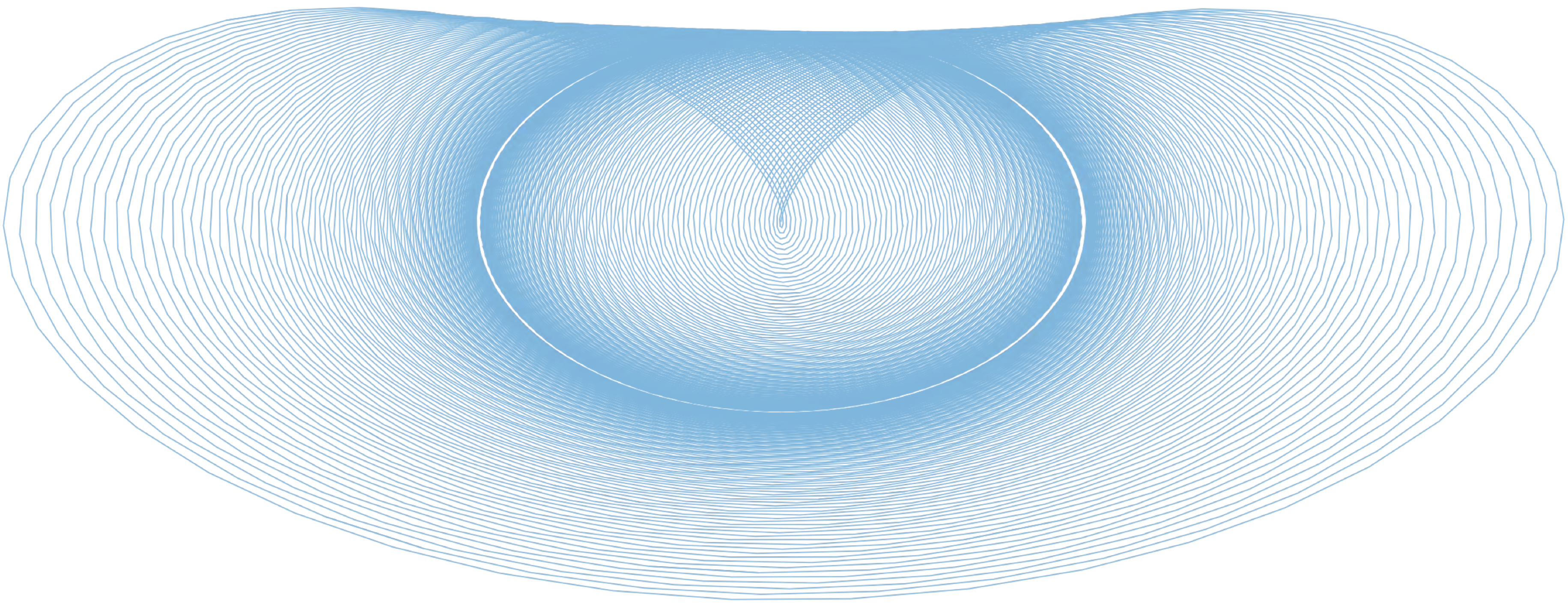
PHYS 1420 (F19)
Physics with Applications to Life Sciences



2019.09.16

Relevant reading:

Kesten & Tauck ch.3.4

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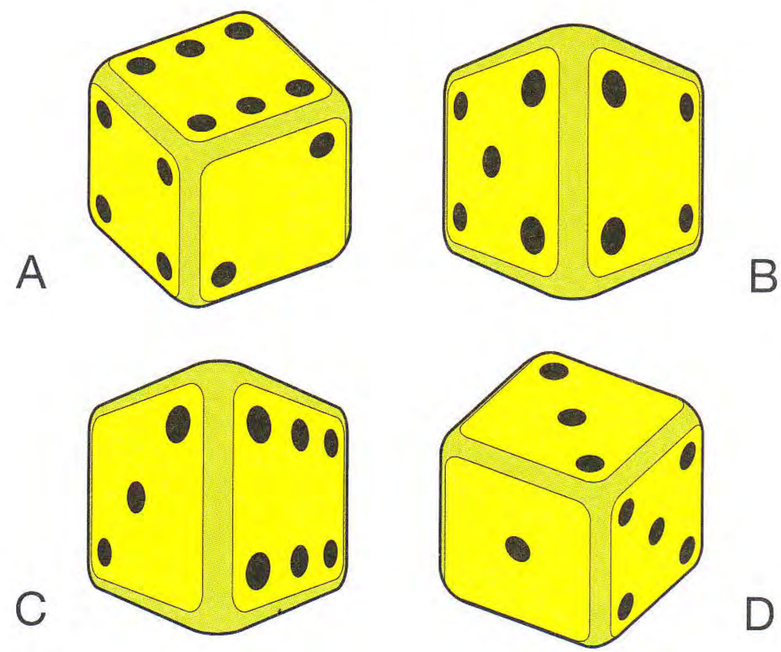
Ref. (re images):

Wolfson (2007), Knight (2017)

77. Four Dice



The four dice are identical. Which one does not belong?

A B C D

Announcements & Key Concepts (re Today)

→ (online) HW #3 posted → Due next Monday, 9/23

→ (written) HW #1 (posted later today) → Due next Wednesday, 9/25

→ Labs: Start this week! (Sept.16-20)
https://www.yorku.ca/menary/courses/firstyrlabs/2019/main.html

Some relevant underlying concepts of the day…

- 2-D kinematics: vector "components" of x, v, and a

- projectile motion

Connecting Vectors to Mechanics

3.2 Velocity and Acceleration Vectors

We defined velocity in one dimension as the rate of change of position. In two or three dimensions it's the same thing, except now the change in position—displacement—is a vector. So we write

$$\bar{\vec{v}} = \frac{\Delta \vec{r}}{\Delta t} \quad \text{(average velocity vector)} \tag{3.3}$$

for the average velocity, in analogy with Equation 2.1. Here division by Δt simply means multiplying by $1/\Delta t$. As before, instantaneous velocity is given by a limiting process:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \quad \text{(instantaneous velocity vector)} \tag{3.4}$$

Again, that derivative $d\vec{r}/dt$ is shorthand for the result of the limiting process, taking ever smaller time intervals Δt and the corresponding displacements $\Delta \vec{r}$. Another way to look at Equation 3.4 is in terms of components. If $\vec{r} = x\hat{\imath} + y\hat{\jmath}$, then we can write

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} = v_x\hat{\imath} + v_y\hat{\jmath}$$

where the velocity components v_x and v_y are the derivatives of the position components.

Acceleration is the rate of change of velocity, so we write

$$\bar{\vec{a}} = \frac{\Delta \vec{v}}{\Delta t} \quad \text{(average acceleration vector)} \tag{3.5}$$

for the average acceleration and

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \quad \text{(instantaneous acceleration vector)} \tag{3.6}$$

for the instantaneous acceleration. We can also express instantaneous acceleration in components, as we did for velocity:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} = a_x\hat{\imath} + a_y\hat{\jmath}$$

→ Our previous (1-D) expressions can be generalized to 2-D (or higher) via vector notation

$$\vec{v} = \vec{v}_0 + \vec{a}t \quad \text{(for constant acceleration only)} \tag{3.8}$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2 \quad \text{(for constant acceleration only)} \tag{3.9}$$

Wolfson

Connecting Vectors to Mechanics

$$\vec{v} = \vec{v}_0 + \vec{a}t \qquad \text{(for constant acceleration only)} \qquad (3.8)$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2 \qquad \text{(for constant acceleration only)} \qquad (3.9)$$

→ You should be comfortable bouncing back & forth between the both versions of the eqns.

$$\left.\begin{aligned} v_x &= v_{x0} \\ v_y &= v_{y0} - gt \\ x &= x_0 + v_{x0}t \\ y &= y_0 + v_{y0}t - \tfrac{1}{2}gt^2 \end{aligned}\right\} \begin{array}{l}\text{(for constant} \\ \text{gravitational} \\ \text{acceleration)}\end{array}$$

