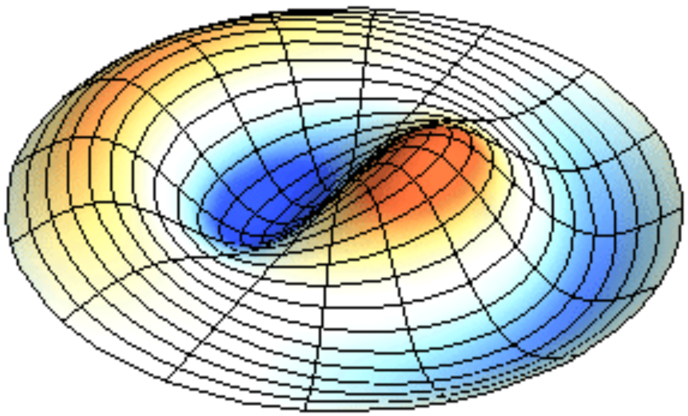
(1,1) mode



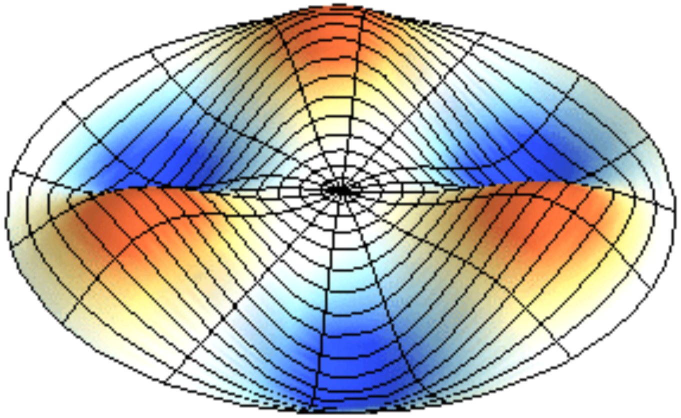
http://www.acs.psu.edu/drussell/demos/membranecircle/circle.html

