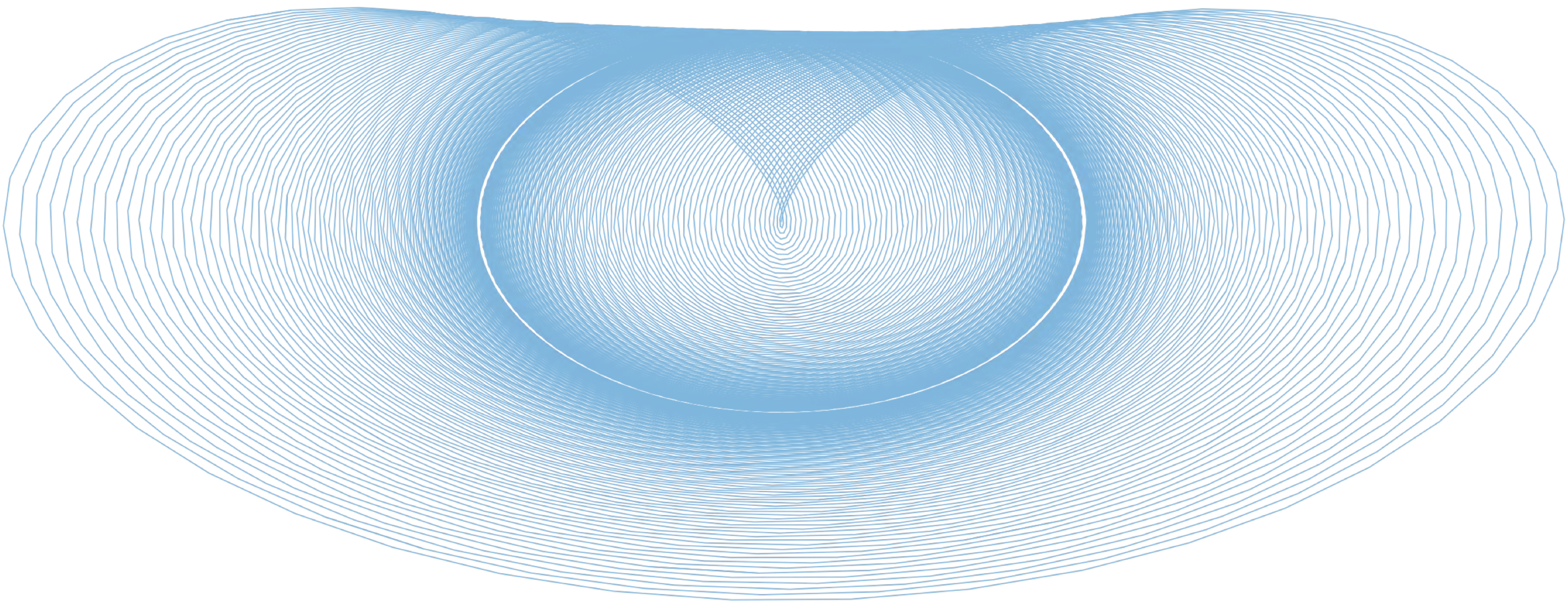
PHYS 1420 (F19)
Physics with Applications to Life Sciences



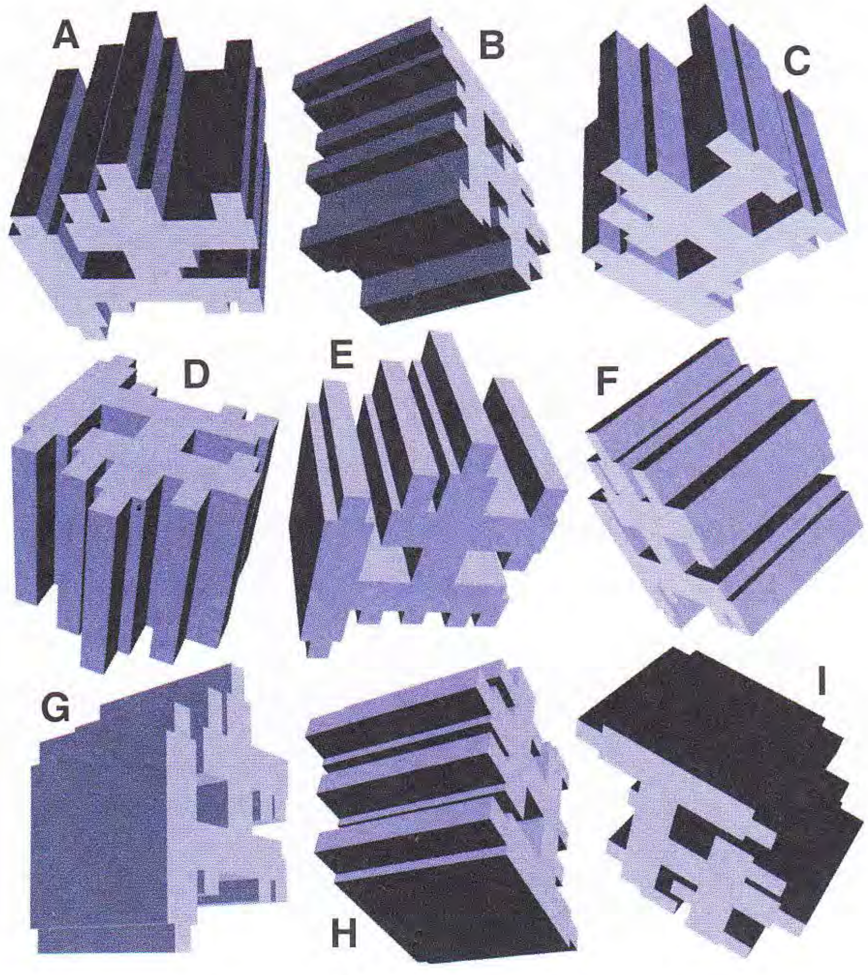
2019.09.18
Relevant reading:
Kesten & Tauck ch.3.5

Christopher Bergevin
York University, Dept. of Physics & Astronomy
Office: Petrie 240   Lab: Farq 103
cberge@yorku.ca

Ref. (re images):
Wolfson (2007), Knight (2017)

Where's the Pair?

Only two of the shapes below are exactly the same – can you find the matching pair?