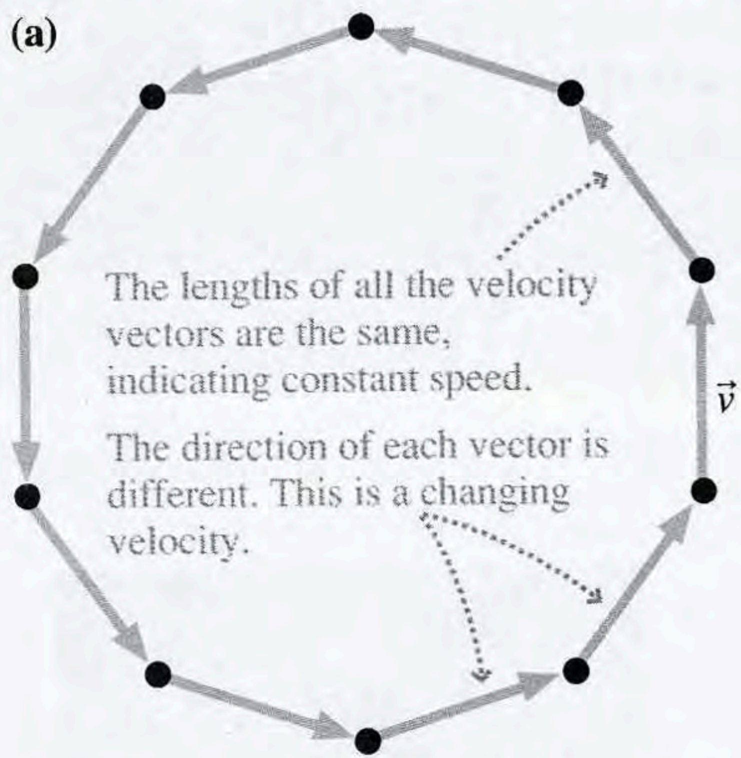
Wolfson

Ex.

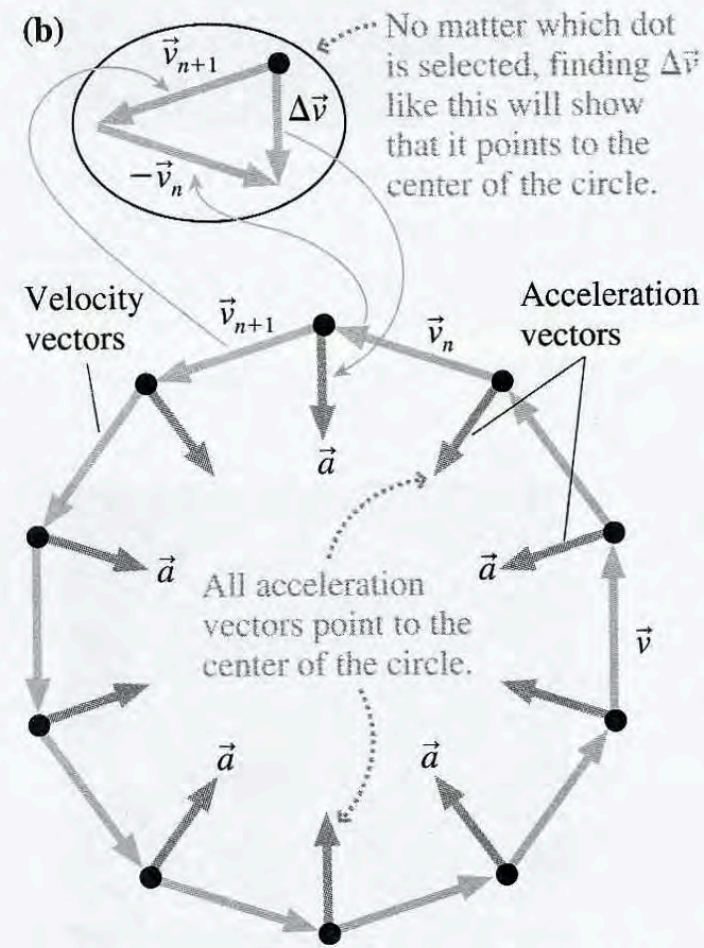
George Washington Gale Ferris Jr.

A classic example: “Maria” riding a Ferris wheel

FIGURE 4.1 Using Tactics Box 4.1 to find Maria’s acceleration on the Ferris wheel.



Maria moves at constant speed but *not* at constant velocity. Thus she is accelerating.



Maria’s acceleration is an acceleration of changing direction, not of changing speed.

Knight

Ex.

STOP TO THINK 4.1 This acceleration will cause the particle to

a. Speed up and curve upward.
b. Speed up and curve downward.
c. Slow down and curve upward.
d. Slow down and curve downward.
e. Move to the right and down.
f. Reverse direction.

Ex. (SOL)

STOP TO THINK 4.1 This acceleration will cause the particle to

a. Speed up and curve upward.
b. Speed up and curve downward.
c. Slow down and curve upward.
d. Slow down and curve downward.
e. Move to the right and down.
f. Reverse direction.

d

Projectile Motion

- Ok, now we have the pieces in place to turn a hard problem into an easier one...

Akira Kurosawa's Throne of Blood

Katniss Everdeen

Daryl Dixon

Projectile Motion

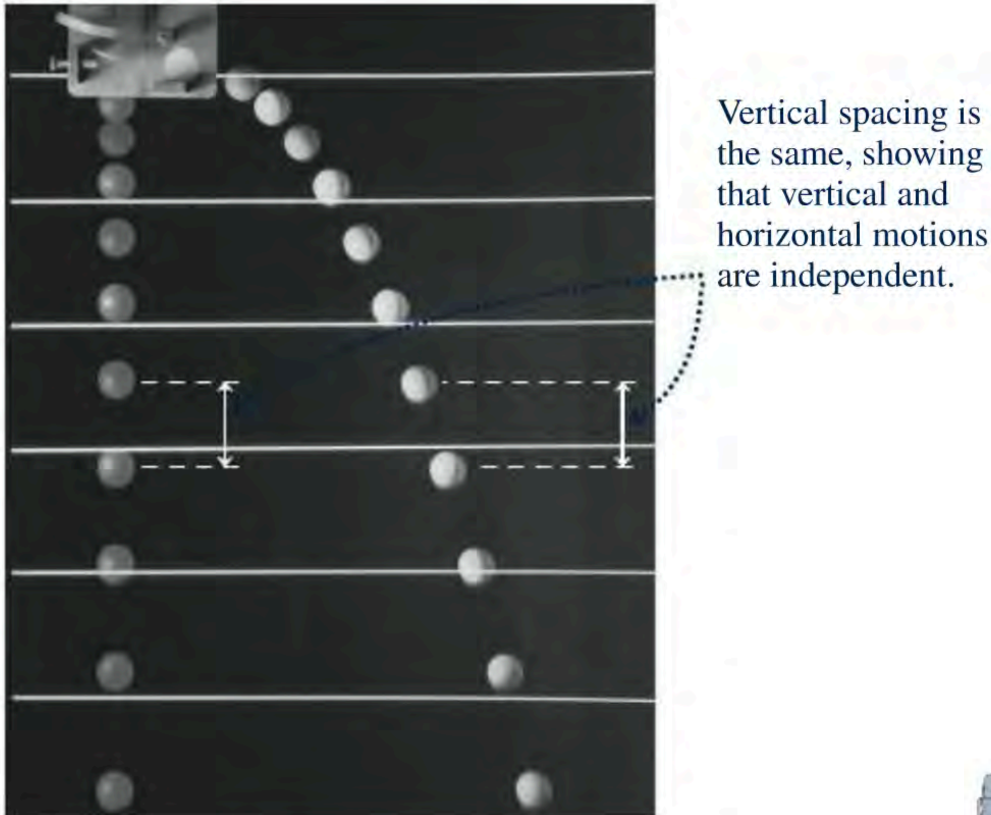


FIGURE 3.11 Two marbles, one dropped and the other projected horizontally.

