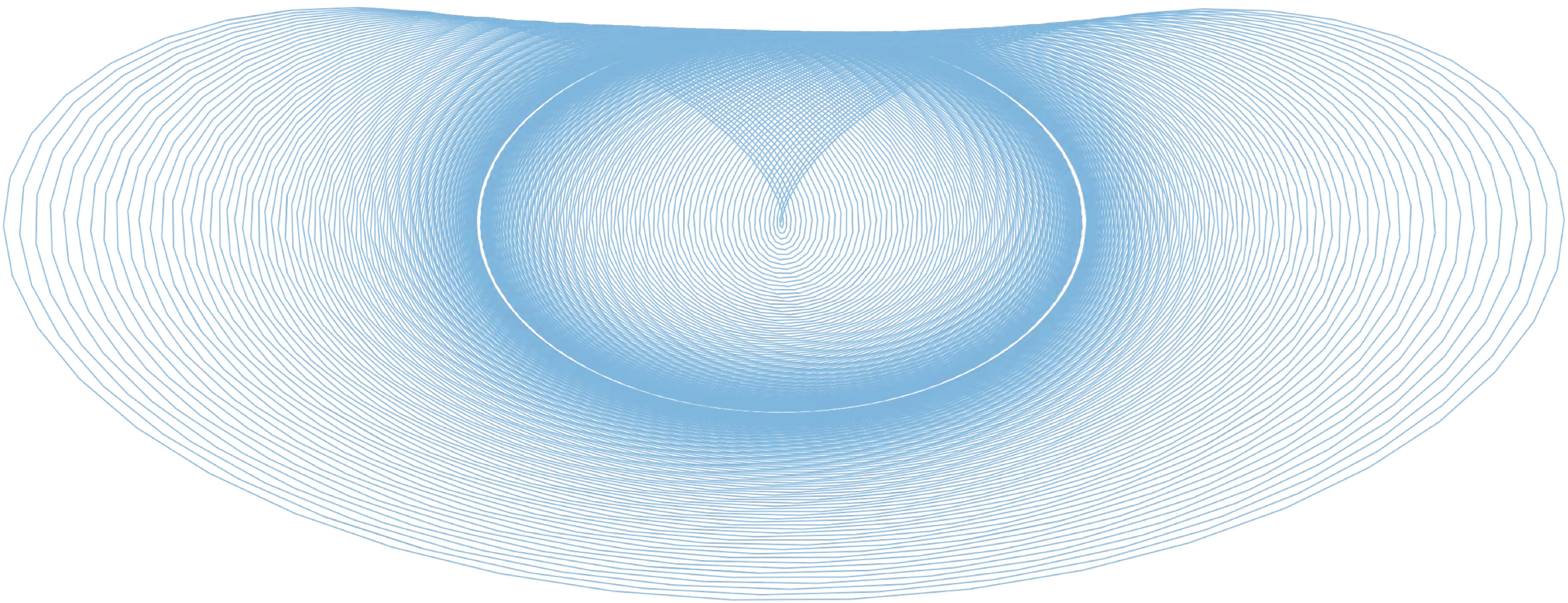
PHYS 1420 (F19)
Physics with Applications to Life Sciences



2019.09.13

Relevant reading:

Kesten & Tauck ch.3.1-3.3

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Ref. (re images):
Wolfson (2007), Knight (2017)

What is the missing number?

9 6 10 4 15 2 13 7 __ 8

Announcements & Key Concepts (re Today)

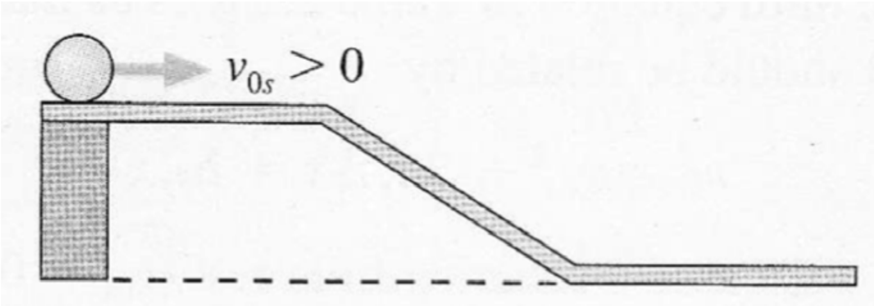
→ (online) HW #2 due later today

→ Labs: Start next week! (Sept.16-20)
https://www.yorku.ca/menary/courses/firstyrlabs/2019/main.html

Some relevant underlying concepts of the day…

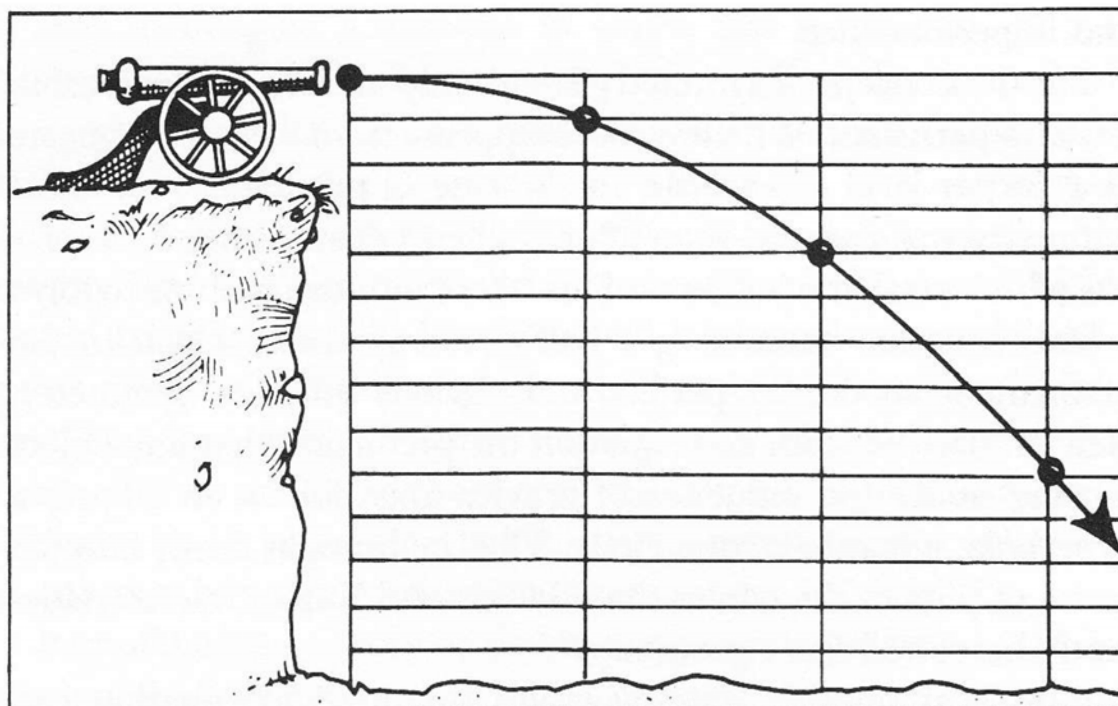
- motion in 2-D
- scalars vs vectors
- vector algebra
- vector "components" of x, v, and a

<u>Mechanics:</u> Shifting to higher dimensions....