Announcements & Key Concepts (re Today)

→ Online HW #3 posted and due Monday (9/23)

→ Written HW #1 posted and due 9/25 (in class at start of lecture)

Some relevant underlying concepts of the day…

- Uniform circular motion
- Equilibrium → Forces

Ex

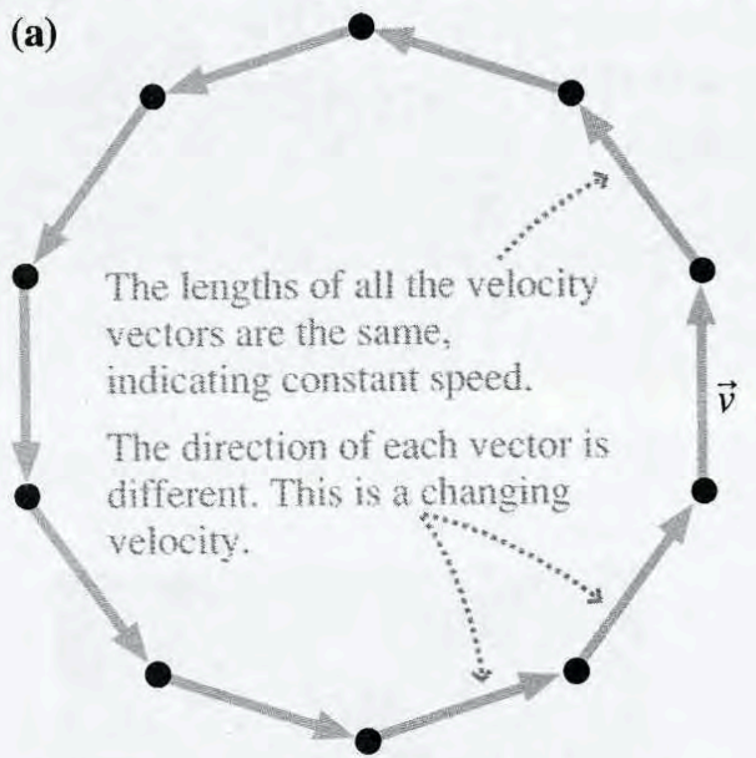
GOT IT? 3.5 Two projectiles are launched simultaneously from the same point on a horizontal surface, one at 45° to the horizontal and the other at 60°. Their launch speeds are different and are chosen so that the two projectiles travel the same horizontal distance before landing. Which of the following statements is true? (a) A and B land at the same time; (b) B's launch speed is lower than A's and B lands sooner; (c) B's launch speed is lower than A's and B lands later; (d) B's launch speed is higher than A's and B lands sooner; or (e) B's launch speed is higher than A's and B lands later.

Ex.

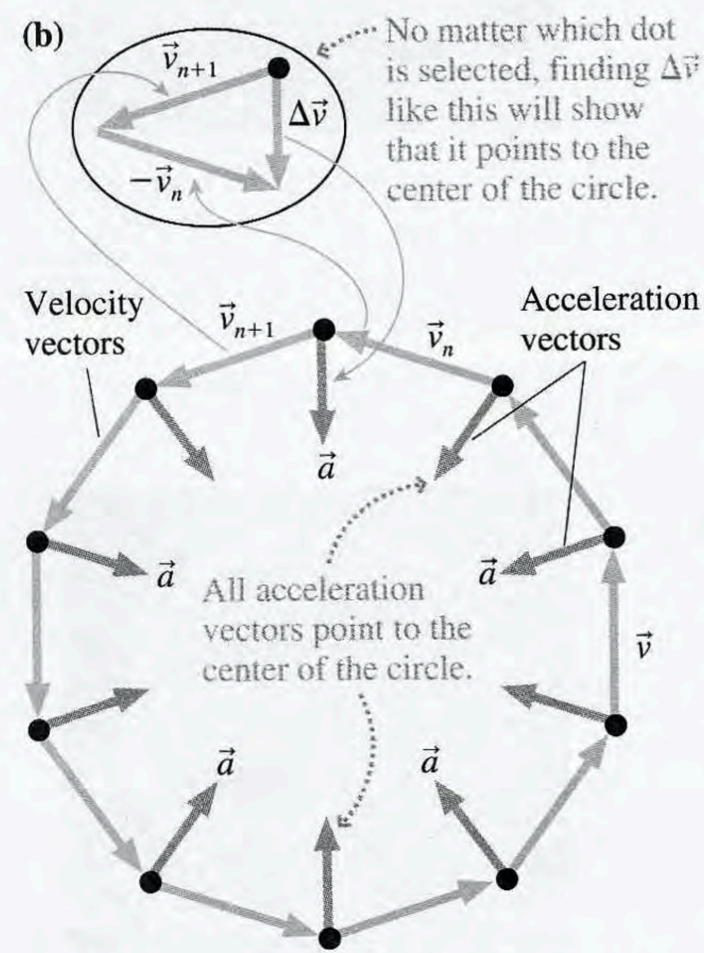
George Washington Gale Ferris Jr.

A classic example: “Maria” riding a Ferris wheel

FIGURE 4.1 Using Tactics Box 4.1 to find Maria’s acceleration on the Ferris wheel.



Maria moves at constant speed but *not* at constant velocity. Thus she is accelerating.

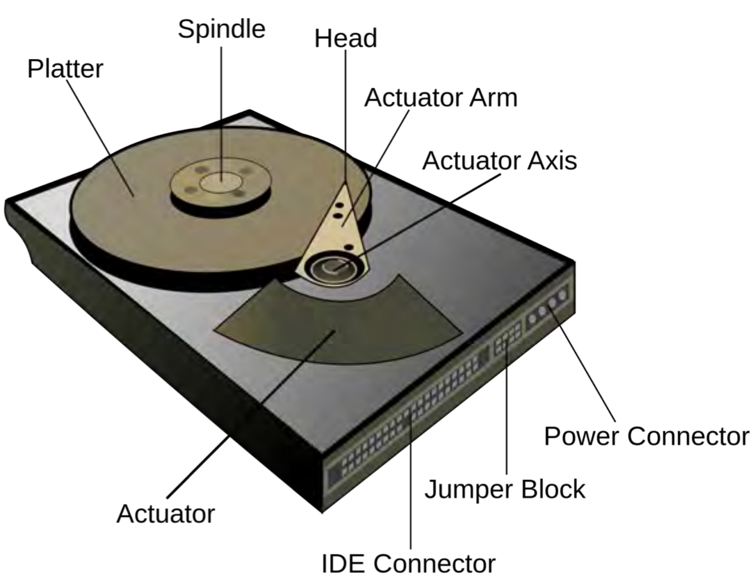


Maria’s acceleration is an acceleration of changing direction, not of changing speed.