Wolfson

Classic example of how the x and y components of motion are independent (as gravity only works along the vertical here)

Remarkably, the same principle applies for a bullet fired from a gun

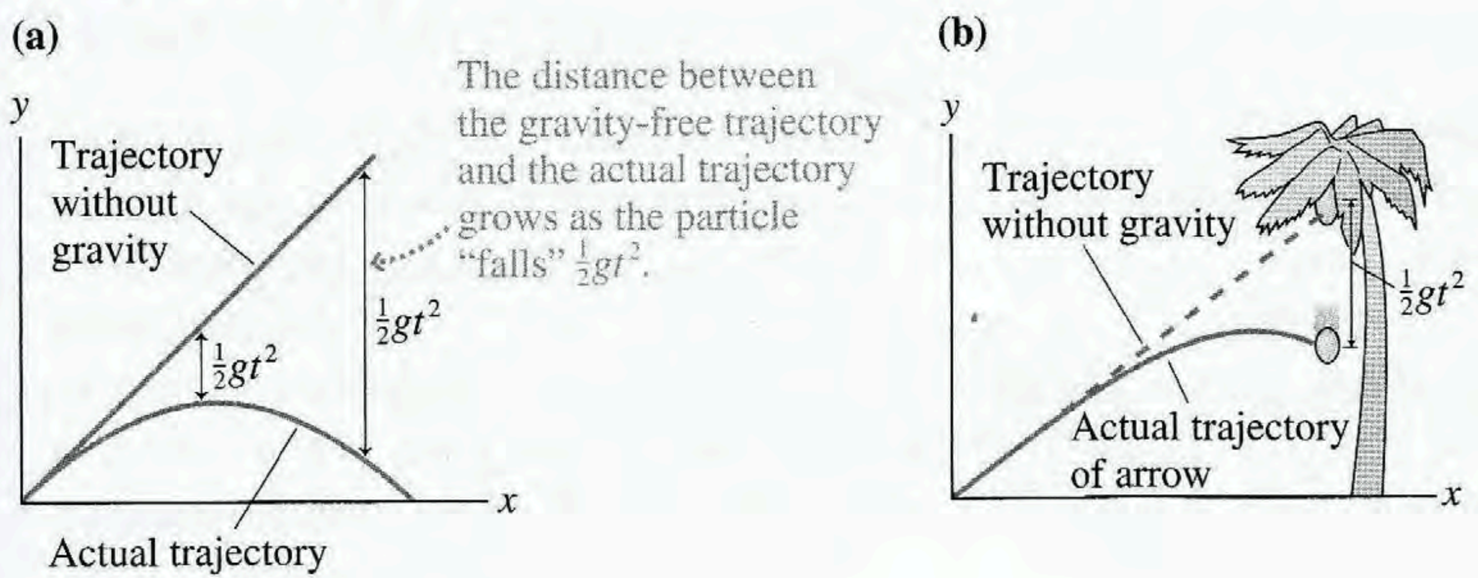
(i.e., it takes the same amount of time for a bullet to fall to the ground independent of whether you shoot it or drop it!)

Be careful re intuition....

A hungry bow-and-arrow hunter in the jungle wants to shoot down a coconut that is hanging from the branch of a tree. He points his arrow directly at the coconut, but as luck would have it, the coconut falls from the branch at the *exact* instant the hunter releases the string. Does the arrow hit the coconut?

Yes, it does(!!!). Why/how?

FIGURE 4.18 A projectile follows a parabolic trajectory because it "falls" a distance $\frac{1}{2}gt^2$ below a straight-line trajectory.



Knight

Projectile Motion

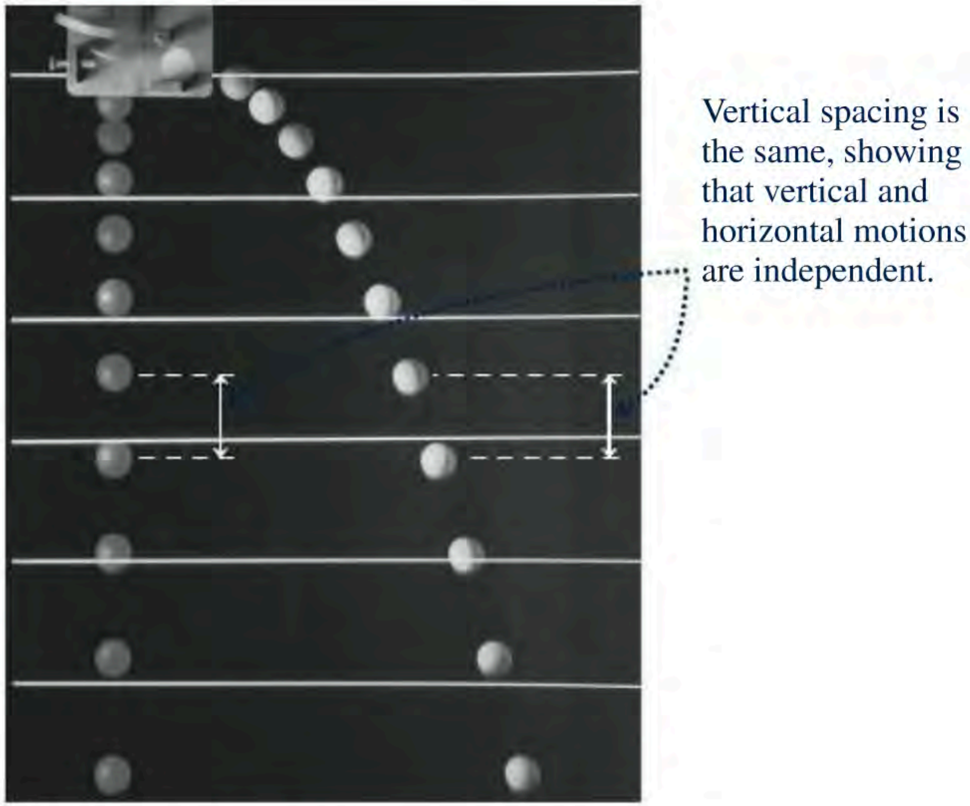


FIGURE 3.11 Two marbles, one dropped and the other projected horizontally.

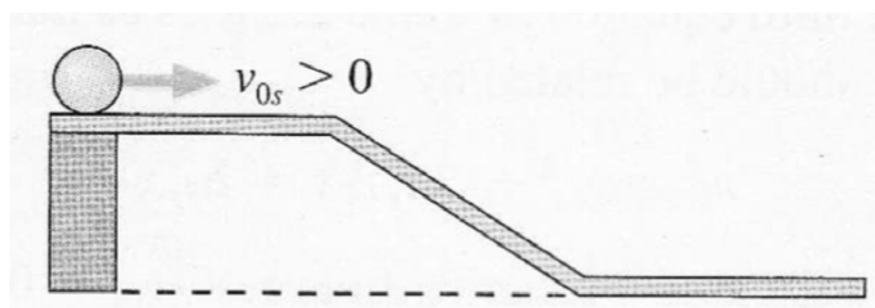
Much easier to study a jet of water than a falling ball (they behave the same!)

FIGURE 3.14 Water droplets–each an individual projectile–combine to form graceful parabolic arcs in this fountain.

Wolfson

Our ball on the track is a bit different. Why?

(we'll come back to this soon)



Projectile Motion

Vector representation of the problem

FIGURE 4.14 A projectile launched with initial velocity $\vec{v}_0$.