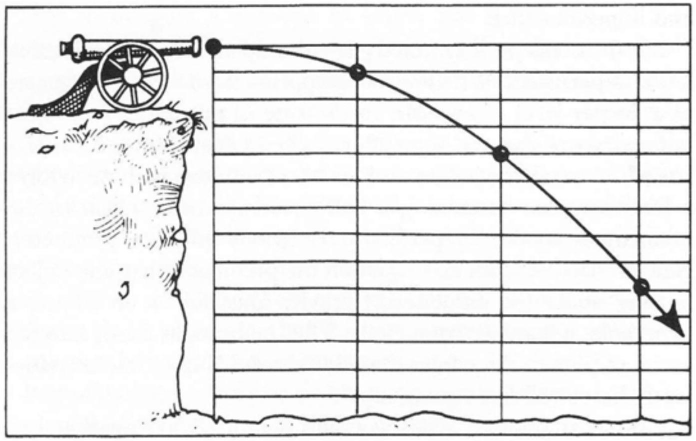
- Perhaps for our ball <u>*confined*</u> to a track, a one-dimensional (1-D) description is sufficient....

- ... but maybe for the cannonball, a higher dimensionality is needed (e.g., both horizontal and vertical position matters)

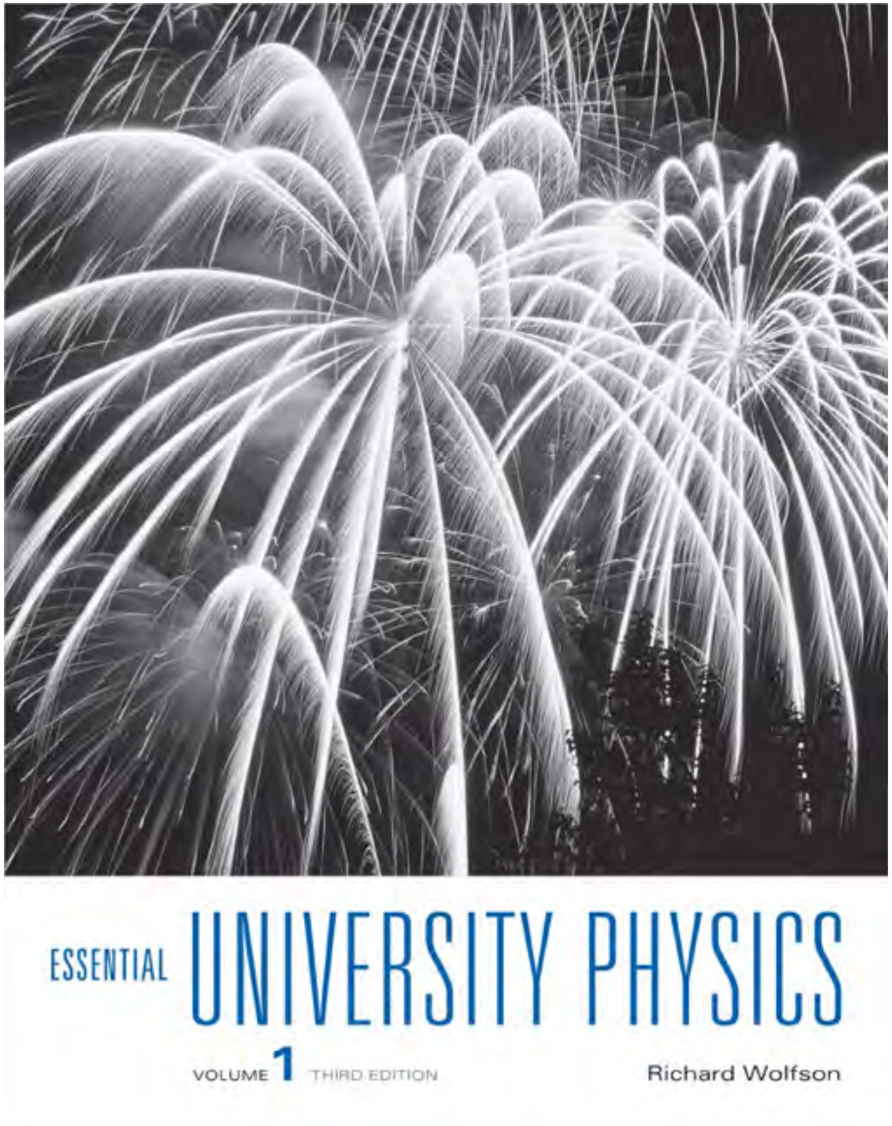


von Baeyer

Caution: Be careful re dimensionality



2-D problem



How many dimensions here?

Note: Despite being very different physical scenarios, both exhibit similar behavior (e.g., parabolic trajectories)

Vectors

- For 1-D (i.e., motion constrained to a line), the *sign* is sufficient to determine "direction"

- For 2-D and higher, things get a bit more complicated and "*vectors*" are utilized

- Conceptually, vectors arise in many ways....

e.g., a bunch of arrows

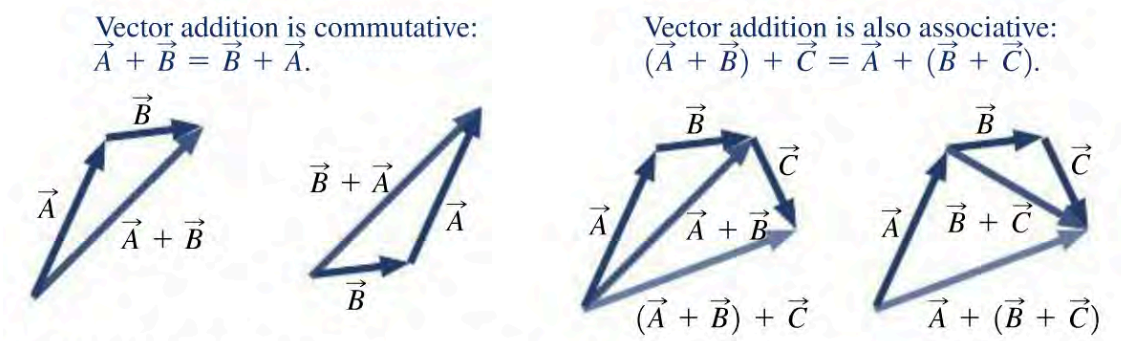


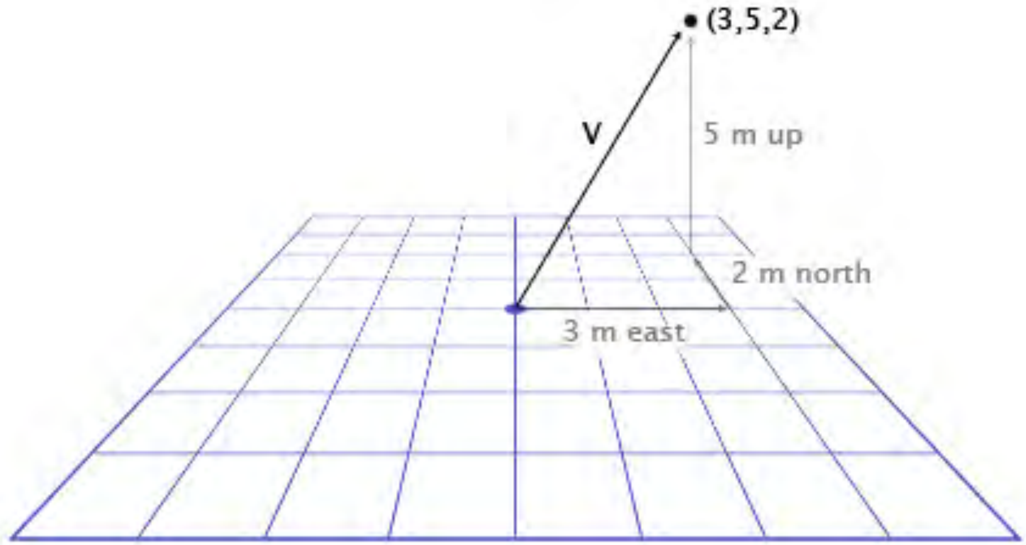
FIGURE 3.3 Vector addition is commutative and associative.