Knight

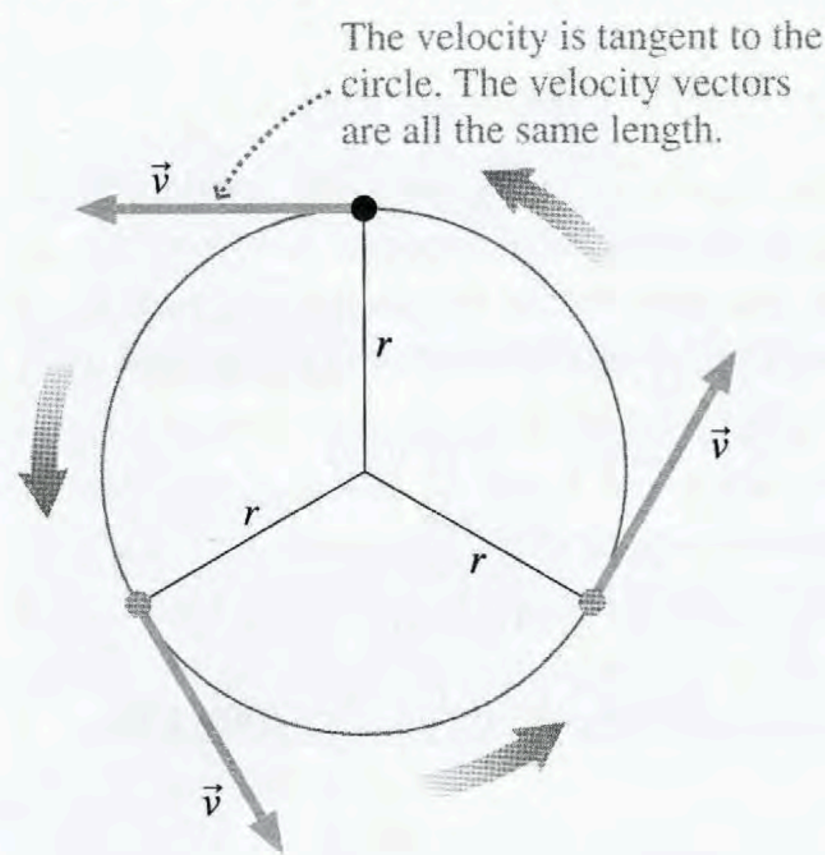
Uniform Circular Motion

→ Circular motion arises everywhere (though, what about this "uniform" business?)



https://en.wikipedia.org/wiki/Hard_disk_drive

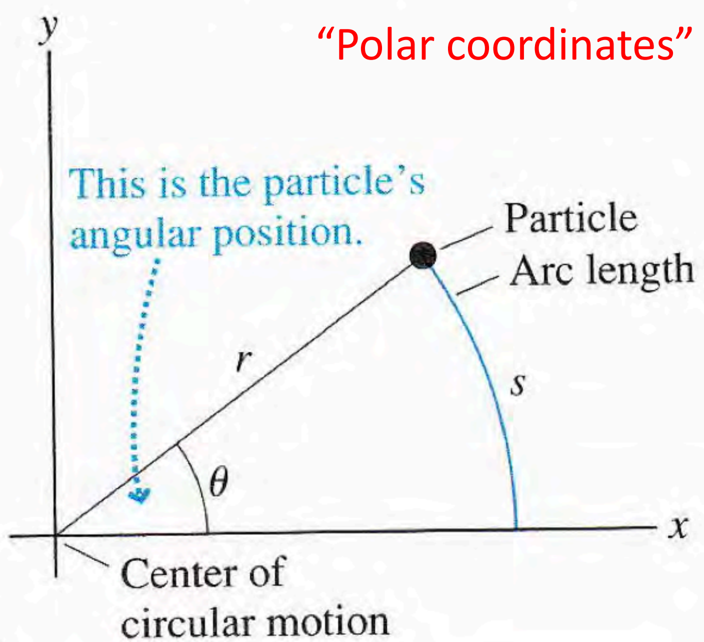
FIGURE 4.25 A particle in uniform circular motion.



Knight (2013)

<u>Circular Motion</u>: Change our coordinate system a bit... → Polar Coordinates

A particle's position is described by distance r and angle θ.



$$\theta(\text{radians}) \equiv \frac{s}{r}$$

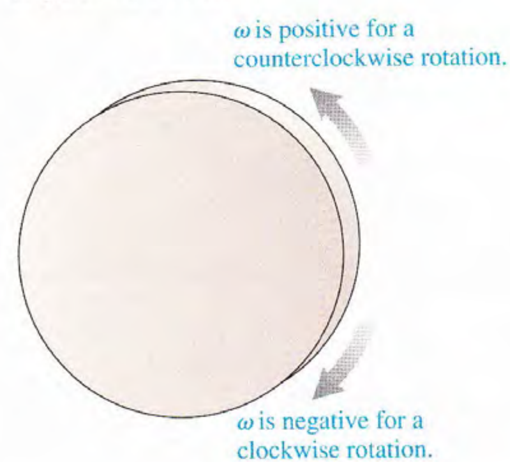
$$1 \text{ rev} = 360^\circ = 2\pi \text{ rad}$$

$$1 \text{ rad} = 1 \text{ rad} \times \frac{360^\circ}{2\pi \text{ rad}} = 57.3^\circ$$

arc length

$$s = r\theta \qquad (\text{with } \theta \text{ in rad})$$

Positive and negative angular velocities.



Just like a sign convention for 1-D motion, when *moving* we need to make a similar choice here:

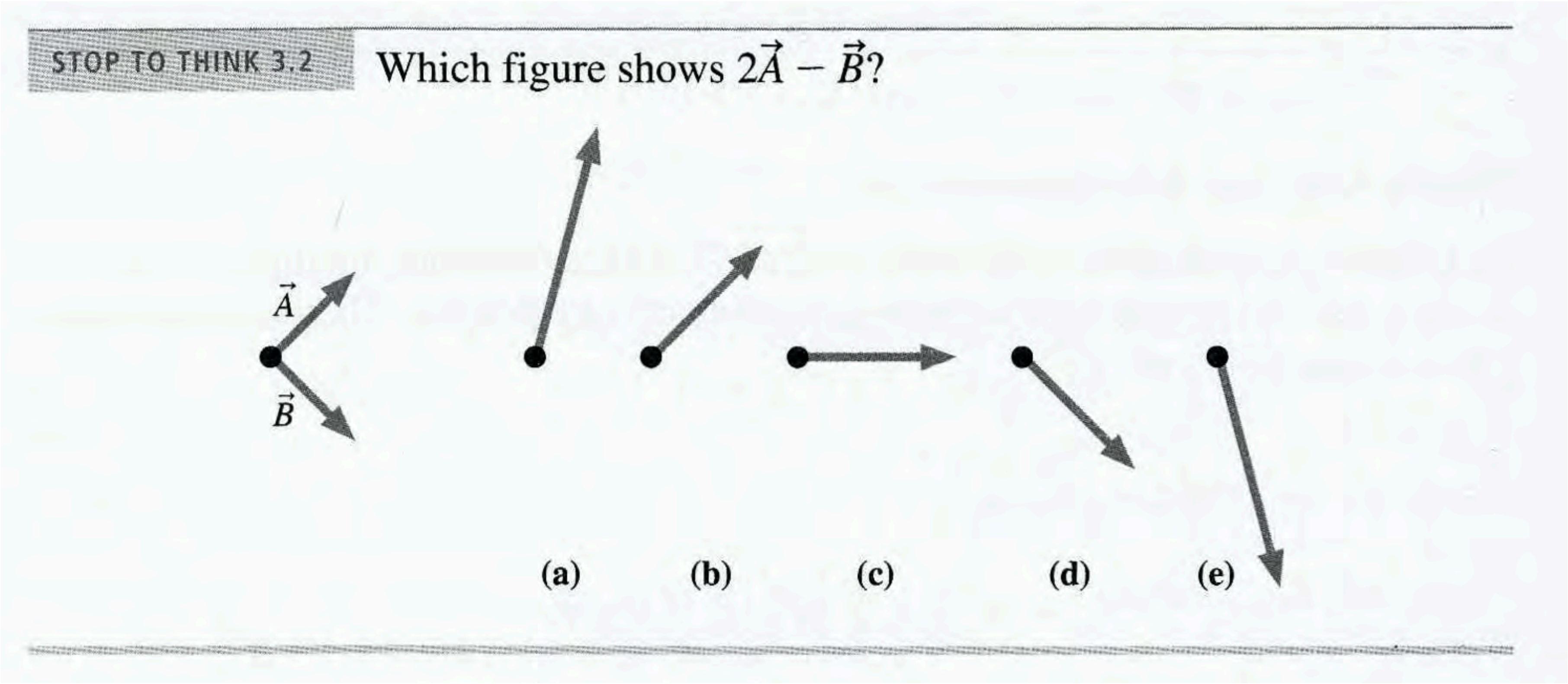
- CCW > 0
- CW < 0

Knight (2013)

Consider watching what is going on on a carousel when standing off it versus riding it

→ Probably an easier problem in polar coordinates (rather than Catesian)

Ex. (REVISTED)



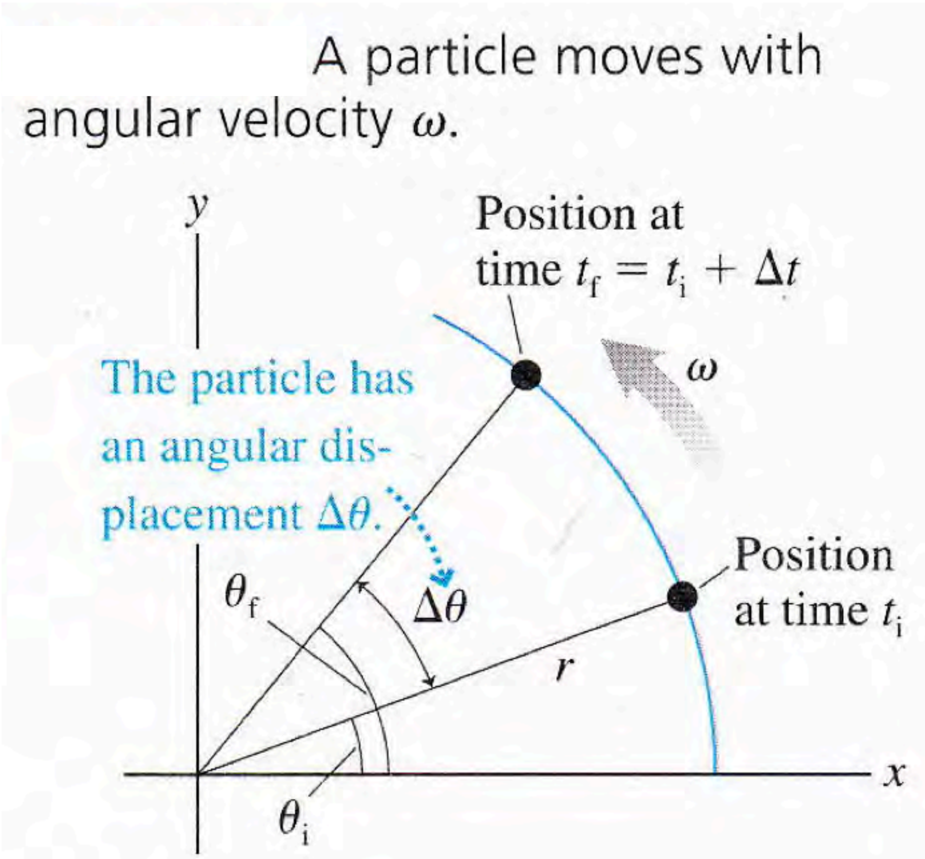
- Much easier when using component notation!

$$\bar{A} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \bar{E} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad 2\bar{A}\text{-}\bar{E} = \begin{bmatrix} 2 - 1 \\ 2 - (-1) \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

→ Recasting problems in a different framework can make them much easier!

Knight

Circular Motion: Velocity



$$\text{average angular velocity} \equiv \frac{\Delta\theta}{\Delta t}$$

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

angular velocity

$$v_t = \frac{ds}{dt} = r\frac{d\theta}{dt}$$

$$v_t = \omega r \qquad (\text{with } \omega \text{ in rad/s})$$

tangential velocity

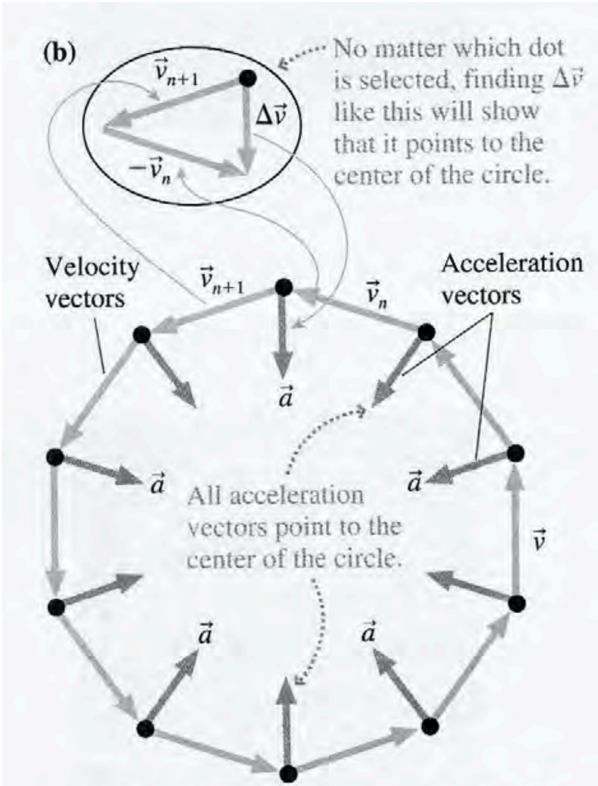
$\omega =$ slope of the θ-versus-t graph at time t

$\theta_f = \theta_i +$ area under the ω-versus-t curve between t_i and t_f

$= \theta_i + \omega\Delta t$

Knight (2013)

Uniform Circular Motion: Acceleration



Maria's acceleration is an acceleration of changing direction, not of changing speed.