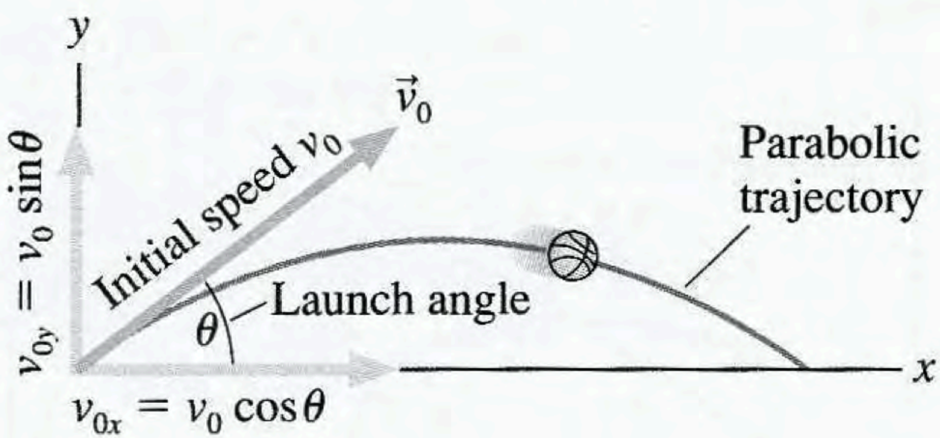
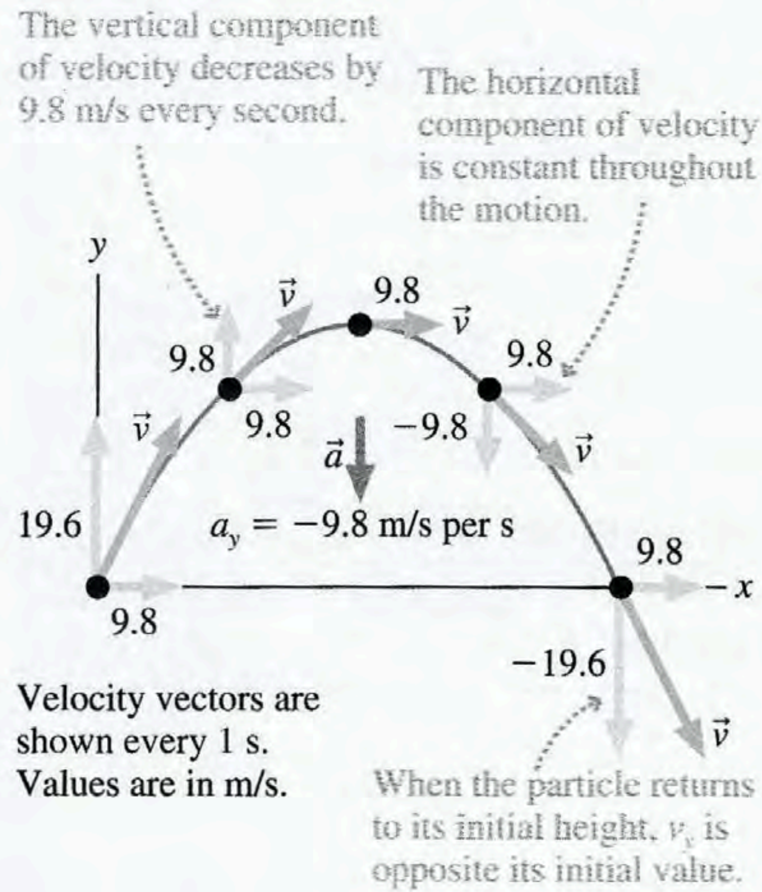


FIGURE 4.15 The velocity and acceleration vectors of a projectile moving along a parabolic trajectory.



Breaking it up into components

Knight

Projectile Motion

$$x = \frac{v_0^{\,2}}{g}\sin 2\theta_0 \qquad \text{(horizontal range)}$$

$$y = x\tan\theta_0 - \frac{g}{2v_0^{\,2}\cos^2\theta_0}x^2 \qquad \text{(projectile trajectory)}$$

→ You should feel comfortable deriving these formulae...

Note: $\sin 2A = 2\sin A\cos A$

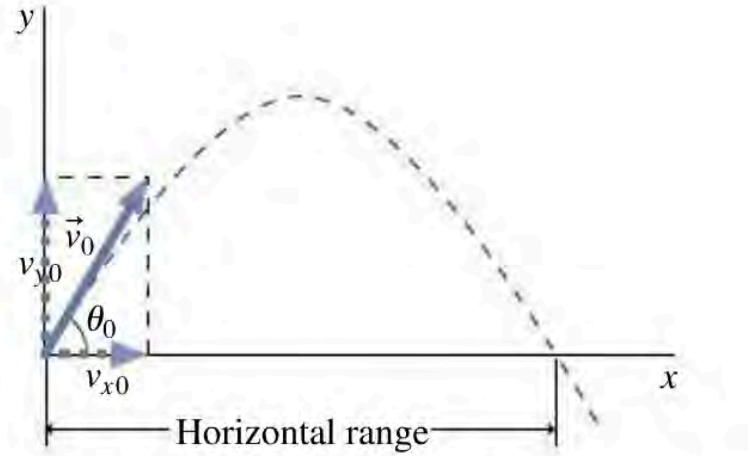
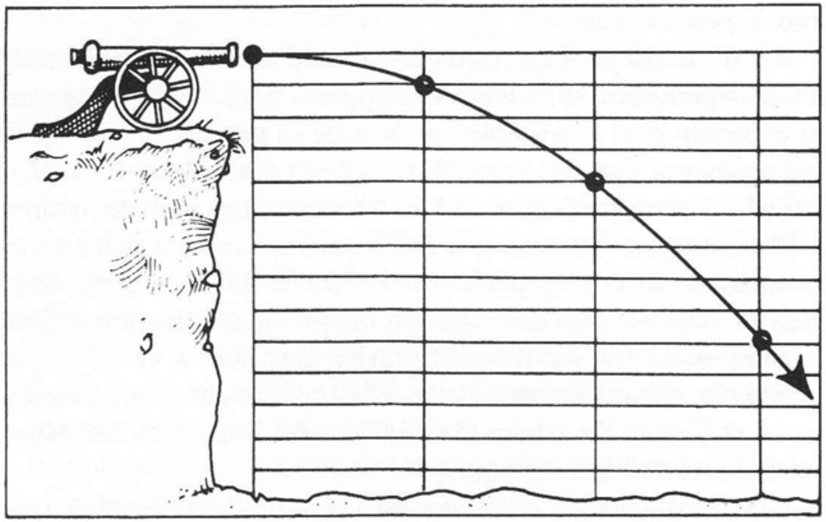


FIGURE 3.17 Parabolic trajectory of a projectile.

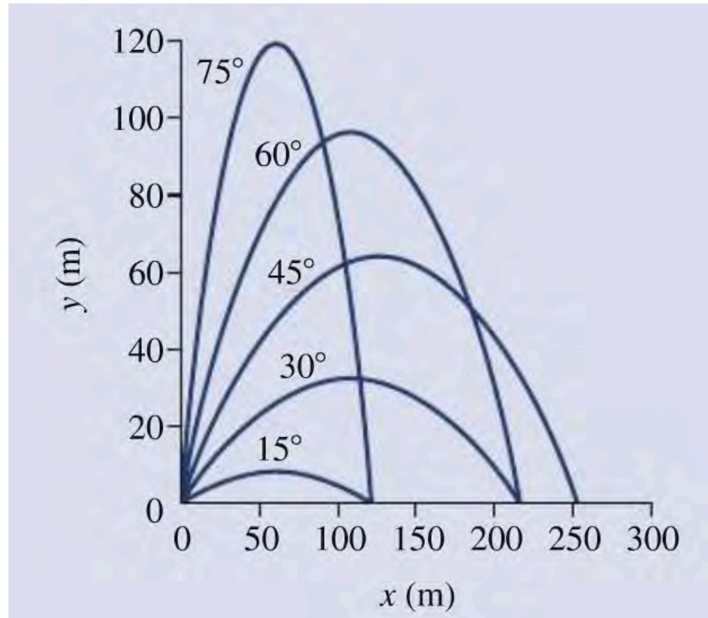
Wolfson



... and Tartaglia's discovery should also be readily apparent

Projectile Motion

$$x = \frac{{v_0}^2}{g}\sin 2\theta_0 \qquad \text{(horizontal range)}$$



Now you have the basic pieces & w/ some practice, we can do harder problems:

- What happens if there is “drag” (i.e., air resistance)?
- Circular motion...
- Force, energy, and all that....
- How does one launch a rocket to Mars?

Ex.

A bird flies 100 m due east from a tree, then 200 m northwest (that is, 45° north of west). What is the bird's net displacement?