<u>Standing waves</u>

(1,2) mode



(3,1) mode

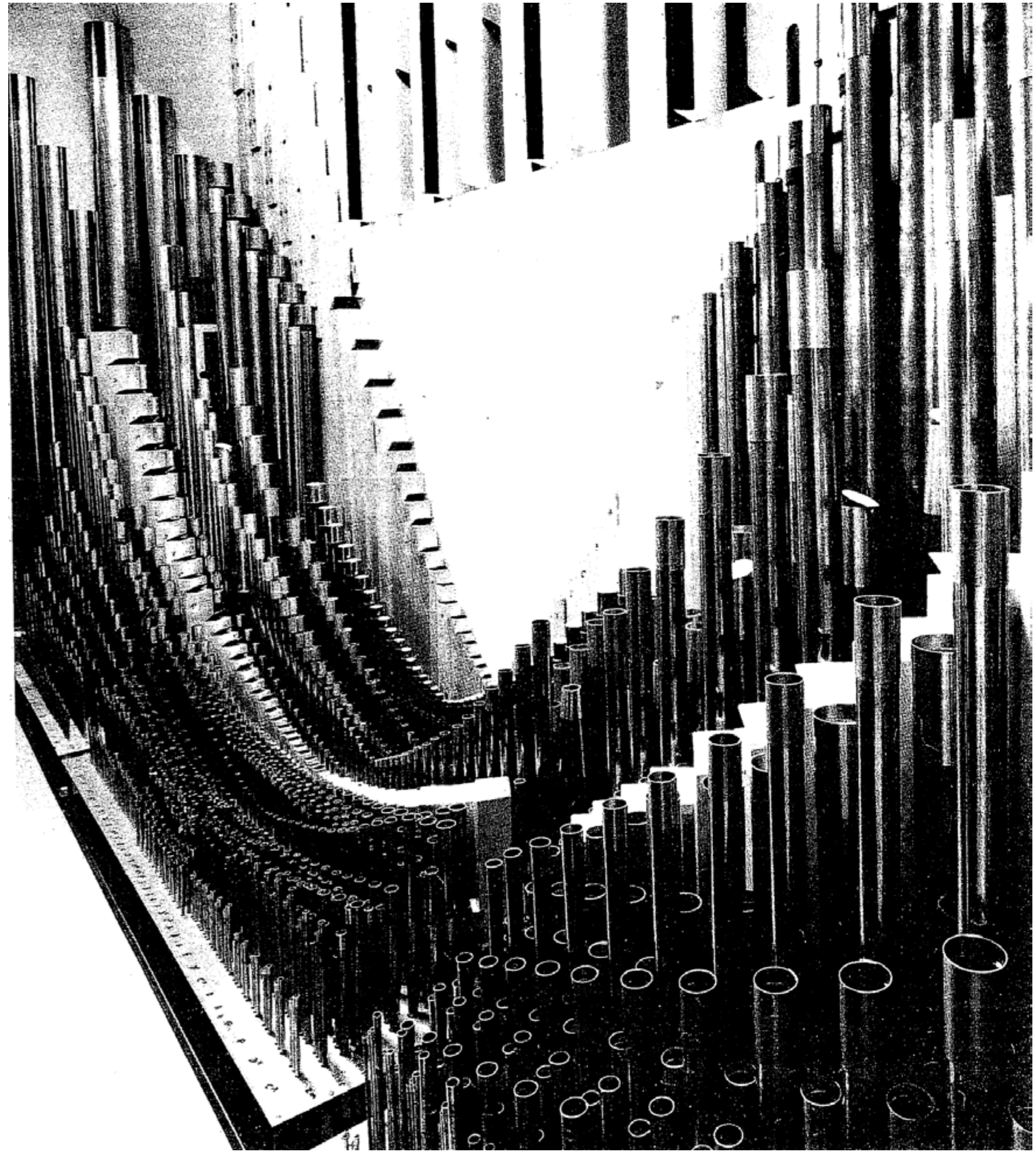


→ Note clear presence of nodes (chief characteristic of standing waves)

http://www.acs.psu.edu/drussell/demos/membranecircle/circle.html

# Standing waves

**INTERIOR VIEW OF ORGAN** at the Sydney Opera House shows some of its 26 ranks of pipes, most of which are of metal but some of which are of wood. The length of the speaking part of each pipe doubles at every 12th pipe; the pipe diameter doubles at about every 16th pipe. Through long experience master organ builders arrived at the proportions necessary for achieving balanced tone quality.



# Looking ahead: Light as a wave....

Review: A traveling wave is a broad term, but in a general sense can be defined as occurring when a "condition of some kind is transmitted from one place to another by means of a medium, but the medium itself is not transported"

- EM waves are a bit special in that they are not entirely consistent w/ our definition of a wave....

- We will need to develop further mathematical tools and physical concepts (e.g., electric fields, magnetism) to properly understand, classically at least, EM waves

Maxwell's equations

| | |
|---|---|
| $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$ | Gauss's law |
| $\oint \vec{B} \cdot d\vec{A} = 0$ | Gauss's law for magnetism |
| $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt}$ | Faraday's law |
| $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{through} + \epsilon_0 \mu_0 \frac{d\Phi_e}{dt}$ | Ampère-Maxwell law |

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ (Lorentz force law)

- **Gauss's law:** Charged particles create an electric field.
- **Faraday's law:** An electric field can also be created by a changing magnetic field.
- **Gauss's law for magnetism:** There are no isolated magnetic poles.
- **Ampère-Maxwell law, first half:** Currents create a magnetic field.
- **Ampère-Maxwell law, second half:** A magnetic field can also be created by a changing electric field.
- **Lorentz force law, first half:** An electric force is exerted on a charged particle in an electric field.
- **Lorentz force law, second half:** A magnetic force is exerted on a charge moving in a magnetic field.