Knight

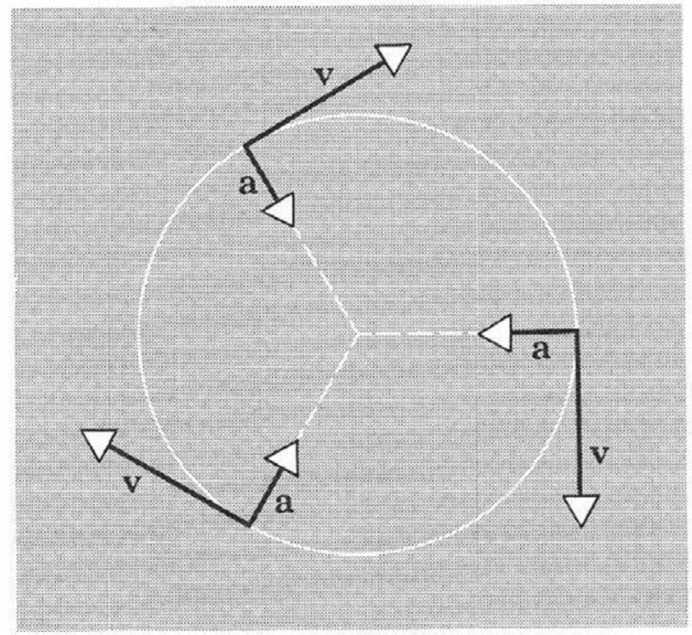


Fig. 4–6 In uniform circular motion the acceleration **a** is always directed toward the center of the circle and hence is perpendicular to **v**.

Resnick & Halliday (1966)

$$a = \frac{v^2}{r} \qquad \text{(uniform circular motion)}$$

Wolfson

- Ok, we got the direction between a and v down. But functionally how are they related?

- v is the tangential velocity
- a is the radial (or centripetal) acceleration

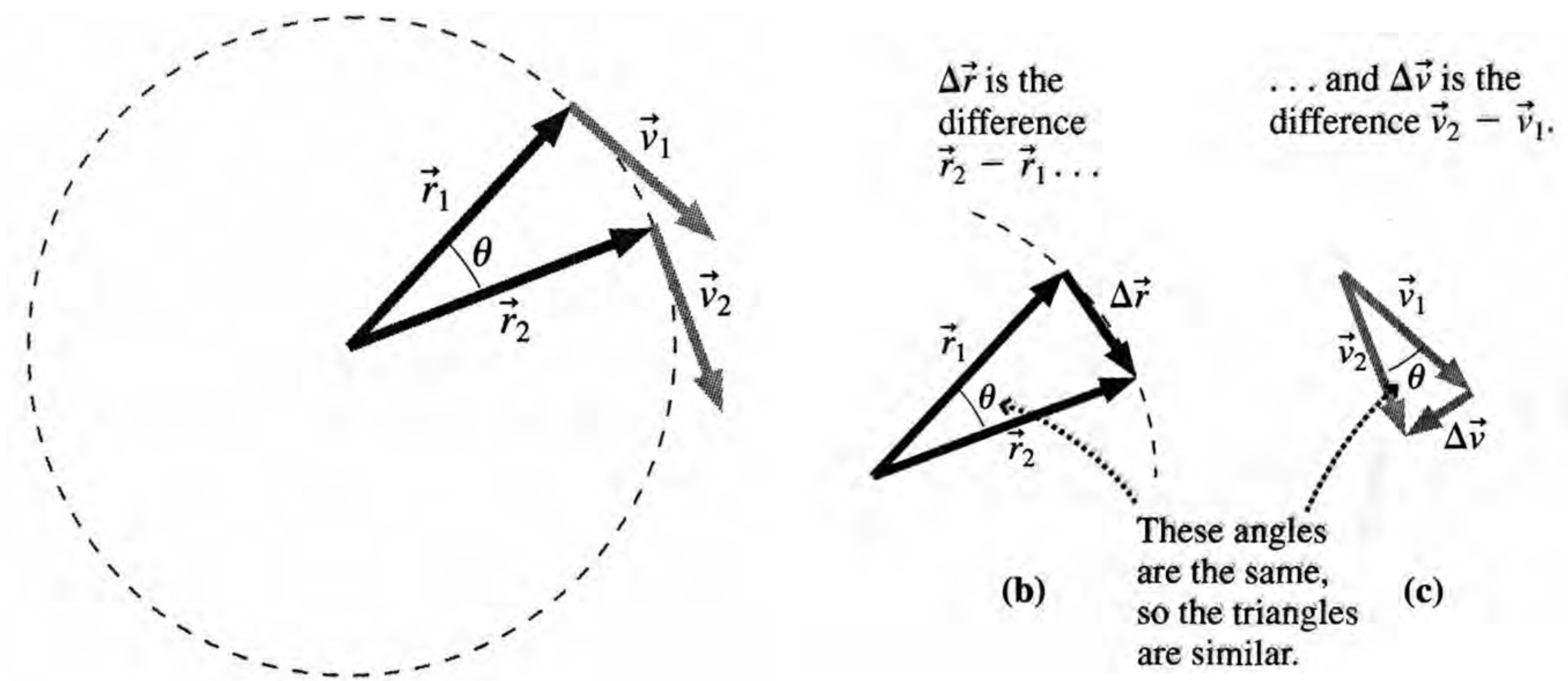
Circular Motion: Acceleration

Handful of approaches to “derive” this relationship…

$$a = \frac{v^2}{r} \qquad \text{(uniform circular motion)}$$

Wolfson

- v is the tangential velocity
- a is the radial (or centripetal) acceleration



Knight (2013)

Circular Motion: Acceleration (alternative derivation I)

Two alternative derivations (re Kesten & Tauck)

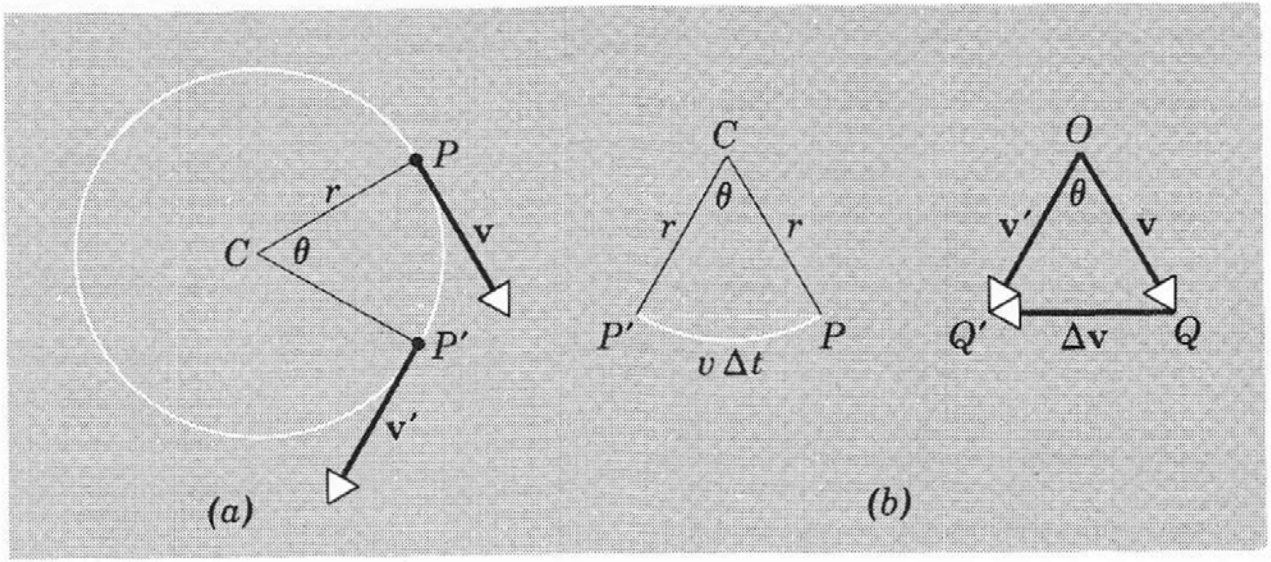


Fig. 4–5 Uniform circular motion. The particle travels around a circle at constant speed. Its velocity at two points P and P' is shown. Its change in velocity in going from P to P' is $\Delta\mathbf{v}$.