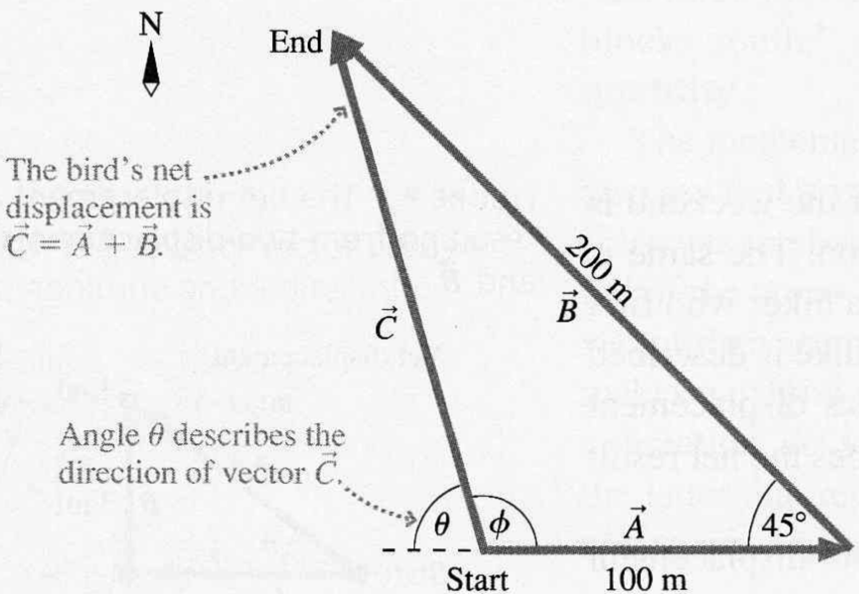
Knight

Ex. (SOL)

A bird flies 100 m due east from a tree, then 200 m northwest (that is, 45° north of west). What is the bird's net displacement?

VISUALIZE **FIGURE 3.4** shows the two individual displacements, which we've called $\vec{A}$ and $\vec{B}$. The net displacement is the vector sum $\vec{C} = \vec{A} + \vec{B}$, which is found graphically.

FIGURE 3.4 The bird's net displacement is $\vec{C} = \vec{A} + \vec{B}$.



SOLVE The two displacements are $\vec{A} = (100 \text{ m, east})$ and $\vec{B} = (200 \text{ m, northwest})$. The net displacement $\vec{C} = \vec{A} + \vec{B}$ is found by drawing a vector from the initial to the final position. But describing $\vec{C}$ is a bit trickier than the example of the hiker because $\vec{A}$ and $\vec{B}$ are not at right angles. First, we can find the magnitude of $\vec{C}$ by using the law of cosines from trigonometry:

$$\begin{aligned} C^2 &= A^2 + B^2 - 2AB\cos 45^\circ \\ &= (100 \text{ m})^2 + (200 \text{ m})^2 - 2(100 \text{ m})(200 \text{ m})\cos 45^\circ \\ &= 21{,}720 \text{ m}^2 \end{aligned}$$

Thus $C = \sqrt{21{,}720 \text{ m}^2} = 147$ m. Then a second use of the law of cosines can determine angle ϕ (the Greek letter phi):

$$B^2 = A^2 + C^2 - 2AC\cos\phi$$

$$\phi = \cos^{-1}\left[\frac{A^2 + C^2 - B^2}{2AC}\right] = 106^\circ$$

It is easier to describe $\vec{C}$ with the angle $\theta = 180^\circ - \phi = 74^\circ$. The bird's net displacement is

$$\vec{C} = (147 \text{ m, } 74^\circ \text{ north of west})$$

Knight

Ex.

Carolyn drives her car north at 30 km/h for 1 hour, east at 60 km/h for 2 hours, then north at 50 km/h for 1 hour. What is Carolyn's net displacement?

Ex. (SOL)

Carolyn drives her car north at 30 km/h for 1 hour, east at 60 km/h for 2 hours, then north at 50 km/h for 1 hour. What is Carolyn's net displacement?

SOLVE Chapter 1 defined velocity as

$$\vec{v} = \frac{\Delta \vec{r}}{\Delta t}$$

so the displacement $\Delta \vec{r}$ during the time interval Δt is $\Delta \vec{r} = (\Delta t)\vec{v}$. This is multiplication of the vector $\vec{v}$ by the scalar Δt. Carolyn's velocity during the first hour is $\vec{v}_1 = (30 \text{ km/h, north})$, so her displacement during this interval is

$$\Delta \vec{r}_1 = (1 \text{ hour})(30 \text{ km/h, north}) = (30 \text{ km, north})$$

Similarly,

$$\Delta \vec{r}_2 = (2 \text{ hours})(60 \text{ km/h, east}) = (120 \text{ km, east})$$

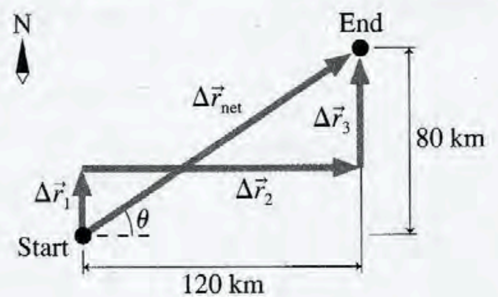
$$\Delta \vec{r}_3 = (1 \text{ hour})(50 \text{ km/h, north}) = (50 \text{ km, north})$$

In this case, multiplication by a scalar changes not only the length of the vector but also its units, from km/h to km. The direction, however, is unchanged. Carolyn's net displacement is

$$\Delta \vec{r}_{\text{net}} = \Delta \vec{r}_1 + \Delta \vec{r}_2 + \Delta \vec{r}_3$$

This addition of the three vectors is shown in FIGURE 3.8, using the tip-to-tail method. $\Delta \vec{r}_{\text{net}}$ stretches from Carolyn's initial position

FIGURE 3.8 The net displacement is the vector sum $\Delta \vec{r}_{\text{net}} = \Delta \vec{r}_1 + \Delta \vec{r}_2 + \Delta \vec{r}_3$.



to her final position. The magnitude of her net displacement is found using the Pythagorean theorem:

$$r_{\text{net}} = \sqrt{(120 \text{ km})^2 + (80 \text{ km})^2} = 144 \text{ km}$$

The direction of $\Delta \vec{r}_{\text{net}}$ is described by angle θ, which is

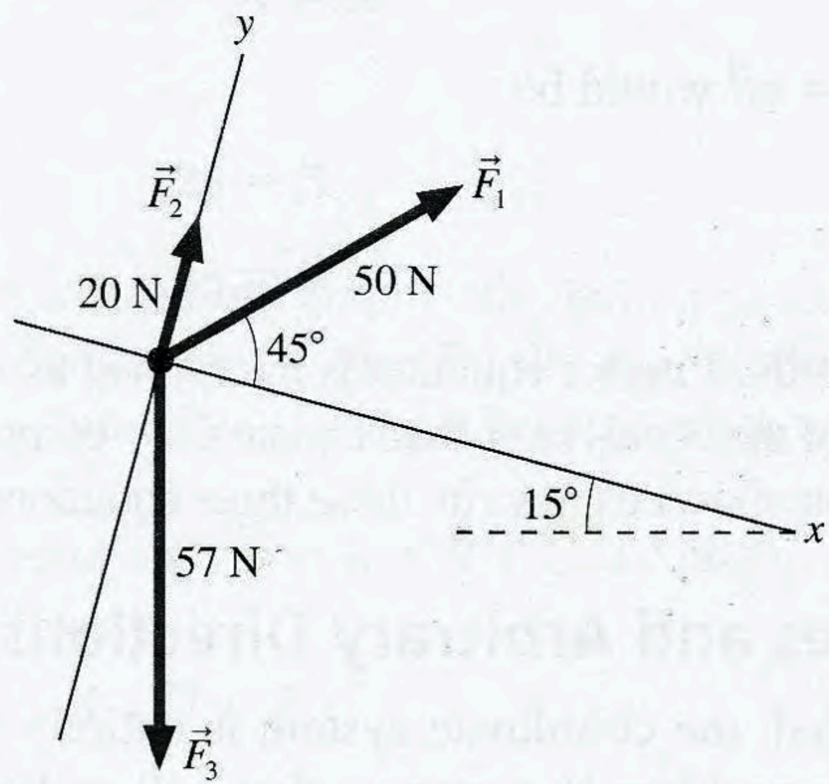
$$\theta = \tan^{-1}\left(\frac{80 \text{ km}}{120 \text{ km}}\right) = 34°$$

Thus Carolyn's net displacement is $\Delta \vec{r}_{\text{net}} = (144 \text{ km, } 34° \text{ north of east})$.