→ Buried in all this is an even more basic notion: ***Oscillations***

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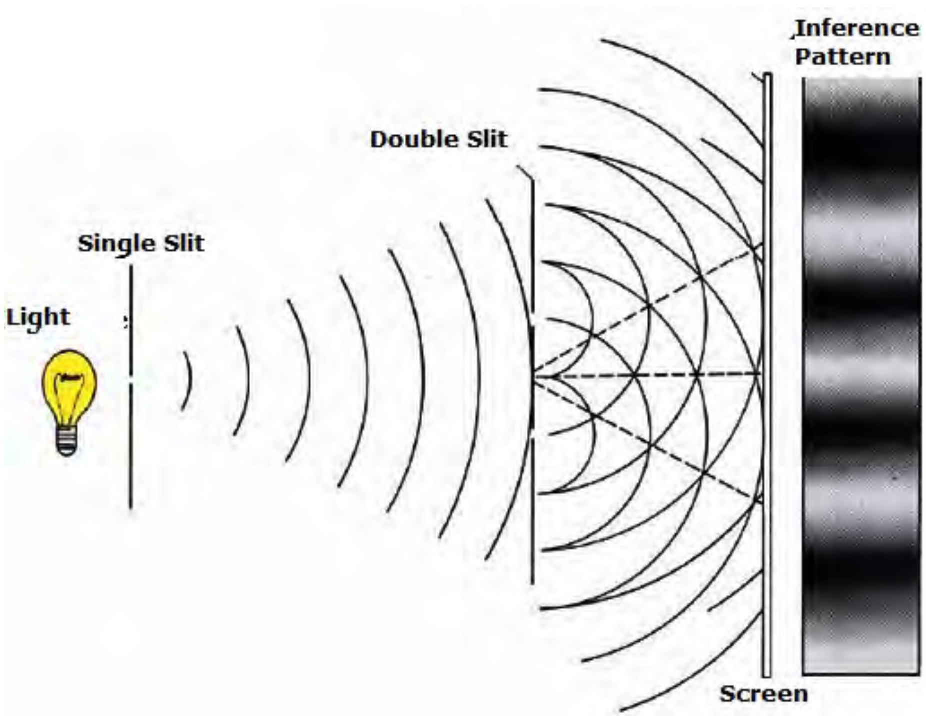
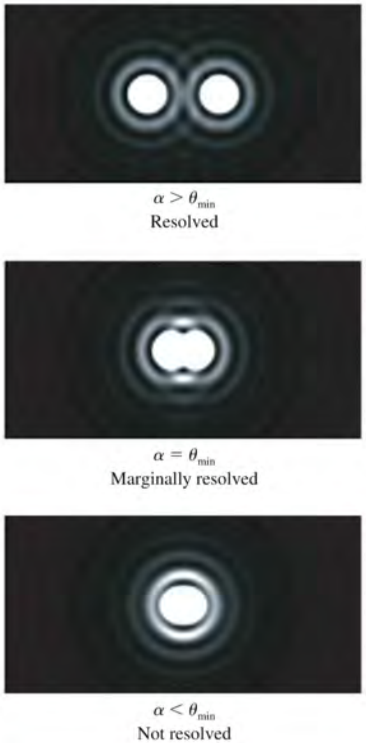
# Interference & Diffraction

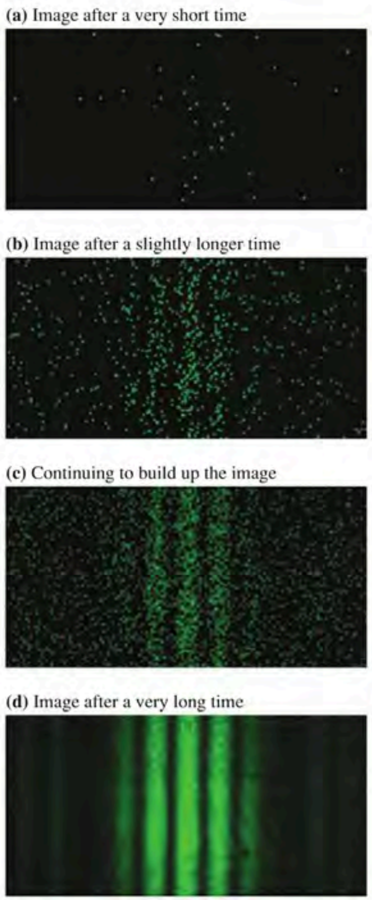
Review: Sometimes the waves interfere (i.e., add up) *constructively*, other times it is *destructively*