The situation is shown in Fig. 4–5*a*. Let P be the position of the particle at the time t and P' its position at the time $t + \Delta t$. The velocity at P is $\mathbf{v}$, a vector tangent to the curve at P. The velocity at P' is $\mathbf{v}'$, a vector tangent to the curve at P'. Vectors $\mathbf{v}$ and $\mathbf{v}'$ are equal in magnitude, the speed being constant, but their directions are different. The length of path traversed during Δt is the arc length PP', which is equal to $v\ \Delta t$, v being the constant speed.

Now redraw the vectors $\mathbf{v}$ and $\mathbf{v}'$, as in Fig. 4–5*b*, so that they originate at a common point. We are free to do this as long as the magnitude and

Resnick & Halliday (1966)

Circular Motion: Acceleration (alternative derivation I)

direction of each vector are the same as in Fig. 4–5*a*. This diagram (Fig. 4–5*b*) enables us to see clearly the *change in velocity* as the particle moved from P to P'. This change, $\mathbf{v}' - \mathbf{v} = \Delta\mathbf{v}$, is the vector which must be added to $\mathbf{v}$ to get $\mathbf{v}'$. Notice that it points inward, approximately toward the center of the circle.

Now the triangle OQQ' formed by $\mathbf{v}$, $\mathbf{v}'$, and $\Delta\mathbf{v}$ is similar to the triangle CPP' formed by the chord PP' and the radii CP and CP'. This is so because both are isosceles triangles having the same vertex angle; the angle θ between $\mathbf{v}$ and $\mathbf{v}'$ is the same as the angle PCP' because $\mathbf{v}$ is perpendicular to CP and $\mathbf{v}'$ is perpendicular to CP'. We can therefore write

$$\frac{\Delta v}{v} = \frac{v\,\Delta t}{r}, \qquad \text{approximately,}$$

the chord PP' being taken equal to the arc length PP'. This relation becomes more nearly exact as Δt is diminished, since the chord and the arc then approach each other. Notice also that $\Delta\mathbf{v}$ approaches closer and closer to a direction perpendicular to $\mathbf{v}$ and $\mathbf{v}'$ as Δt is diminished and therefore approaches closer and closer to a direction pointing to the exact center of the circle. It follows from this relation that

$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}, \qquad \text{approximately,}$$

and in the limit when $\Delta t \rightarrow 0$ this expression becomes exact. We therefore obtain

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{v^2}{r} \qquad (4\text{–}9)$$

as the magnitude of the acceleration. The direction of $\mathbf{a}$ is instantaneously along a radius inward toward the center of the circle.

Resnick & Halliday (1966)

Circular Motion: Acceleration (alternative derivation II)

Note: Switch to a different coordinate system (Cartesian → circular)

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{v^2}{r} \qquad (4\text{–}9)$$

Let us now derive Eq. 4–9 using vector methods. Figure 4–8*a* shows a particle in uniform circular motion about the origin O of a reference frame. For this motion the polar coordinates r, θ are more useful than the rectangular coordinates x, y because r remains constant throughout the motion and θ increases in a simple linear way with time; the behavior of x and y during such motion is more complex. The two sets of coordinates are related by

$$r = \sqrt{x^2 + y^2} \qquad \text{and} \qquad \theta = \tan^{-1} y/x \qquad (4\text{–}10a)$$

or by the reciprocal relations

$$x = r \cos\theta \qquad \text{and} \qquad y = r \sin\theta. \qquad (4\text{–}10b)$$

In rectangular reference frames we used the unit vectors $\mathbf{i}$ and $\mathbf{j}$ to describe motion in the x-y plane. Here we find it more convenient to introduce two new unit vectors $\mathbf{u}_r$ and $\mathbf{u}_\theta$. These, like $\mathbf{i}$ and $\mathbf{j}$, have unit length and are dimensionless; they designate direction only.

The unit vector $\mathbf{u}_r$ at any point is in the direction of increasing $\mathbf{r}$ at that point; it is directed radially outward from the origin. The unit vector $\mathbf{u}_\theta$ at any point is in the direction of increasing θ at that point; it is always tangent to a circle

Resnick & Halliday (1966)

Circular Motion: Acceleration (alternative derivation II)

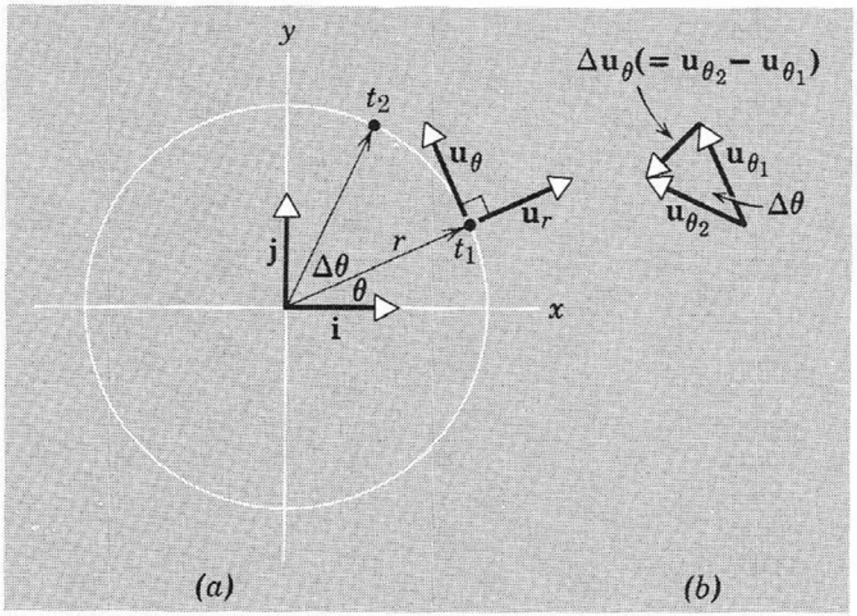


Fig. 4–8 (*a*) A particle moving counterclockwise in a circle of radius r. (*b*) The unit vectors $\mathbf{u}_{\theta_1}$ and $\mathbf{u}_{\theta_2}$ at times t_1 and t_2 respectively, and the change $\Delta\mathbf{u}_\theta$ $(= \mathbf{u}_{\theta_2} - \mathbf{u}_{\theta_1})$.