Windsock

- Ultimately, vectors are a compact means to represent/manipulate information

Vectors: Matlab, Linear Algebra, & Beyond...

- Vectors have “components” and can be expressed in a variety of ways



↔

$$\begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$$

Note: Here, the spatial layout implicitly contains specific and key information

→ Linear algebra is a powerful branch of mathematics that capitalizes upon vectors

(e.g., *eigenvalues* tell you something about how a mass-on-a-spring will behave)

http://blog.wolfire.com/2009/07/linear-algebra-for-game-developers-part-1/

Vectors: Matlab, Linear Algebra, & Beyond...

Defining a Vector

Matlab is a software package that makes it easier for you to enter matrices and vectors, and manipulate them. The interface follows a language that is designed to look a lot like the notation use in linear algebra. In the following tutorial, we will discuss some of the basics of working with vectors.

If you are running windows or Mac OSX, you can start matlab by choosing it from the menu. To start matlab on a unix system, open up a unix shell and type the command to start the software: *matlab*. This will start up the software, and it will wait for you to enter your commands. In the text that follows, any line that starts with two greater than signs (>>) is used to denote the matlab command line. This is where you enter your commands.

Almost all of Matlab's basic commands revolve around the use of vectors. A vector is defined by placing a sequence of numbers within square braces:

```
>> v = [3 1]

v =

     3     1
```

"Arrays" are just:

- A bunch of numbers organized together (e.g., a grid of #s)
- A bunch of vectors stacked together
- Also called matrices (hence the "Mat" in Matlab)

http://www.cyclismo.org/tutorial/matlab/vector.html

Vector Representations

- Vectors are very useful for representing various physical quantities....

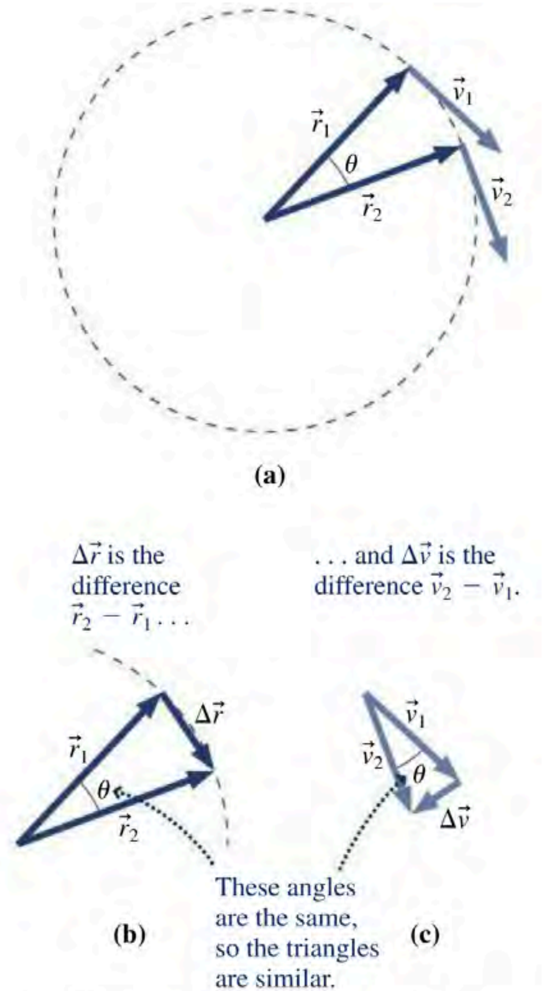


FIGURE 3.22 Position and velocity vectors for two nearby points on the circular path.

Position, velocity, acceleration

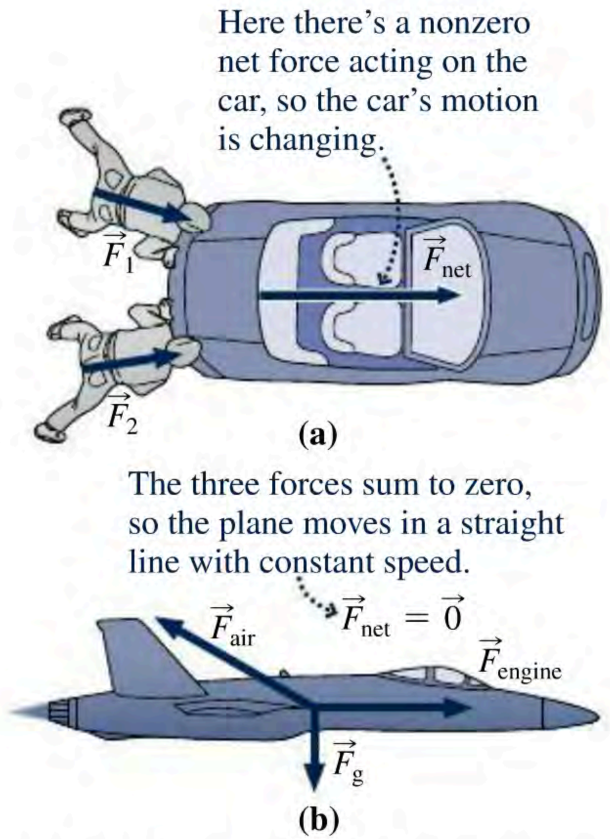


FIGURE 4.2 The net force determines the change in an object's motion.

Forces

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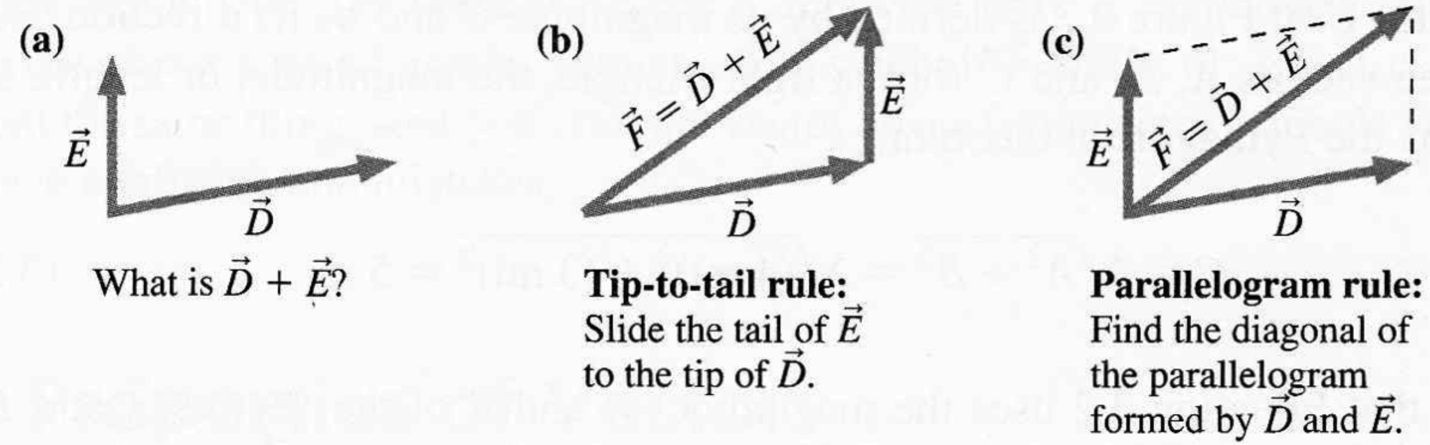
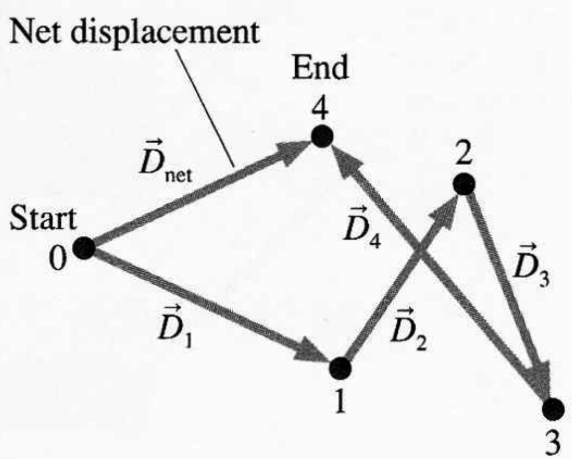
Vector Algebra

FIGURE 3.5 Two vectors can be added using the tip-to-tail rule or the parallelogram rule.