Ex.

FIGURE 3.23 shows three forces acting at one point. What is the net force $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$?

FIGURE 3.23 Three forces.



Note: Be a bit carful here. Notice that the coord. system is tilted....

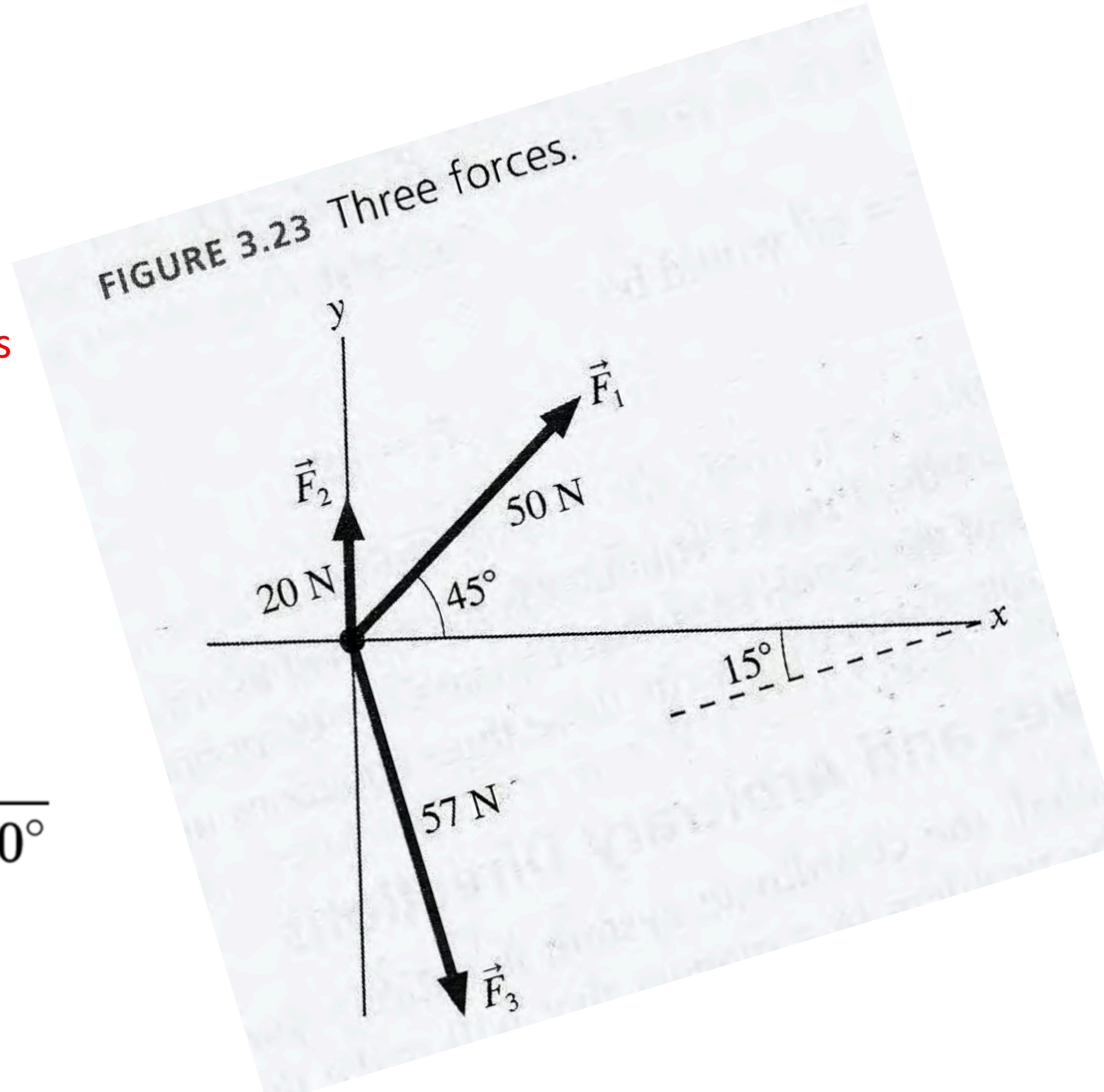
Ex.

One strategy is to "rotate" the given coordinate system…

… and then re-express the vectors

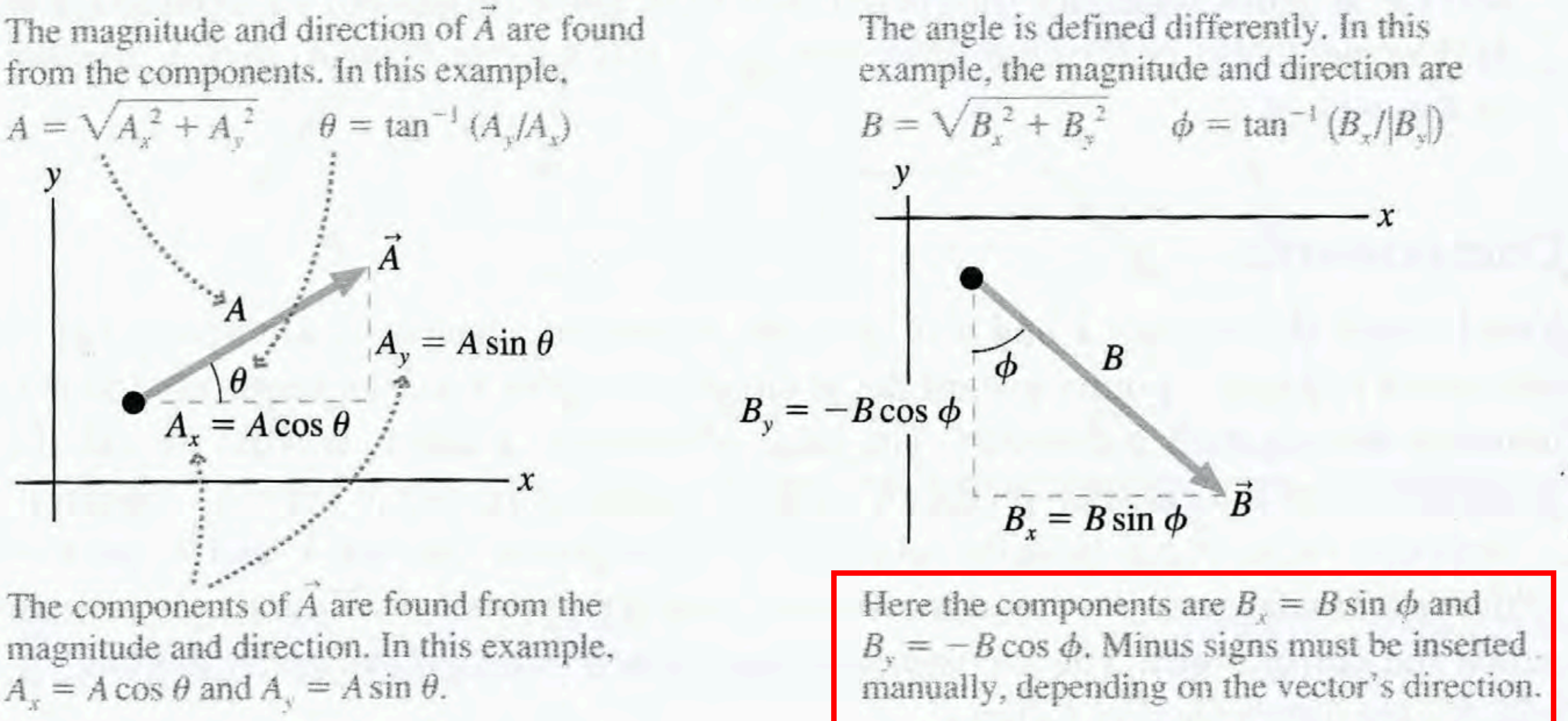
Note: Be careful w/ angle in degrees versus radians

$$\textbf{radians} = \textbf{degrees} \cdot \frac{\pi}{180^\circ}$$



Vector Components & Unit Vectors

FIGURE 3.12 Moving between the geometric representation and the component representation.