Note: Use of unit vectors here, though from a radial frame of reference!

through the point in a counterclockwise direction. As Fig. 4–8*a* shows, $\mathbf{u}_r$ and $\mathbf{u}_\theta$ are at right angles to each other. The unit vectors $\mathbf{u}_r$ and $\mathbf{u}_\theta$ differ from the unit vectors $\mathbf{i}$ and $\mathbf{j}$ in that the directions of $\mathbf{u}_r$ and $\mathbf{u}_\theta$ vary from point to point in the plane; the unit vectors $\mathbf{u}_r$ and $\mathbf{u}_\theta$ are thus *not constant* vectors.

In terms of $\mathbf{u}_r$ and $\mathbf{u}_\theta$ the motion of a particle moving counterclockwise at uniform speed v in a circle about the origin in Fig. 4–8*a* can be described by the vector equation

$$\mathbf{v} = \mathbf{u}_\theta v. \qquad (4\text{–}11)$$

This relation tells us, correctly, that the direction of $\mathbf{v}$ (which is the same as the direction of $\mathbf{u}_\theta$) is tangent to the circle and that the magnitude of $\mathbf{v}$ is the constant quantity v (because the magnitude of $\mathbf{u}_\theta$ is unity).

To find the acceleration we combine Eqs. 4–3 and 4–11, yielding

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d\mathbf{u}_\theta}{dt}v. \qquad (4\text{–}12)$$

Resnick & Halliday (1966)

Circular Motion: Acceleration (alternative derivation II)

Note that v in Eq. 4–11 is a constant, but $\mathbf{u}_\theta$ is not since its direction changes as the particle moves. To evaluate $d\mathbf{u}_\theta/dt$, consider Fig. 4–8*b* which shows the unit vectors $\mathbf{u}_{\theta_1}$ and $\mathbf{u}_{\theta_2}$ corresponding to an elapsed time Δt $(= t_2 - t_1)$ for the moving particle. The vector $\Delta\mathbf{u}_\theta$ $(= \mathbf{u}_{\theta_2} - \mathbf{u}_{\theta_1})$ points radially inward toward the origin in the limiting case as $\Delta t \to 0$. In other words, $d\mathbf{u}_\theta$ at any point has the direction of $-\mathbf{u}_r$. The angle between $\mathbf{u}_{\theta_2}$ and $\mathbf{u}_{\theta_1}$ in the figure is $\Delta\theta$, which is the angle swept out by a radial line from the origin to the particle in time Δt. The magnitude of $\Delta\mathbf{u}_\theta$ is simply $\Delta\theta$; bear in mind that the vectors $\mathbf{u}_{\theta_1}$ and $\mathbf{u}_{\theta_2}$ in Fig. 4–8*b* have the magnitude unity. Thus

$$\frac{d\mathbf{u}_\theta}{dt} = -\mathbf{u}_r \lim_{\Delta t \to 0} \frac{\Delta\theta}{dt} = -\mathbf{u}_r \frac{d\theta}{dt}$$

and, from Eq. 4–12,

$$\mathbf{a} = \frac{d\mathbf{u}_\theta}{dt} v = -\mathbf{u}_r \frac{d\theta}{dt} v. \qquad (4\text{–}13)$$

Now, $d\theta/dt$ is the uniform angular rotation rate of the particle and is given by

$$\frac{d\theta}{dt} = \frac{2\pi \text{ radians}}{\text{time for one revolution}} = \frac{2\pi}{2\pi r/v} = \frac{v}{r}.$$

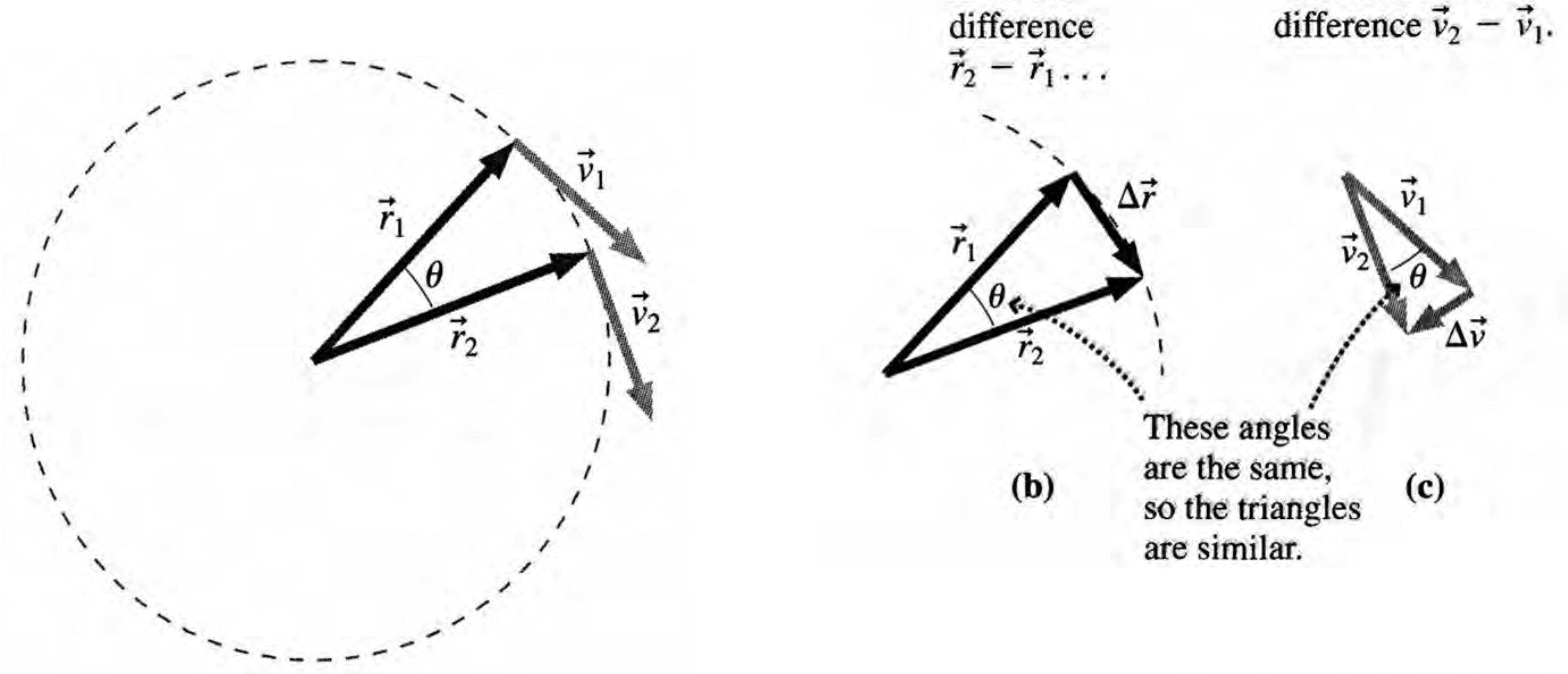
Putting this into Eq. 4–13 leads us finally to

$$\mathbf{a} = -\mathbf{u}_r \frac{v^2}{r} \qquad (4\text{–}14)$$

which tells us that the acceleration in uniform circular motion has a magnitude v^2/r (see Eq. 4–9) and points radially inward (note the factor $-\mathbf{u}_r$). The vector relation Eq. 4–14 thus tells us *both* the magnitude *and* the direction of the centripetal acceleration $\mathbf{a}$. Note that, as expected, $\mathbf{a}$ has a constant magnitude but changes continually in direction because $\mathbf{u}_r$ changes continually in direction.