Different "visual" ways to think about vector addition

FIGURE 3.6 The net displacement after four individual displacements.



Addition extends to more than just two vectors

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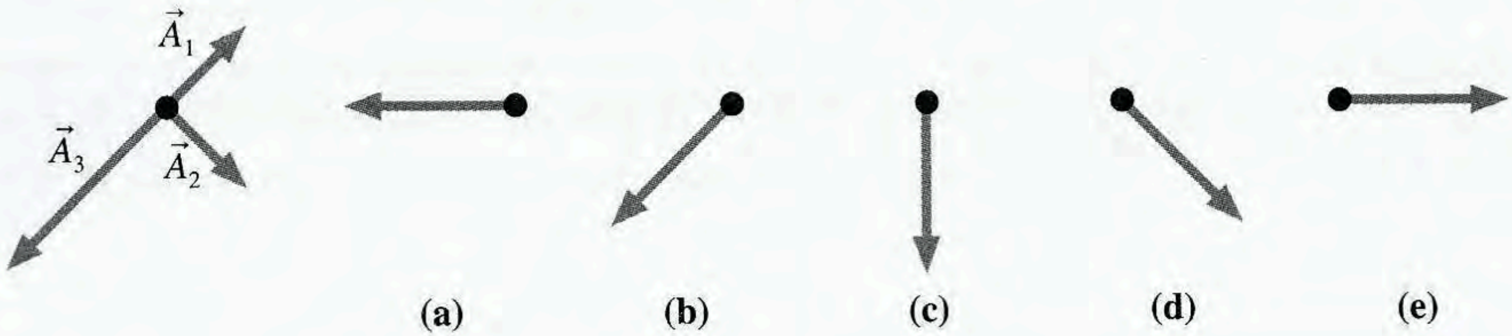
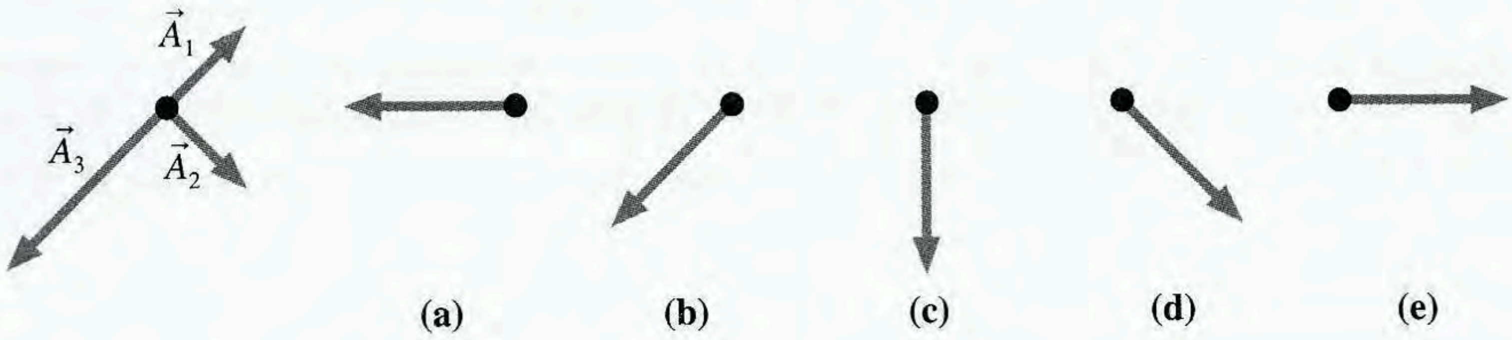
Ex.

STOP TO THINK 3.1 Which figure shows $\vec{A}_1 + \vec{A}_2 + \vec{A}_3$?



Ex. (SOL)

STOP TO THINK 3.1 Which figure shows $\vec{A}_1 + \vec{A}_2 + \vec{A}_3$?