→ Same idea applies here re the "double slit" experiments previously described




http://www.mysearch.org.uk/website1/html/546.Double-Slit.html

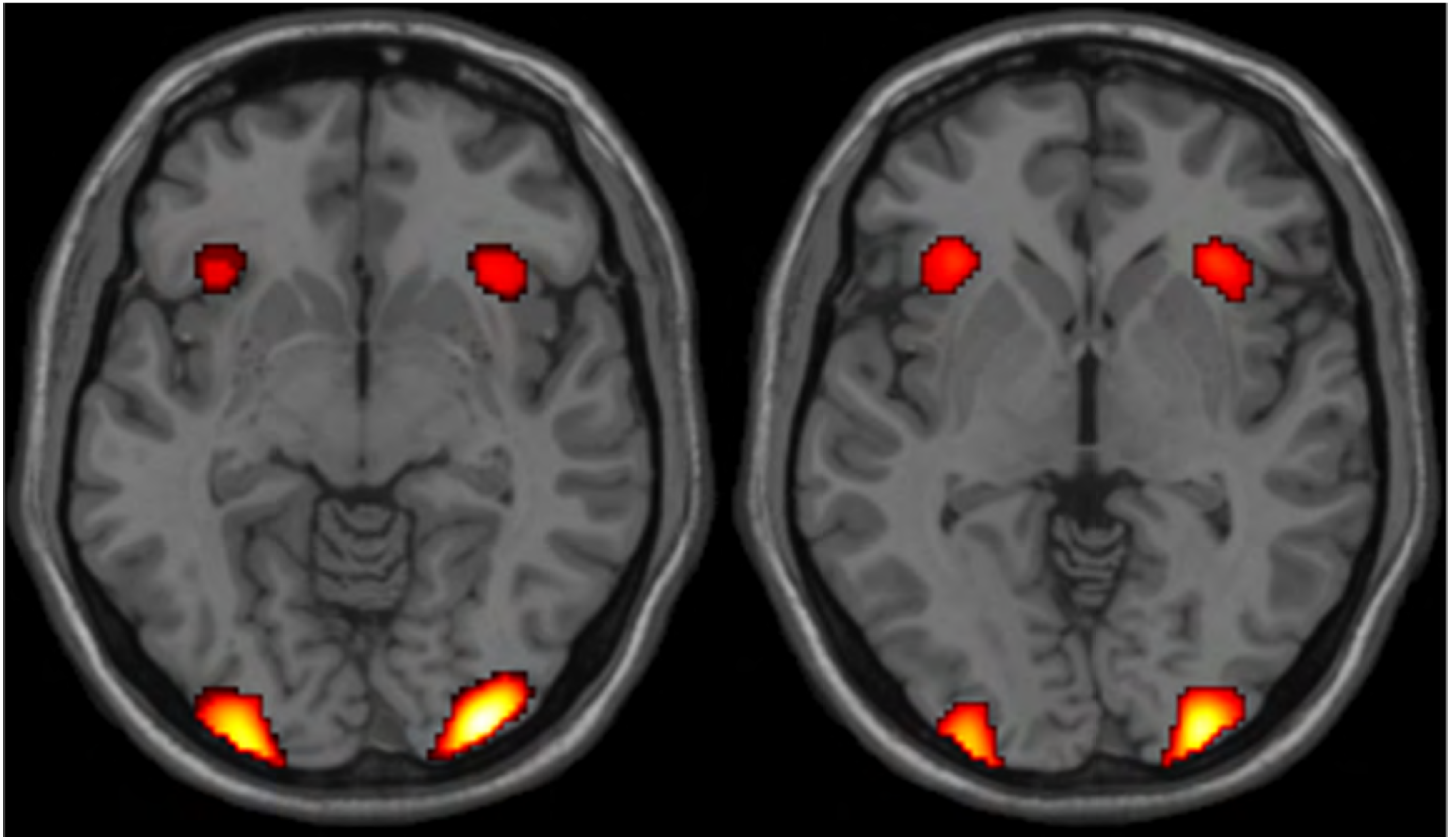



Note: Relevant concept here is known as ***Huygens-Fresnel principle***

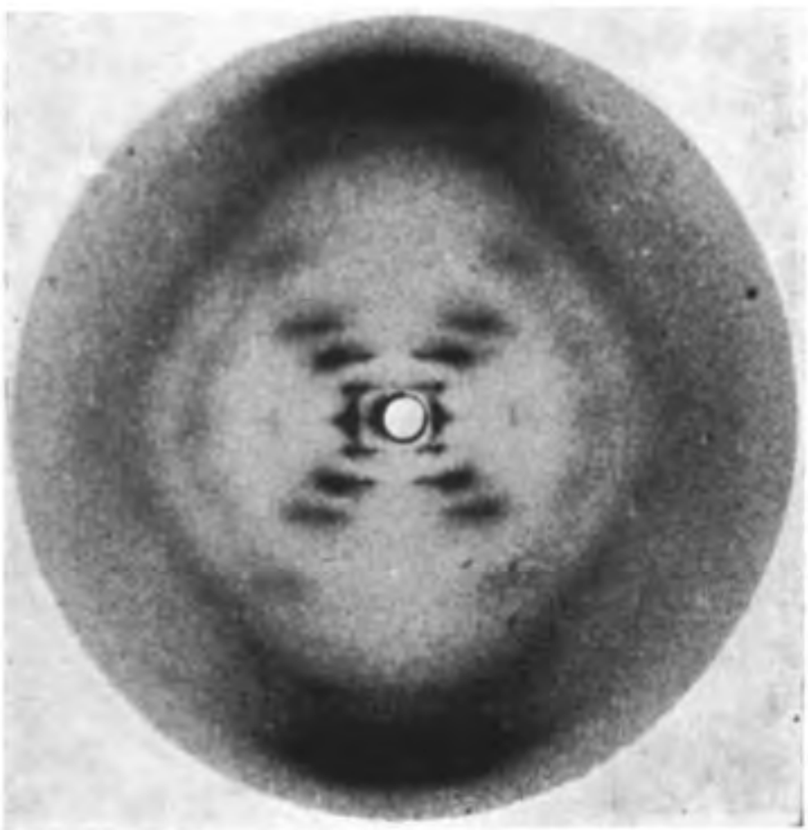
Question

How might we “see” brain? How it “functions”?



## Question

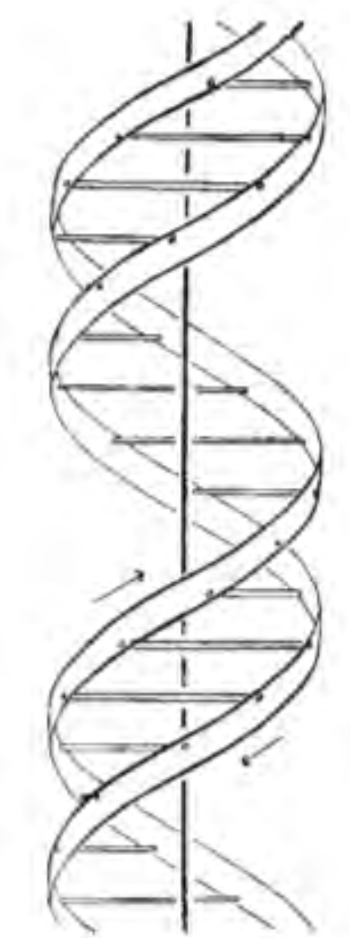
How do we make a 3-D image of really small things (e.g., molecular structure)?



Sodium deoxyribose nucleate from calf thymus. Structure *B*

Franklin & Gosling (1953)

→ How did Watson and Crick actually figure out a "double helix" for DNA?



This figure is purely diagrammatic. The two ribbons symbolize the two phosphate—sugar chains, and the horizontal rods the pairs of bases holding the chains together. The vertical line marks the fibre axis

Watson & Crick (1953)

Caveat

Even "simple" questions can be hard…

e.g., How many mountain *peaks* are there in this image?

# PHYS 1420 Course "Philosophy" (WOT version)

1. Learn the basics of 1st year physics and some applications to the life sciences

→ ”Physics" is generally equated w/ a branch of critical inquiry trying to track down the "universal rules" that govern how the universe works

2. Develop/refine "quantitative problem solving" skills

→ Ultimately an issue of "***attitude***" such that you feel comfortable tacking the unknown (as these sorts of skills are invaluable and will open doors for you downstream). by and large, this involves refining/developing your mathematical-based reasoning skills