Resnick & Halliday (1966)

Circular Motion: Acceleration



$$\frac{\Delta v}{v} = \frac{\Delta r}{r} \longrightarrow \frac{\Delta v}{v} \simeq \frac{v\,\Delta t}{r} \longrightarrow \bar{a} = \frac{\Delta v}{\Delta t} \simeq \frac{v^2}{r}$$

$$a = \frac{v^2}{r} \quad \text{(uniform circular motion)}$$

Knight (2013)

Summary (re 2-D velocity and acceleration)

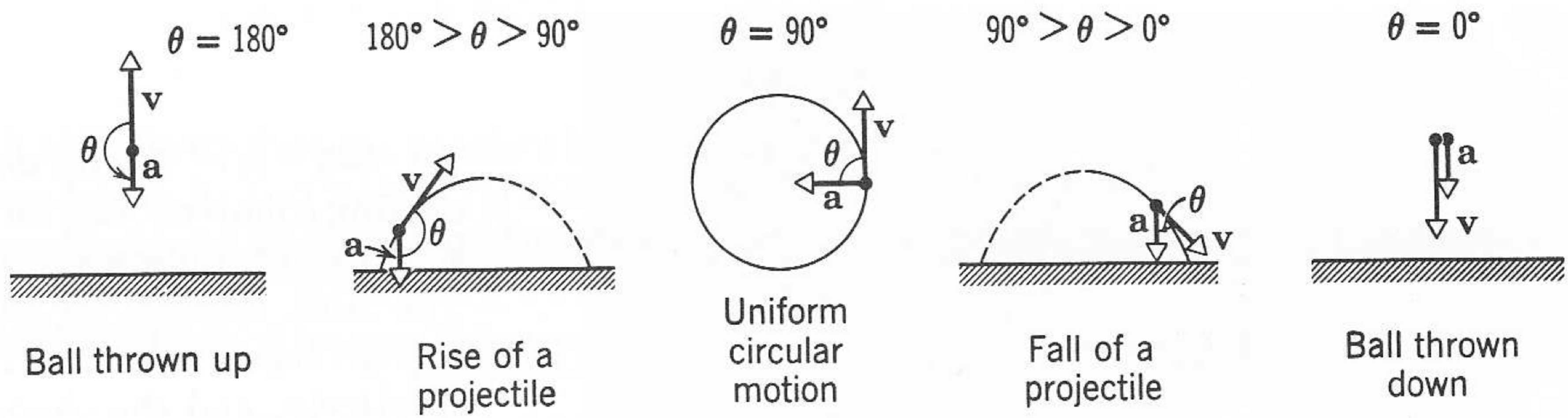


Fig. 4–7 Showing the relation between **v** and **a** for various motions.

- Different angles between a and v result in different types of motion

Resnick & Halliday (1966)

2 Motion in a Straight Line | 3 Motion in Two and Three Dimensions | **4** | 5 Using Newton's Laws | 6 Energy, Work, and Power

Force and Motion

- There are a **LOT** of deep/important concepts introduced in this chapter
- The most fundamental comes right off the bat (pun!): *change*

4.1 The Wrong Question

The Right Question

Our first question—about why the spacecraft keeps moving—is the wrong question. So what's the right question? It's the second one, about why the baseball's motion *changed*. Dynamics isn't about what causes motion itself; it's about what causes *changes* in motion.

Ex

Find the orbital period (the time to complete one orbit) of the International Space Station in its circular orbit at altitude 400 km, where the acceleration of gravity is 89% of its surface value.

Ex

An engineer is designing a flat, horizontal road for an 80 km/h speed limit (that's 22.2 m/s). If the maximum acceleration of a vehicle on this road is 1.5 m/s^2, what's the minimum safe radius for curves in the road?

Ex.

A hoop of radius R rolls without slipping along a horizontal plane with constant speed v. What is the acceleration of different points on the hoop's circumference?

Shaskol'Skaya & El'Tsin (1963)

Ex. (SOL)

→ So the answer is the same as if the hoop was not rolling along the surface (i.e., there was no "translational" component to the motion)

Note: A key concept here is the notion of the "*uniformly*" (the translational motion was specified to be at "constant speed"). We'll come back to this shortly....

We may consider the motion of a hoop rolling uniformly without slipping along a horizontal plane thus. Suppose that any two positions of the hoop be given. Then the hoop can be moved from one position to the other by moving it from the first position to the second in a straight line with a velocity equal to that of the centre, and by rotating the hoop uniformly about its centre with the same linear velocity so that all points on the circumference reach the places corresponding with its second position (Fig. 160). Since this is true for any two positions of the hoop, it is true for any two positions however close together. Therefore we can consider uniform rolling without slipping as a combination of two simultaneous motions: uniform motion in a straight line with the velocity of the centre, and uniform rotation about the centre with the same linear velocity for the points on the circumference. But there is no acceleration in uniform linear motion, while in motion in a circle all points on the circumference have one and the same centripetal acceleration, which equals v^2/R.

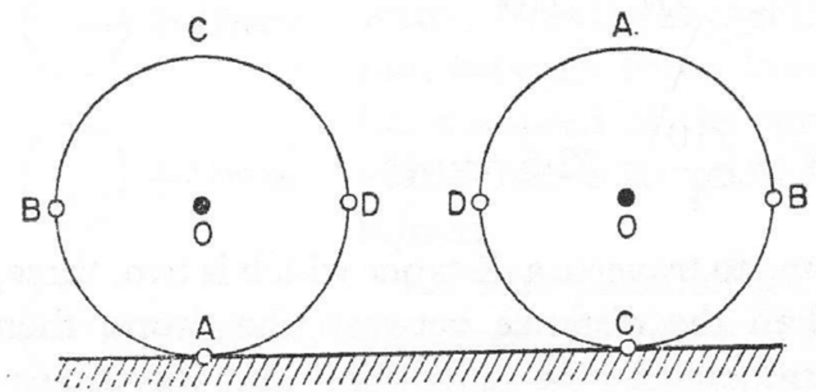


FIG. 160

Shaskol'Skaya & El'Tsin (1963)