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Vector Algebra

Vector algebra follows familiar rules

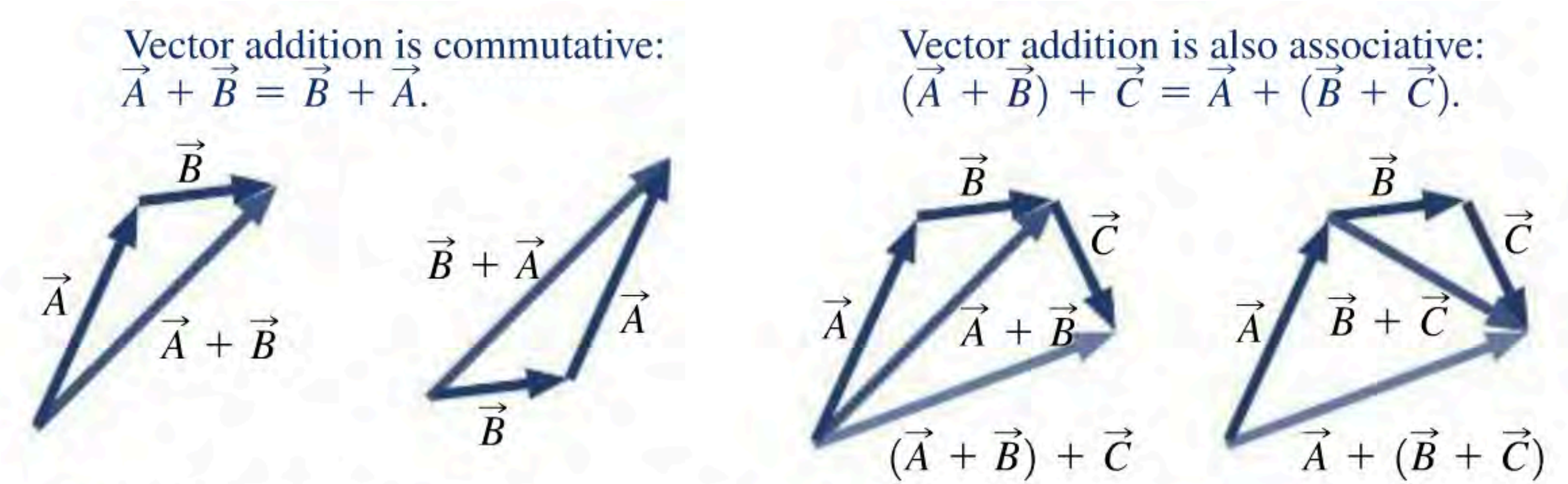
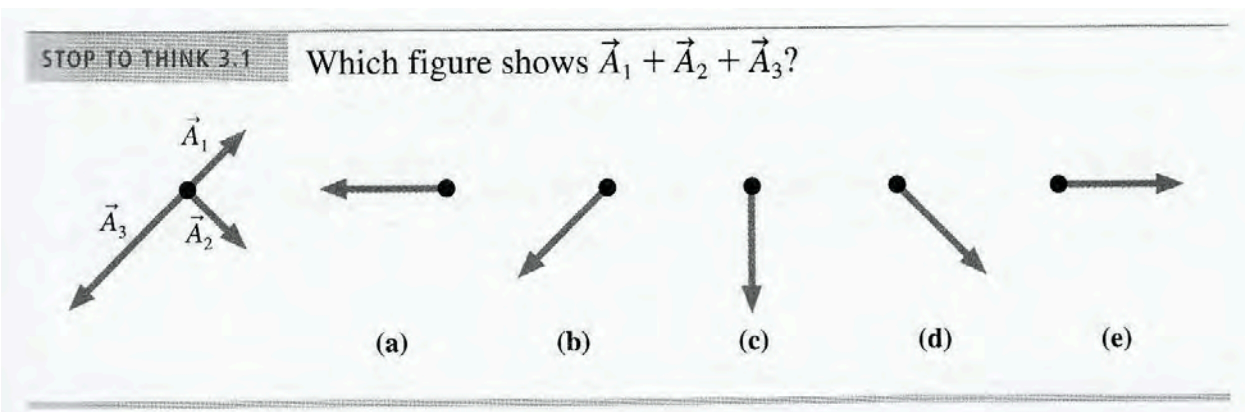
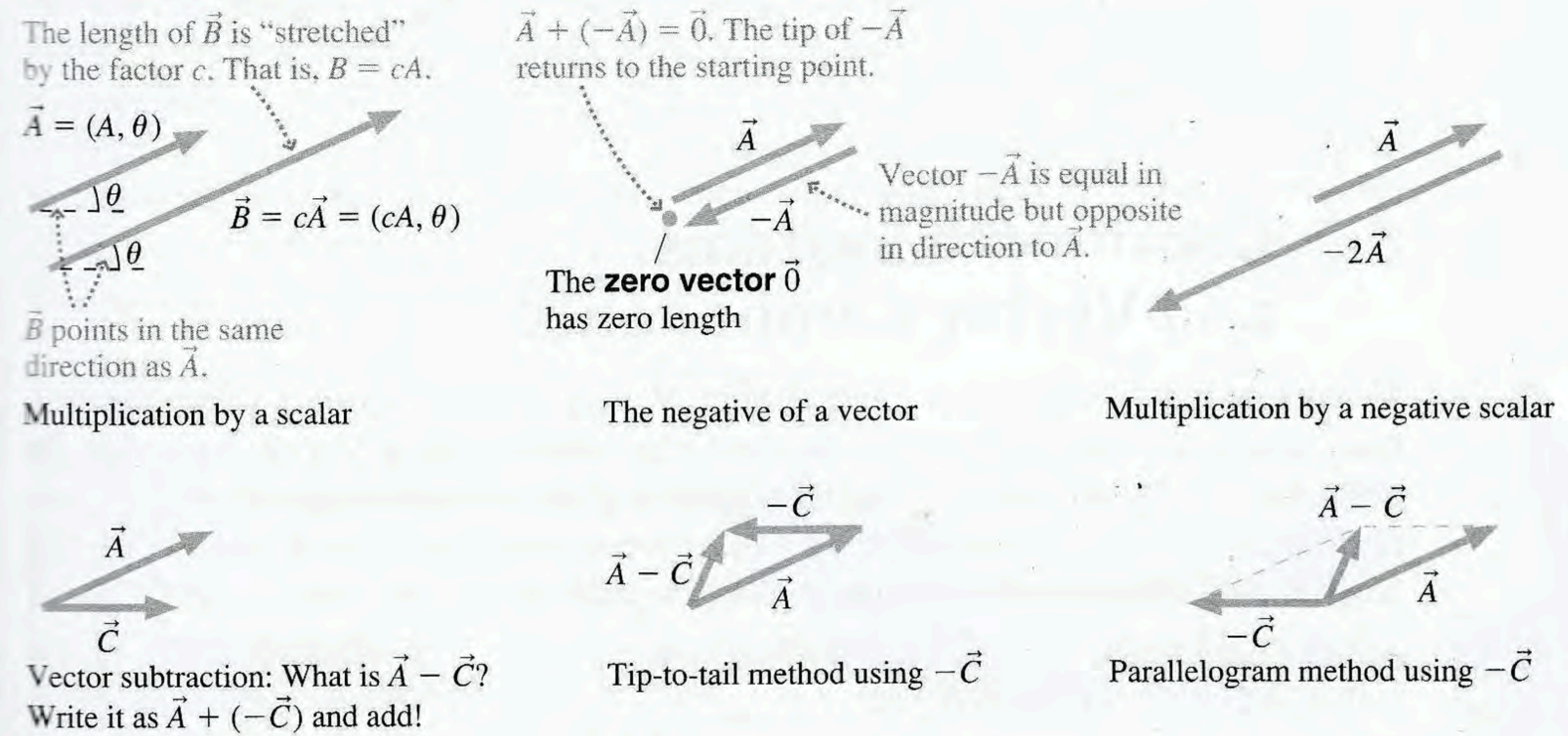


FIGURE 3.3 Vector addition is commutative and associative.



Vector algebra follows familiar rules (cont)

FIGURE 3.7 Working with vectors.



Note: Multiplication of two vectors is a bit trickier (we'll get there later in the semester)

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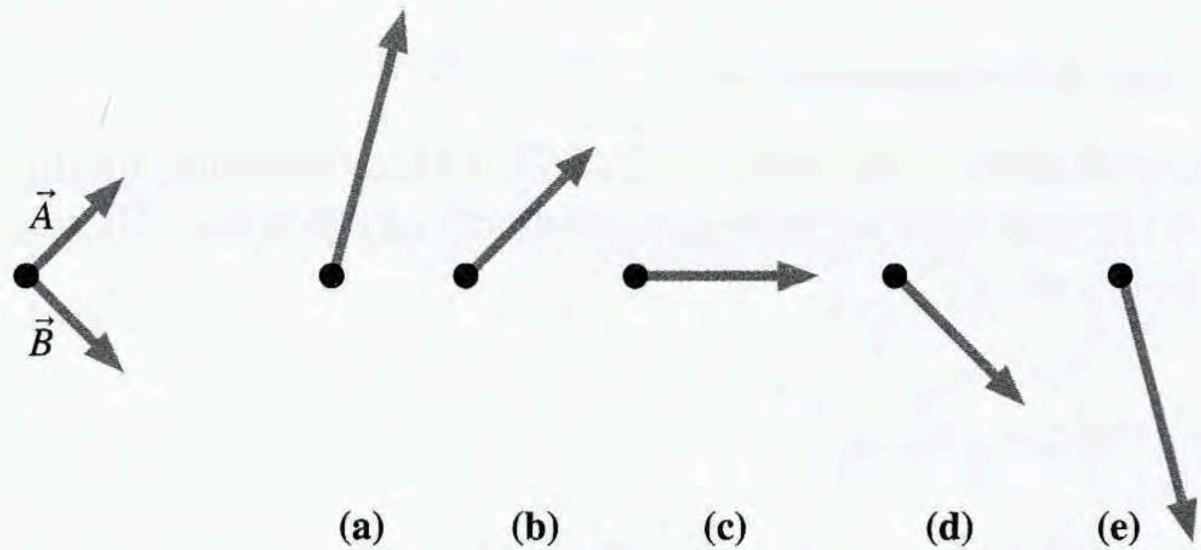
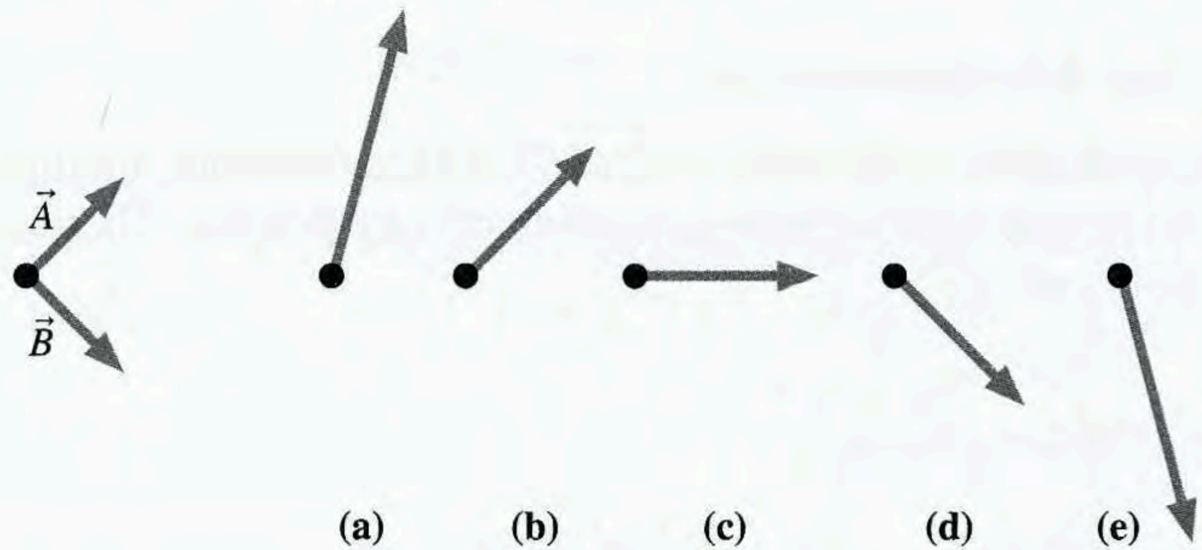
Ex.