Polya

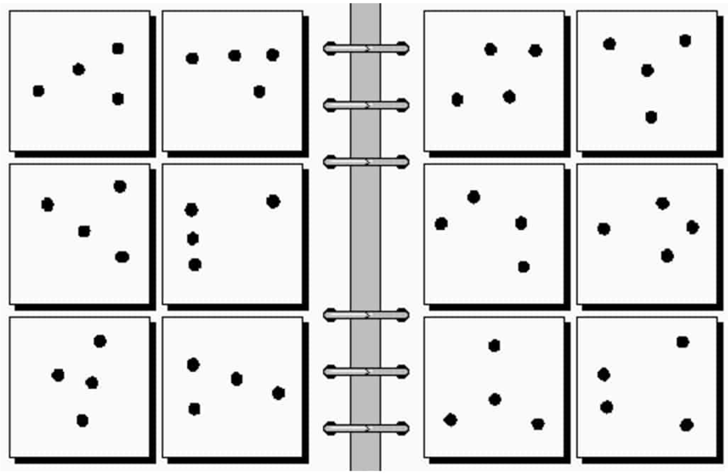
# How To Solve It

## *A New Aspect of Mathematical Method*

G. POLYA

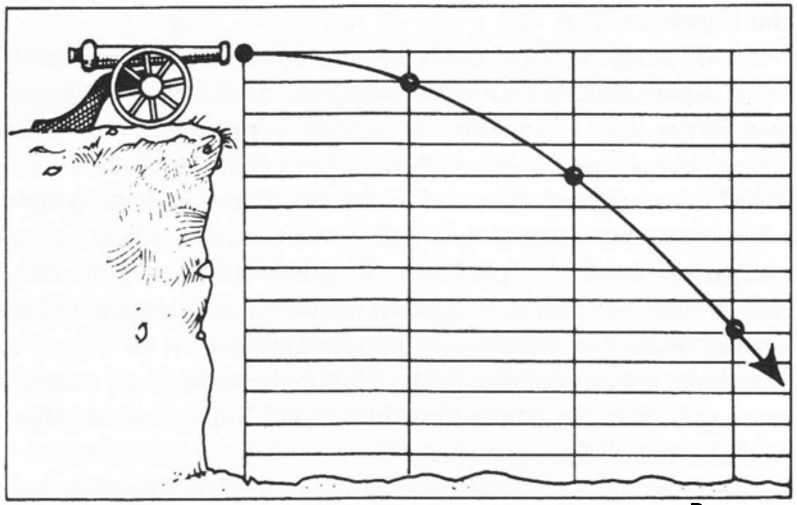
*Stanford University*

*How To Solve It* was originally published by Princeton University Press in 1945. The Anchor Books edition is published by arrangement with Princeton University Press.



Theme: Problem-solving......

A man lives on the tenth floor of a building. Every day he takes the elevator to go down to the ground floor to go to work or to go shopping. When he returns he takes the elevator to the seventh floor and walks up the stairs to reach his apartment on the tenth floor. Why does he do this?



von Baeyer

e.g., “classical mechanics”

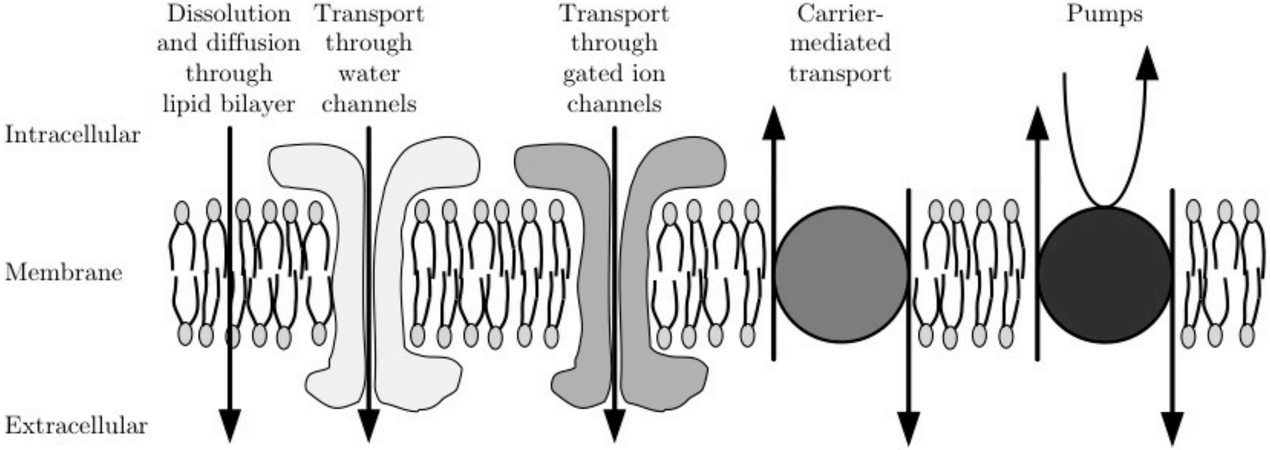
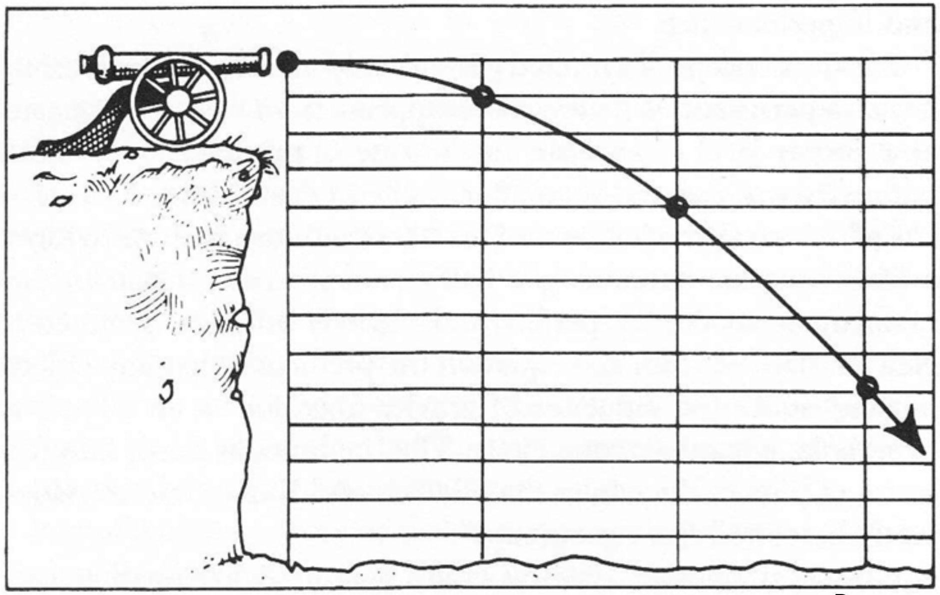