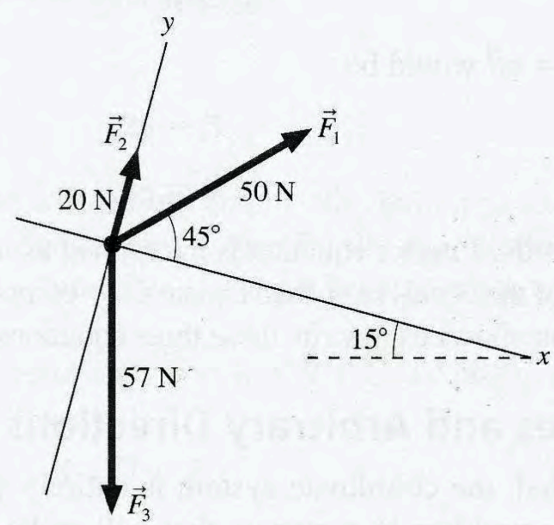


→ Be careful! How you define the angle and/or keep track of "bookkeeping" can have huge consequences!

Knight

Ex. (SOL)

FIGURE 3.23 Three forces.



VISUALIZE Figure 3.23 show the forces and establishes a tilted coordinate system.

SOLVE The vector equation $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$ is really two simultaneous equations:

$$(F_{net})_x = F_{1x} + F_{2x} + F_{3x}$$

$$(F_{net})_y = F_{1y} + F_{2y} + F_{3y}$$

The components of the forces are determined with respect to the axes. Thus

$$F_{1x} = F_1 \cos 45° = (50 \text{ N}) \cos 45° = 35 \text{ N}$$

$$F_{1y} = F_1 \sin 45° = (50 \text{ N}) \sin 45° = 35 \text{ N}$$

$\vec{F}_2$ is easier. It is pointing along the y-axis, so $F_{2x} = 0$ N and $F_{2y} = 20$ N. To find the components of $\vec{F}_3$, we need to recognize—because $\vec{F}_3$ points straight down—that the angle between $\vec{F}_3$ and the x-axis is 75°. Thus

$$F_{3x} = F_3 \cos 75° = (57 \text{ N}) \cos 75° = 15 \text{ N}$$

$$F_{3y} = -F_3 \sin 75° = -(57 \text{ N}) \sin 75° = -55 \text{ N}$$

The minus sign in F_{3y} is critical, and it appears not from some formula but because we recognized—from the figure—that the y-component of $\vec{F}_3$ points in the $-y$-direction. Combining the pieces, we have

$$(F_{net})_x = 35 \text{ N} + 0 \text{ N} + 15 \text{ N} = 50 \text{ N}$$

$$(F_{net})_y = 35 \text{ N} + 20 \text{ N} + (-55 \text{ N}) = 0 \text{ N}$$

Thus the net force is $\vec{F}_{net} = 50\hat{\imath}$ N. It points along the x-axis of the tilted coordinate system.

ASSESS Notice that all work was done with reference to the axes of the coordinate system, not with respect to vertical or horizontal.

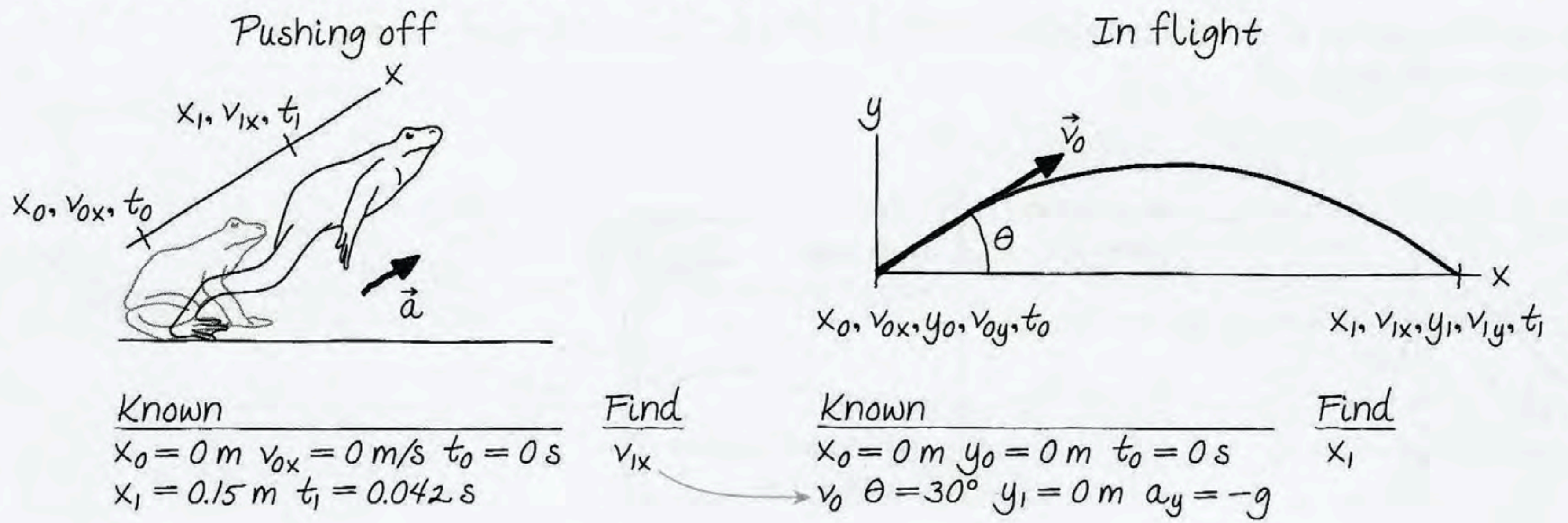
Ex.

Frogs, with their long, strong legs, are excellent jumpers. And thanks to the good folks of Calaveras County, California, who have a jumping frog contest every year in honor of a Mark Twain story, we have very good data on how far a determined frog can jump.

High-speed cameras show that a good jumper goes into a crouch, then rapidly extends his legs by typically 15 cm during a 42 ms push off, leaving the ground at a 30° angle. How far does this frog leap?

Ex. (SOL)

FIGURE 4.19 Pictorial representations of the jumping frog.



Knight

Ex. (SOL)

MODEL Represent the frog as a particle. Model the push off as linear motion with constant acceleration. A bullfrog is fairly heavy and dense, so ignore air resistance and consider the leap to be projectile motion.

VISUALIZE This is a two-part problem: linear acceleration followed by projectile motion. A key observation is that **the final velocity for pushing off the ground becomes the initial velocity of the projectile motion.** FIGURE 4.19 shows a separate pictorial representation for each part. Notice that we've used different coordinate systems for the two parts; coordinate systems are our choice, and for each part of the motion we've chosen the coordinate system that makes the problem easiest to solve.

SOLVE While pushing off, the frog travels 15 cm = 0.15 m in 42 ms = 0.042 s. We could find his speed at the end of pushing off if we knew the acceleration. Because the initial velocity is zero, we can find the acceleration from the position-acceleration-time kinematic equation:

$$x_1 = x_0 + v_{0x}\,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2 = \tfrac{1}{2}a_x(\Delta t)^2$$

$$a_x = \frac{2x_1}{(\Delta t)^2} = \frac{2(0.15\text{ m})}{(0.042\text{ s})^2} = 170\text{ m/s}^2$$

This is a substantial acceleration, but it doesn't last long. At the end of the 42 ms push off, the frog's velocity is

$$v_{1x} = v_{0x} + a_x\Delta t = (170\text{ m/s}^2)(0.042\text{ s}) = 7.14\text{ m/s}$$

We'll keep an extra significant figure here to avoid round-off error in the second half of the problem.