STOP TO THINK 3.2 Which figure shows $2\vec{A} - \vec{B}$?



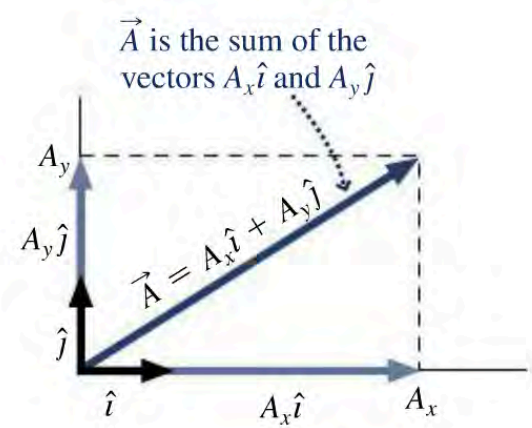
Ex. (SOL)

STOP TO THINK 3.2 Which figure shows $2\vec{A} - \vec{B}$?



a

→ These problems become a bit easier to deal with when shifting to a different representation....



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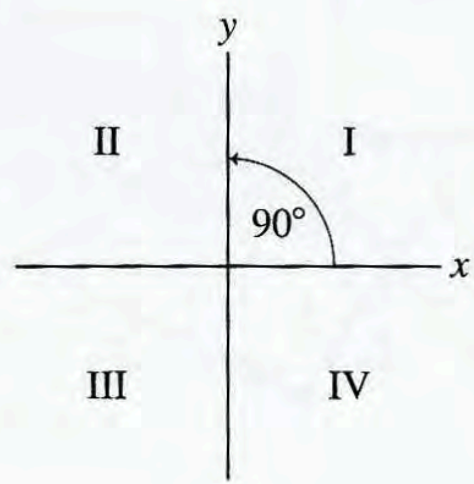
Vector Components & Unit Vectors

- Choose a coordinate system (such provides a key *frame of reference*)

FIGURE 3.9 A conventional xy-coordinate system and the quadrants of the xy-plane.



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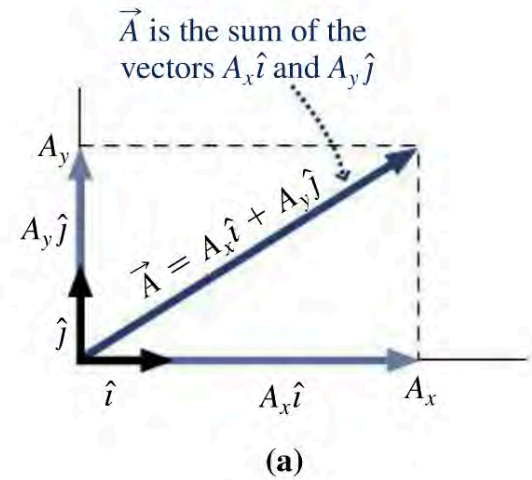
→ Cartesian system is a good starting choice for 2-D problems



Note: 3-D pictures can appear a bit crowded

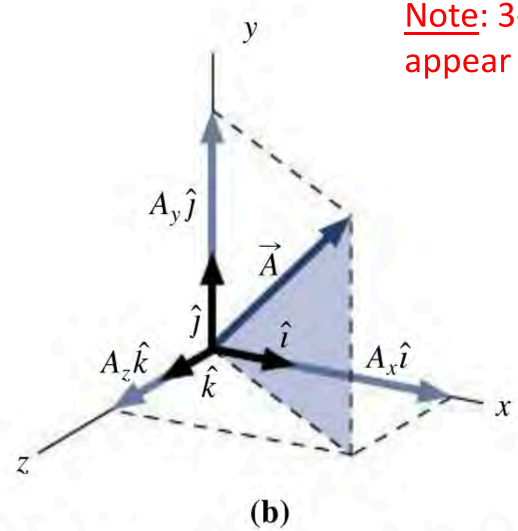


FIGURE 3.5 Vectors in (a) a plane and (b) space, expressed using unit vectors.

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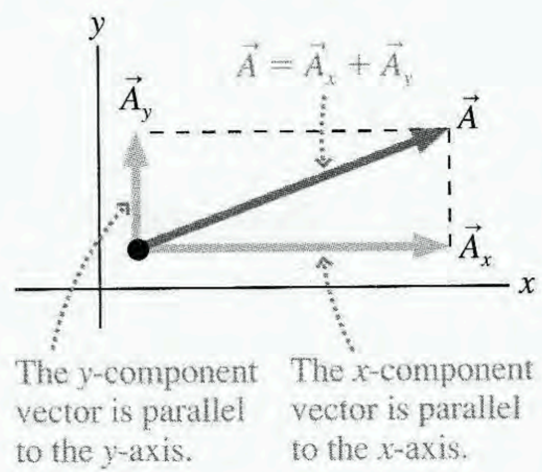
Vector Components & Unit Vectors

- Unit vectors stem directly from the chosen frame and allow a compact way to express vectors via *components*

FIGURE 3.10 Component vectors $\vec{A}_x$ and $\vec{A}_y$ are drawn parallel to the coordinate axes such that $\vec{A} = \vec{A}_x + \vec{A}_y$.