Figure 2.19

e.g., "classical mechanics"



von Baeyer

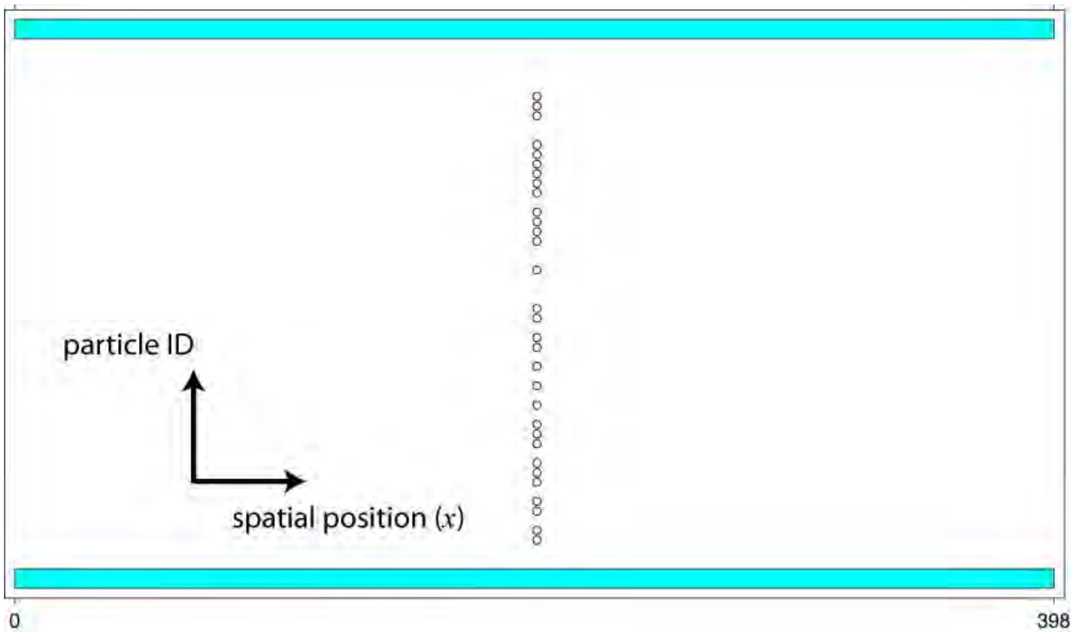
Note: Studying how stuff "moves" across a cell membrane is the same basic thing as the more general "how does stuff move?"

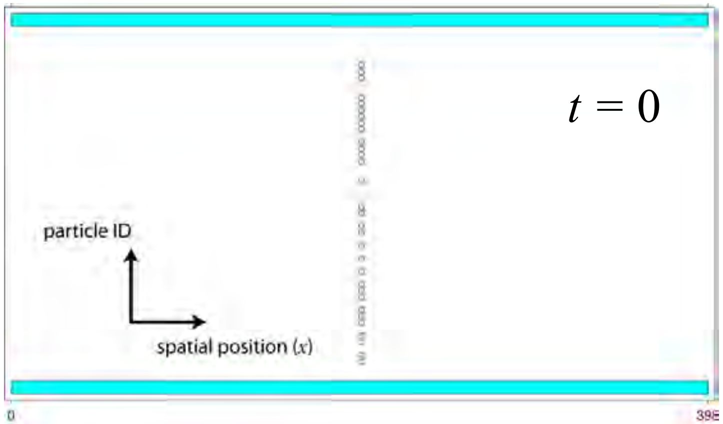
→ We initially delved into "mechanics" and easier sorts of "stuff" to study