Knight

Ex. (SOL)

The end of the push off is the beginning of the projectile motion, so the second part of the problem is to find the distance of a projectile launched with velocity $\vec{v}_0 = (7.14 \text{ m/s}, 30°)$. The initial x- and y-components of the launch velocity are

$$v_{0x} = v_0 \cos\theta \qquad v_{0y} = v_0 \sin\theta$$

The kinematic equations of projectile motion, with $a_x = 0$ and $a_y = -g$, are

$$\begin{aligned} x_1 &= x_0 + v_{0x}\Delta t \\ &= (v_0 \cos\theta)\Delta t \\ y_1 &= y_0 + v_{0y}\Delta t - \tfrac{1}{2}g(\Delta t)^2 \\ &= (v_0 \sin\theta)\Delta t - \tfrac{1}{2}g(\Delta t)^2 \end{aligned}$$

We can find the time of flight from the vertical equation by setting $y_1 = 0$:

$$0 = (v_0 \sin\theta)\Delta t - \tfrac{1}{2}g(\Delta t)^2 = (v_0 \sin\theta - \tfrac{1}{2}g\,\Delta t)\Delta t$$

and thus

$$\Delta t = 0 \qquad \text{or} \qquad \Delta t = \frac{2v_0 \sin\theta}{g}$$

Both are legitimate solutions. The first corresponds to the instant when $y = 0$ at the launch, the second to when $y = 0$ as the frog hits the ground. Clearly, we want the second solution. Substituting this expression for Δt into the equation for x_1 gives

$$x_1 = (v_0 \cos\theta)\frac{2v_0 \sin\theta}{g} = \frac{2v_0^2 \sin\theta \cos\theta}{g}$$

We can simplify this result with the trigonometric identity $2\sin\theta\cos\theta = \sin(2\theta)$. Thus the distance traveled by the frog is

$$x_1 = \frac{v_0^2 \sin(2\theta)}{g}$$

Using $v_0 = 7.14$ m/s and $\theta = 30°$, we find that the frog leaps a distance of 4.5 m.

ASSESS 4.5 m is about 15 feet. This is much farther than a human can jump from a standing start, but it seems believable. In fact, the current record holder, Rosie the Ribeter, made a leap of 6.5 m!

Knight

Ex.

A bomb is dropped from an aeroplane flying horizontally at a constant speed. Where will the aeroplane be when the bomb hits the ground?

Shaskol'Skaya & El'Tsin (1963)

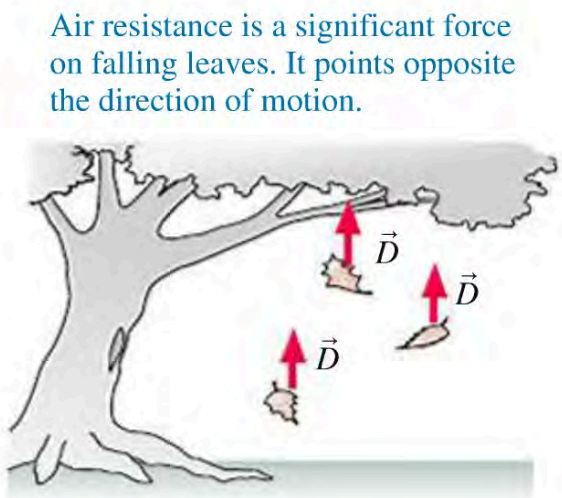
Ex. (SOL)

The plane flies horizontally with constant speed v. The bomb follows the path of a parabola, since its motion is compounded of horizontal motion with initial velocity v and uniformly accelerated vertical fall. If there were no air-resistance, the bomb's horizontal velocity would be no different from that of the plane, and the plane would be directly above the bomb the whole time—in particular, when the bomb hits the ground. But in fact, as a result of air-resistance, the bomb's horizontal velocity is decreasing all the time, and so it falls behind the plane (Fig. 163). Therefore the fall to earth and explosion of the bomb take place not underneath the plane, but considerably behind it.

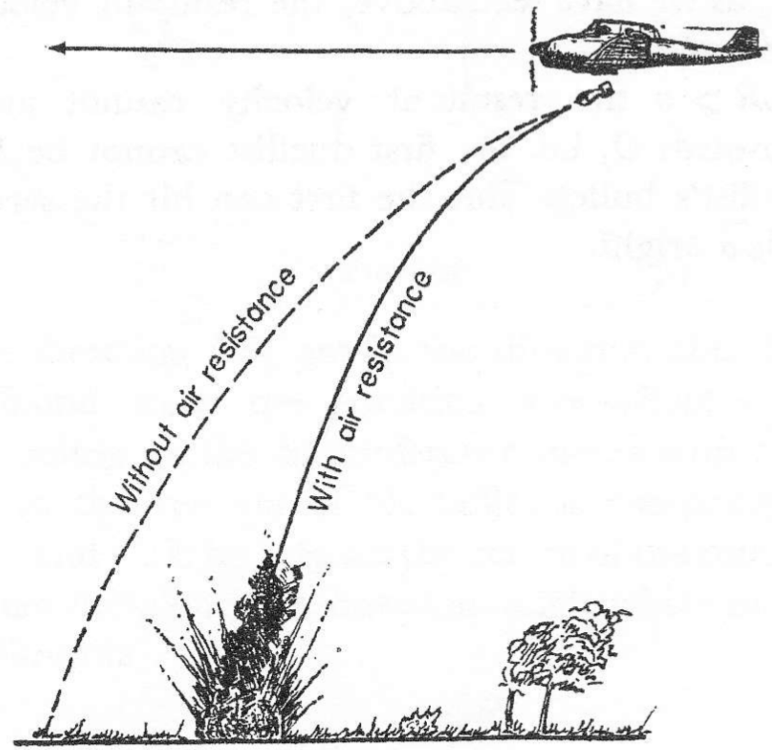
→ Be careful! Does "air resistance" matter?

(we'll return to this soon)

FIGURE 5.11 Air resistance is an example of drag.



Knight (2013)



Shaskol'Skaya & El'Tsin (1963)

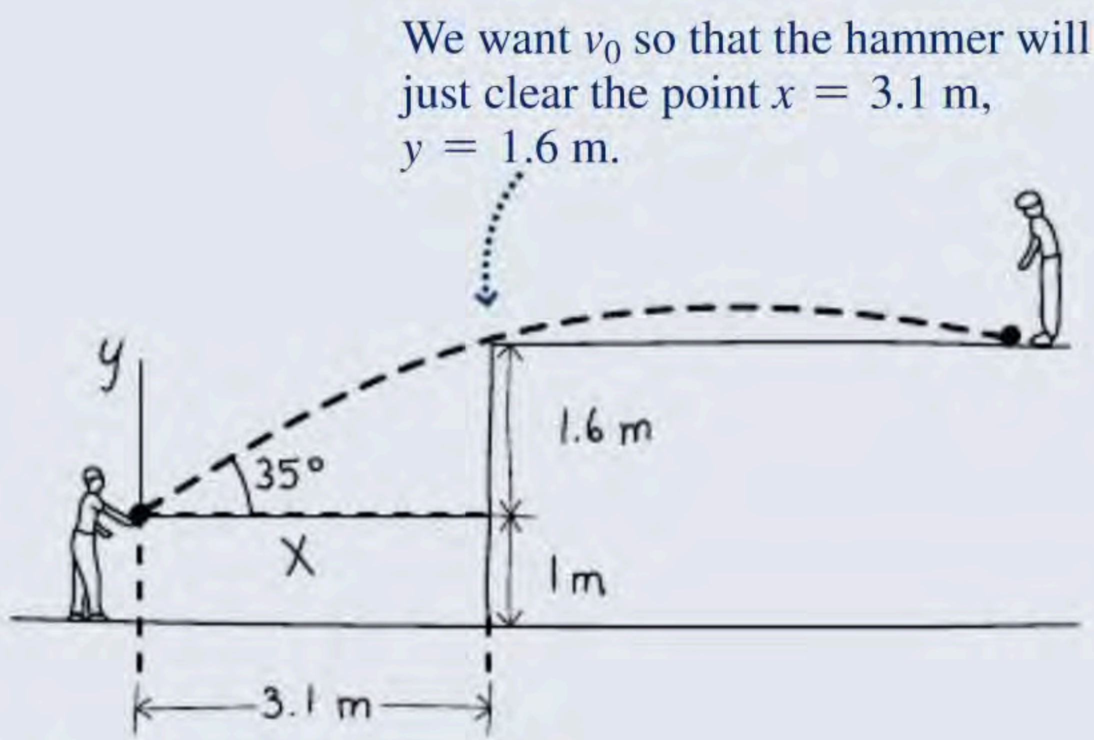


FIGURE 3.18 Our sketch for Example 3.5.

→ Practice these "projectile motion" problems, keeping careful track of what assumptions are stated (or need to be presumed!)