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- "Components" can be vectors or scalars combined w/ the unit vectors

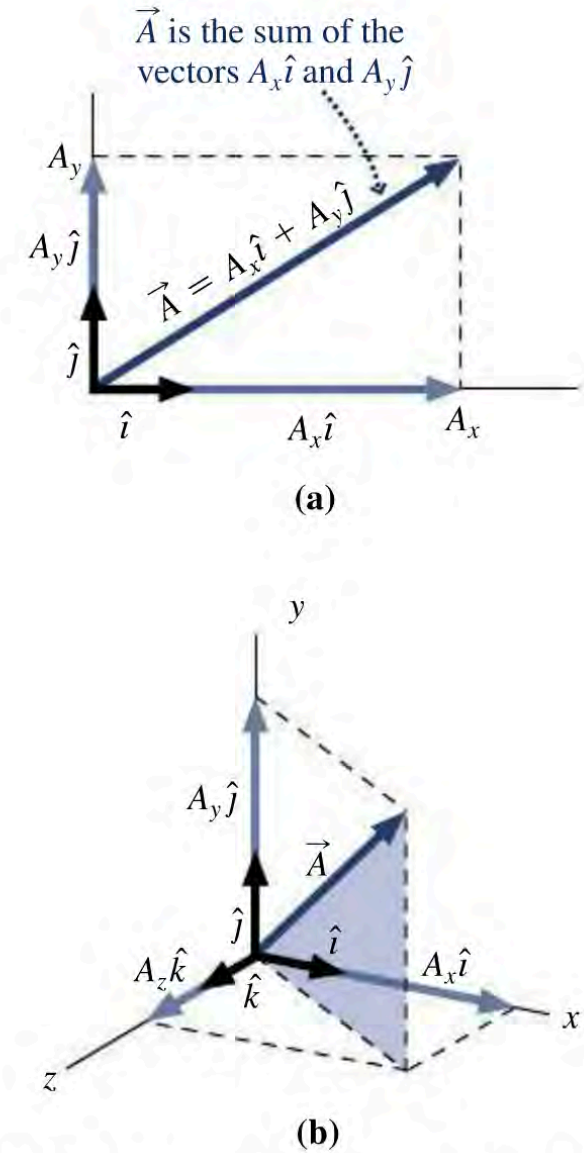

FIGURE 3.5 Vectors in (a) a plane and (b) space, expressed using unit vectors.

Vector Components & Unit Vectors

To summarize:
[2-D] Two pieces of information can be expressed in different ways:

- x—y coordinates
- magnitude & direction (i.e., phase)
- component vectors
- components tied to unit vectors

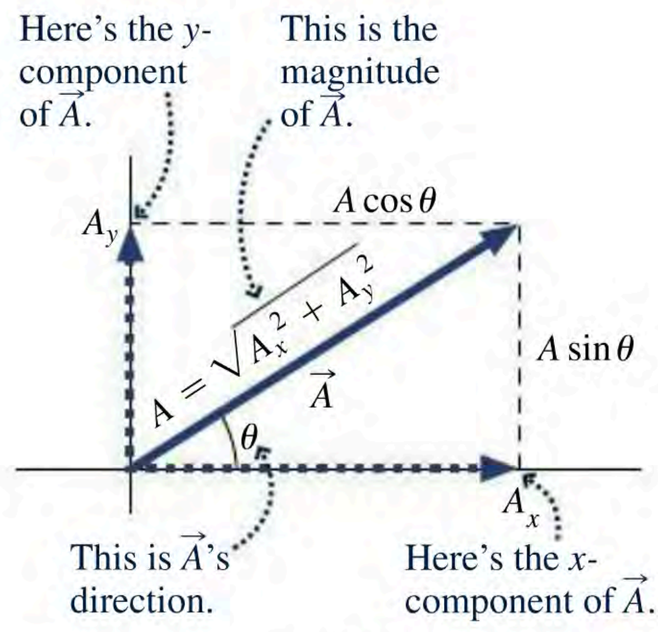

FIGURE 3.4 Magnitude/direction and component representations of vector $\vec{A}$.

A dummy's guide to "component vectors":

TACTICS BOX 3.1 **Determining the components of a vector**

1. The absolute value $|A_x|$ of the x-component A_x is the magnitude of the component vector $\vec{A}_x$.
2. The *sign* of A_x is positive if $\vec{A}_x$ points in the positive x-direction, negative if $\vec{A}_x$ points in the negative x-direction.
3. The y-component A_y is determined similarly.

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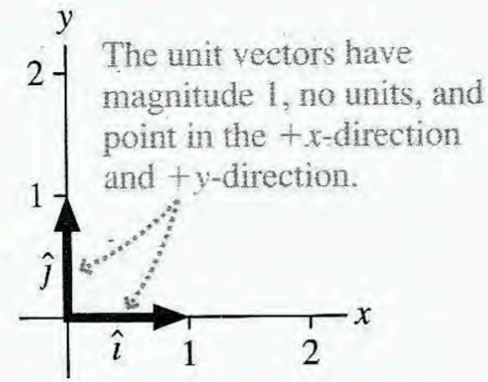
Vector Components & Unit Vectors

FIGURE 3.17 The unit vectors $\hat{\imath}$ and $\hat{\jmath}$.



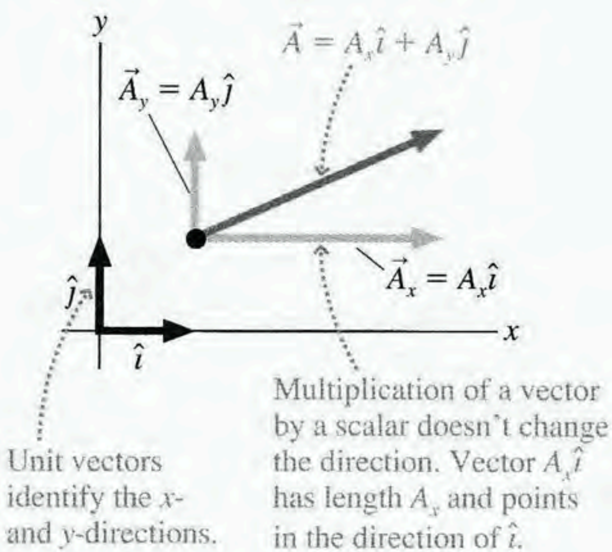
Dealing w/ unit vectors

$$\hat{\imath} \equiv (1, \text{positive } x\text{-direction})$$

$$\hat{\jmath} \equiv (1, \text{positive } y\text{-direction})$$

The notation $\hat{\imath}$ (read "i hat") and $\hat{\jmath}$ (read "j hat") indicates a unit vector with a magnitude of 1. Recall that the symbol $\equiv$ means "is defined as."

FIGURE 3.18 The decomposition of vector $\vec{A}$ is $A_x\hat{\imath} + A_y\hat{\jmath}$.



$$\vec{A}_x = A_x\hat{\imath}$$

$$\vec{A}_y = A_y\hat{\jmath}$$

$$\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{\imath} + A_y\hat{\jmath}$$