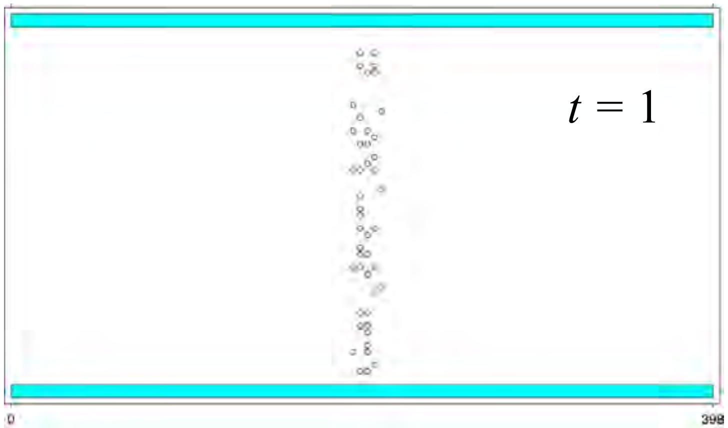
# Theme: Microscopic basis for diffusion

Brownian motion ⇒ 'Random Walker' (1-D)

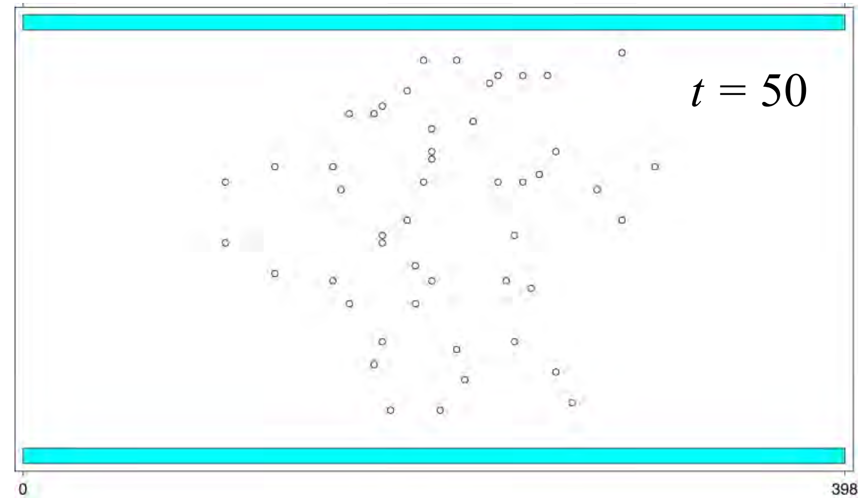
Ensemble of Random Walkers

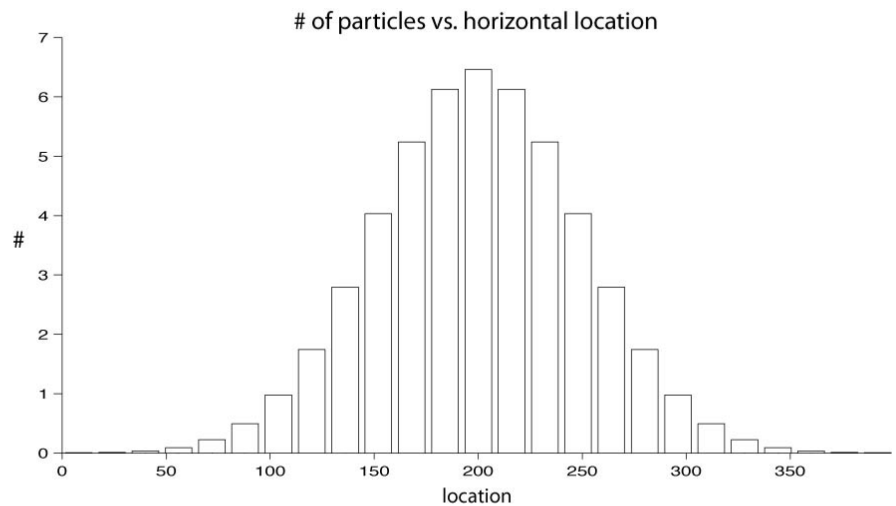









→ Diffusion (for which Brownian motion is the *microscopic* basis)

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