→ These are just different ways of expressing the same thing

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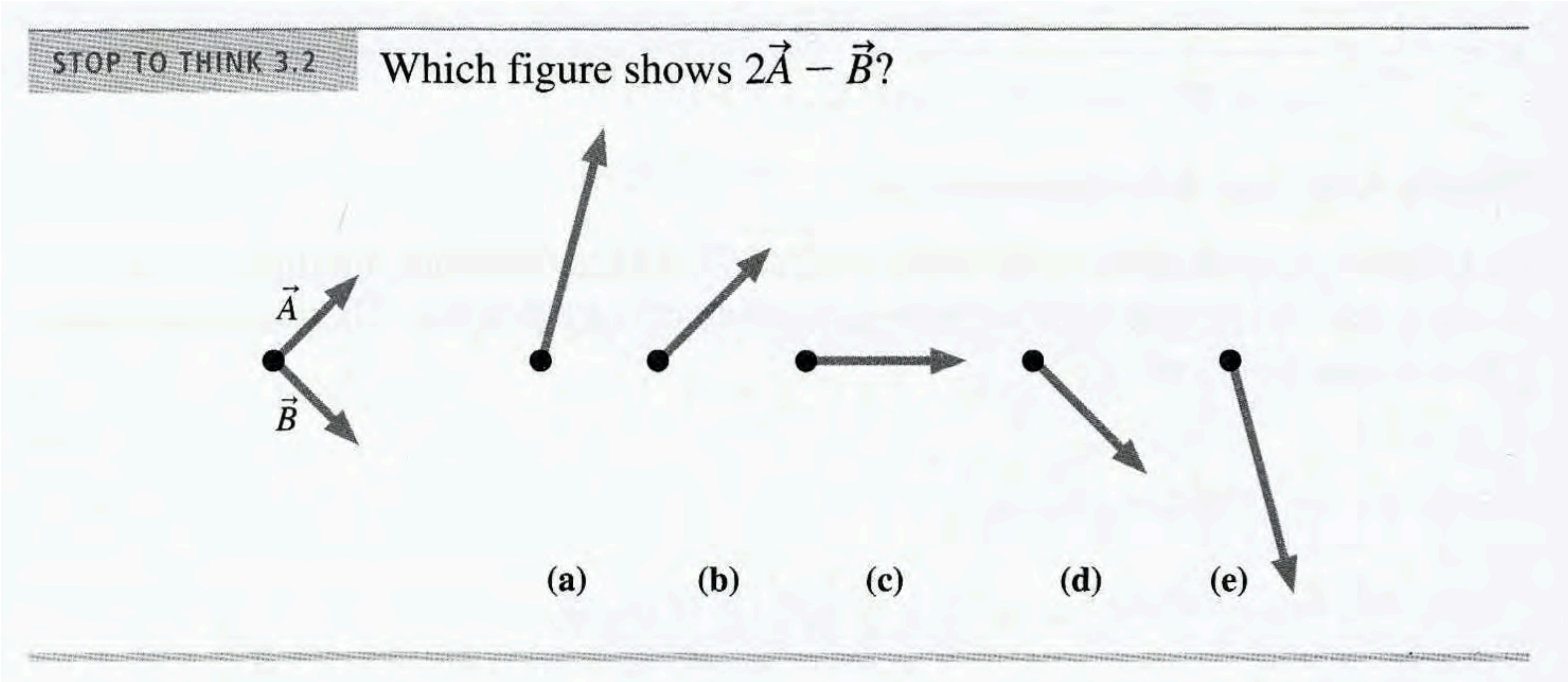
Ex. (REVISTED)



- Much easier when using component notation!

$$\bar{A} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \bar{E} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad 2\bar{A}\text{-}\bar{E} = \begin{bmatrix} 2 - 1 \\ 2 - (-1) \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

→ Same as a!

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Connecting Vectors to Mechanics

3.2 Velocity and Acceleration Vectors

We defined velocity in one dimension as the rate of change of position. In two or three dimensions it's the same thing, except now the change in position—displacement—is a vector. So we write

$$\bar{\vec{v}} = \frac{\Delta \vec{r}}{\Delta t} \quad \text{(average velocity vector)} \tag{3.3}$$

for the average velocity, in analogy with Equation 2.1. Here division by Δt simply means multiplying by $1/\Delta t$. As before, instantaneous velocity is given by a limiting process:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \quad \text{(instantaneous velocity vector)} \tag{3.4}$$

Again, that derivative $d\vec{r}/dt$ is shorthand for the result of the limiting process, taking ever smaller time intervals Δt and the corresponding displacements $\Delta \vec{r}$. Another way to look at Equation 3.4 is in terms of components. If $\vec{r} = x\hat{\imath} + y\hat{\jmath}$, then we can write

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} = v_x\hat{\imath} + v_y\hat{\jmath}$$

where the velocity components v_x and v_y are the derivatives of the position components.

Acceleration is the rate of change of velocity, so we write

$$\bar{\vec{a}} = \frac{\Delta \vec{v}}{\Delta t} \quad \text{(average acceleration vector)} \tag{3.5}$$

for the average acceleration and

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \quad \text{(instantaneous acceleration vector)} \tag{3.6}$$

for the instantaneous acceleration. We can also express instantaneous acceleration in components, as we did for velocity:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} = a_x\hat{\imath} + a_y\hat{\jmath}$$

→ Our previous (1-D) expressions can be generalized to 2-D (or higher) via vector notation

$$\vec{v} = \vec{v}_0 + \vec{a}t \quad \text{(for constant acceleration only)} \tag{3.8}$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2 \quad \text{(for constant acceleration only)} \tag{3.9}$$

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Connecting Vectors to Mechanics

$$\vec{v} = \vec{v}_0 + \vec{a}t \qquad \text{(for constant acceleration only)} \qquad (3.8)$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2 \qquad \text{(for constant acceleration only)} \qquad (3.9)$$

→ You should be comfortable bouncing back & forth between the both versions of the eqns.

$$\left.\begin{aligned} v_x &= v_{x0} \\ v_y &= v_{y0} - gt \\ x &= x_0 + v_{x0}t \\ y &= y_0 + v_{y0}t - \tfrac{1}{2}gt^2 \end{aligned}\right\} \text{(for constant gravitational acceleration)}$$

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(independence of) Vector components

$$\vec{v} = \vec{v}_0 + \vec{a}t \qquad \text{(for constant acceleration only)} \qquad (3.8)$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2 \qquad \text{(for constant acceleration only)} \qquad (3.9)$$

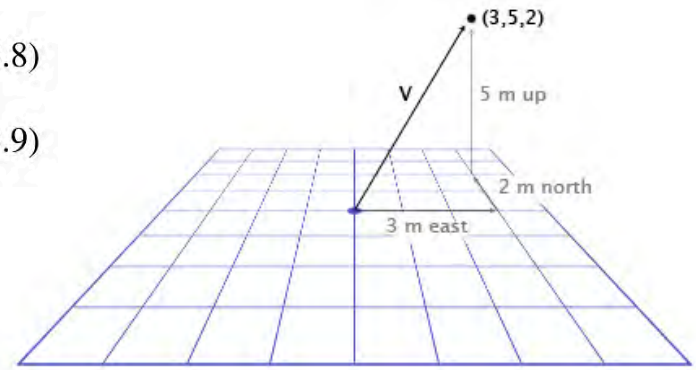

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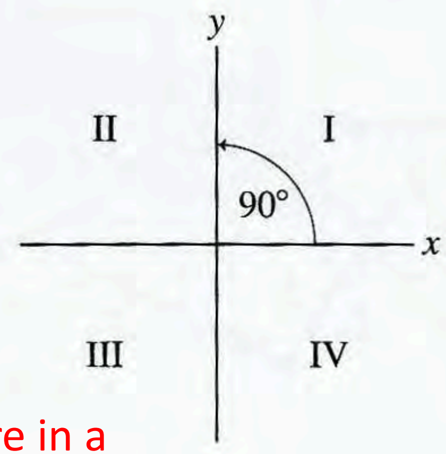
FIGURE 3.9 A conventional *xy*-coordinate system and the quadrants of the *xy*-plane.



Key Conceptual Point

Even though the different components are related (but virtue of being *coupled* together into the same vector), they are inherently independent from one another….

… just as x and y are in a Cartesian coord system (independence of) Vector components

(rough) Analogy: To meet someone in Toronto, you need specify the cross-streets (i.e., x and y), the building floor (z), and a time (t)